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ESTIMATING THE UNEMPLOYMENT RATE USING MATHEMATICAL MODELING

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RESUME

This article proposes and analyzes a nonlinear mathematical model of the unemployment problem, based on the consideration of three main variables: the number of unemployed individuals, employed individuals, and those not officially employed. The model reflects the situation observed in some countries with a transitional economy or in countries with a strong agricultural sector, where the unemployed rarely turn to labor authorities for job placement, which in turn leads to an increase in the rate of hidden unemployment. The results for the mathematical model were obtained using the stability theory of nonlinear differential equations.

Key words: Hidden unemployment, system of nonlinear differential equations, equilibrium point, stability analysis.

The increase in the number of unemployed people in the country is economically dangerous because it can lead to a decline in gross domestic product and an increase in the social burden on the state. From a social perspective, it can result in a rise in poverty levels, which may contribute to an increase in crime.

If we define unemployment, then according to the legislation of the Republic of Uzbekistan on employment, the following individuals are considered unemployed: "Unemployed persons are able-bodied individuals aged sixteen and older, up to the acquisition of the right to pension benefits, who do not have paid work or income-generating activities, are seeking employment and ready to start working as soon as a job is offered to them, or are willing to undergo vocational training, retraining, or skills improvement (except for those studying in educational institutions).

Persons specified in the first part of this article who have applied to local labor authorities for assistance in employment and have been registered by them as job seekers are recognized as unemployed."[5].

Unlike officially registered unemployment, there is hidden unemployment. In countries with a transitional economy or a highly developed agricultural sector, hidden unemployment tends to grow significantly. As K. Mukulsky states, the reason for this is "employment and self-employment in personal households that are not officially recorded."[7] The article examines the dynamics of unemployment and proposes a mechanism to reduce the rate of the hidden economy.

The history of the development of this topic begins with the work of A.K. Misra and Arvind K. Singh, dedicated to the development of a mathematical model of unemployment. In their article, published in 2011, they modeled the problem of unemployment using nonlinear ordinary differential equations [1]. The study [1] considered three main variables: U is the number of unemployed individuals, T is the number of seasonally employed individuals, and R is the number of permanently employed individuals. The model assumes that unemployed individuals can either obtain permanent employment or become temporarily employed. At the same time, temporarily employed individuals attempt to transition from the temporary employment class to the permanent employment class. It is also assumed that permanently employed individuals may be dismissed from their jobs or voluntarily resign, thereby joining the class of unemployed individuals.

This study was motivated by the work of Nikolopoulos and Tsanetis, which developed a model for providing housing to the homeless population in response to a natural disaster [2].

In the future, based on this model, Misra and Singh proposed a nonlinear mathematical model for unemployment control [3]. In [3], they considered three dynamic variables: U - the number of unemployed individuals, E - the number of employed individuals, and V - the number of new job vacancies created through government measures. In this case, the time delay is proportional to the number of newly created vacancies and is denoted as $U(\tau-t)$. In the model [4], Gulbanu Pathan and Bhattachwala also consider three dynamic variables and assume that there is no time delay from the state and the private sector in creating new job vacancies. They also assume that the unemployed attempt to engage in independent economic activities, creating self-employment opportunities, which is essential for their survival.

Given the statements in articles [1], [2], [3], and [4], we proposed a different model of unemployment consisting of three dynamic variables: $U-$ the number of unemployed individuals, $V-$ the number of unemployed individuals who have applied to local labor authorities for employment, $E-$ the number of employed individuals. It is also assumed that employed individuals can lose their jobs and move from the employed class to the unemployed class. Mortality and population migration are taken into account for each class in the model. A distinctive feature of our work is that, in describing the model, we use a closed biological model of infection spread, whereas the researchers mentioned above use an open model [6].

The model was constructed considering three main variables: the number of unemployed individuals $U(t)$, the number of unemployed individuals who have applied to local labor authorities for employment $V(t)$, and the number of employed individuals $E(t)$. When formulating the model, we assume that people enter the unemployed class at a constant rate Λ and may either migrate in search of work or find employment. In order to be employed, unemployed individuals must apply to local labor authorities. The number of applications to labor authorities is assumed to be proportional to the existing job vacancies and the number of unemployed individuals. It is also assumed that the number of newly created jobs is proportional to the number of officially unemployed individuals. Additionally, migration levels are considered proportional to the population size of each class separately. In our study, unlike researchers such as Misra, Singh, Gulbanu Pathan, we used a closed-type model to better reveal the internal mechanisms of employment. However, in the future, we plan to study employment issues using an open model, as employment dynamics need to be analyzed in the long term, taking external factors into account. Furthermore, our model is a modified version of a virus spread model, which allows us to more accurately simulate employment dynamics and hidden unemployment in countries with a developed agricultural sector.

The formulation of the model is as follows:

$$\begin{cases} \frac{dU}{dt} = \Lambda - \varphi UV + \mu E - \gamma U \\ \frac{dV}{dt} = \varphi UV - \delta V - \gamma V \\ \frac{dE}{dt} = \delta V - \mu E - \gamma E \end{cases} \tag{2}$$

The parameters involved in system (1) are detailed in the following table:

Parameter	Descriptions of the parameter
Λ	Rate of increase in the number of unemployed
φ	The rate of unemployed individuals applying to labor authorities for employment.
μ	The rate of job loss for employed individuals
γ	Migration rate
δ	The employment rate after contacting labor authorities.

Initial conditions at time $t = 0$:

$$U(0) = U_0 \geq 0$$

$$V(0) = V_0 \geq 0$$

$$E(0) = E_0 \geq 0$$

$$U(t) + V(t) + E(t) = N(t) = const$$

$$\frac{dU}{dt} + \frac{dV}{dt} + \frac{dE}{dt} = 0$$

Here, $N(t)$ is the number of the working-age population at any moment t.

Theorem 1. *Let for all, the set* $\Omega = \{(U, V, E) \in \mathbb{R}^3_+ : U(0) > 0, V(0) > 0, E(0) > 0, 0 \leq N(t) \leq \frac{\Lambda}{\gamma}\}$ *be positive and bounded, then the system (3.1) has a solution.*

Proof. First, we will check each equation in system (1) for positivity.

$$\frac{dU}{dt} \geq -\varphi UV - \gamma U$$

$$\frac{dU}{dt} \geq (-\varphi UV - \gamma)U$$

$$\int_0^t \frac{dU}{Udt} \geq \int_0^t (-\varphi UV - \gamma)dt$$

$$U(t) \geq U(0)e^{\int_0^t (-\varphi UV - \gamma)dt} \geq 0$$

If we integrate the second and third equations in system (1) in a similar manner from 0 to t, we obtain the following inequalities:

$$V(t) \geq V(0)e^{\int_0^t (\varphi U - \delta - \gamma)dt} \geq 0$$

$$E(t) \geq E(0)e^{\int_0^t (-\mu - \gamma)dt} \geq 0.$$

Let's check for boundedness:

$$\frac{dN}{dt} = \frac{dU}{dt} + \frac{dV}{dt} + \frac{dE}{dt} = \Lambda - \gamma N$$

taking limit supremum, we get

$$\lim_{t \to +\infty} supN(t) \leq \frac{\Lambda}{\gamma}.$$

Let's begin the equilibrium analysis. To do this, we must find the equilibrium point. If we solve each equation in system (1) by setting it to zero, we can find the equilibrium point.

$$\begin{cases} \frac{dU}{dt} = \Lambda - \varphi UV + \mu E - \gamma U = 0 \\ \frac{dV}{dt} = \varphi UV - \delta V - \gamma V = 0 \\ \frac{dE}{dt} = \delta V - \mu E - \gamma E = 0 \end{cases} \tag{3}$$

$$\frac{dV}{dt} = 0 \quad \Rightarrow \quad U^* = \frac{\delta + \gamma}{\varphi}$$

$$\frac{dE}{dt} = 0 \quad \Rightarrow \quad V^* = \frac{(\mu + \gamma)E^*}{\delta}$$

$$\frac{dU}{dt} = 0 \quad \Rightarrow \quad E^* = \frac{\delta\gamma + \gamma^2 - \Lambda\varphi}{\varphi(\mu - (\delta + \gamma)(\mu + \gamma))}$$

Thus, we have found the equilibrium point $E_p(U^*, V^*, E^*)$.

After finding the equilibrium point, we must check its stability. We will perform this check by calculating the Jacobian matrix for system (1).

$$J = \begin{pmatrix} \frac{\partial f}{\partial U} & \frac{\partial f}{\partial V} & \frac{\partial f}{\partial E} \\ \frac{\partial g}{\partial U} & \frac{\partial g}{\partial V} & \frac{\partial g}{\partial E} \\ \frac{\partial h}{\partial U} & \frac{\partial h}{\partial V} & \frac{\partial h}{\partial E} \end{pmatrix} = \begin{pmatrix} -\varphi V^* - \gamma & -\varphi U^* & \mu \\ \varphi V^* & \varphi U^* - \delta - \gamma & 0 \\ 0 & \delta & -\mu - \gamma \end{pmatrix}$$

Let's introduce the following notations in J:

$$a_1 = -\varphi V^* - \gamma$$

$$a_2 = \varphi U^* - \delta - \gamma$$

$$a_3 = -\mu - \gamma$$

To find the characteristic number of matrix J, we must solve the following equation:

$$det(J - \lambda E) = \begin{vmatrix} a_1 - \lambda & -\varphi U^* & \mu \\ \varphi V^* & a_2 - \lambda & 0 \\ 0 & \delta & a_3 - \lambda \end{vmatrix} = 0$$

$$\lambda^3 - (a_1 + a_2 + a_3)\lambda^2 + (a_1 a_2 + a_1 a_3 + a_2 a_3)\lambda - a_1 a_2 a_3 + \varphi V^* U^* (\varphi\mu + \varphi\gamma - \mu) = 0$$

$$b_1 = a_1 + a_2 + a_3$$

$$b_2 = a_1 a_2 + a_1 a_3 + a_2 a_3$$

$$b_3 = -a_1 a_2 a_3 + \varphi V^* U^* (\varphi\mu + \varphi\gamma - \mu)$$

Then we obtain the following characteristic equation:

$$\lambda^3 + b_1 \lambda^2 + b_2 \lambda + b_3 = 0$$

We will check the stability of the system using the Routh-Hurwitz criterion [8]. According to this criterion, if the following conditions are satisfied, the system will be stable:

$$b_1 > 0$$

$$b_3 > 0$$

$$b_1 b_2 - b_3 > 0$$

These conditions are satisfied when $V^* > U^*$. Therefore, the system will be stable when $V^* > U^*$.

From the equilibrium analysis, we can see that there exists an equilibrium point that is stable for certain parameter values. This implies the possibility of achieving a stable level of employment. However, changes in certain coefficients, such as migration or the rate of job applications, may lead to instabilities, which characterize the growth of hidden unemployment. In turn, this suggests that without government employment programs and without state regulatory intervention, the unemployment rate may remain at a high level.

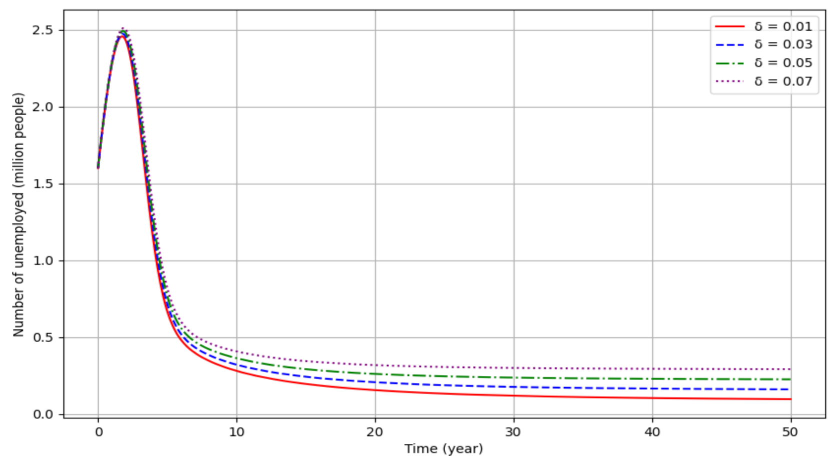
The developed mathematical model can be applied to forecast the long-term consequences of employment policy. For example, by increasing the rate of job placement after an application and decreasing the rate of job loss, it is possible to stabilize employment. In the model, these actions are represented by the parameters δ and μ, respectively. Everything stated above indicates that the model is practically significant.

If we compare the results with previous studies, we can see that the proposed model describes the internal mechanisms of employment more accurately, such as the balance between employed and unemployed individuals. Our further research will focus on studying the balance between seasonally employed workers and those employed on a permanent basis.

In this part, we will consider the following solution to system (1) using the "RK45"method in the Python programming language. In the graphs below we can see a graphical modeling of unemployment levels depending on changes in the parameters affecting it. The values of additional parameters are given below: $\Lambda = 0.1$, $\varphi = 0.02$, $\mu = 0.05$, $\gamma = 0.01$,
$\delta = 0.03$. Initial conditions for a country with a developed agricultural sector, where the population is about 38 million people, with 60 percent being youth.

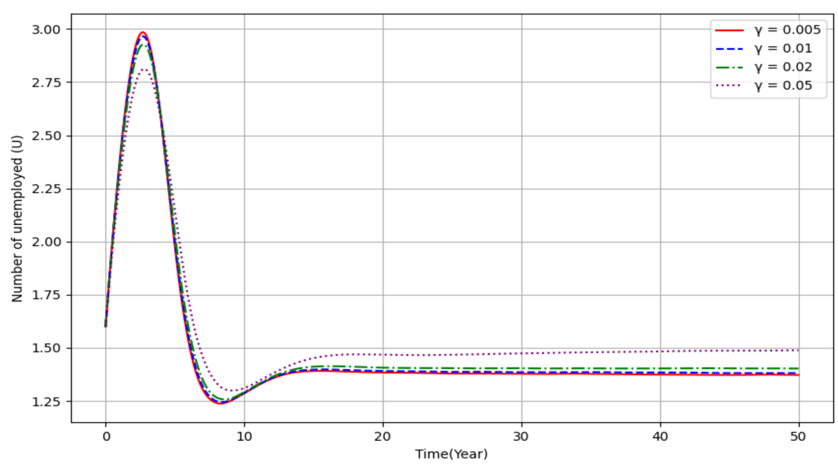
$$U(0) = 1.6 \quad mln., \quad V(0) = 0.45 \quad mln., \quad E(0) = 14.2 \quad mln.$$

In the first graph, we can observe changes in the number of unemployed individuals as the employment rate increases after contacting local labor authorities:



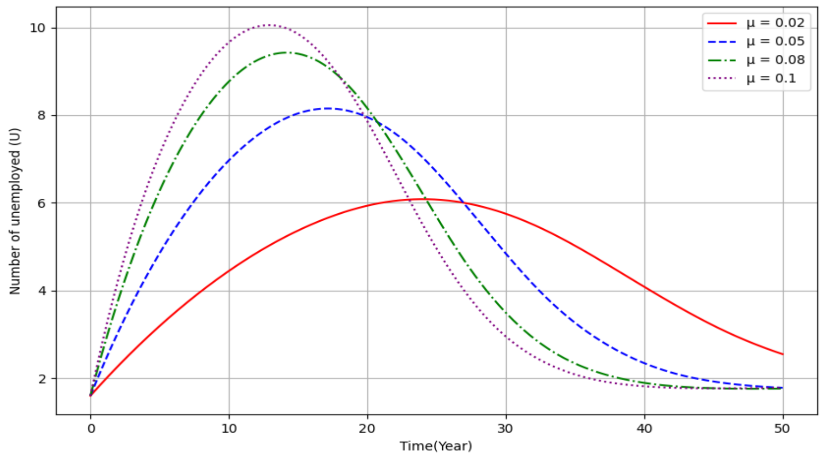
Figur 1: Impact of the employment rate on the unemployment rate

In the second graph, we can see changes in the number of unemployed due to changes in the migration level:



Figur 2: Impact of migration level on unemployment rate

In the third graph, the change in the number of unemployed under the influence of migration is shown:



Figur 3: Impact of the rate of job loss on the unemployment rate

The present article examines a mathematical model describing the dynamics of unemployment. The model considers three main variables: the number of unemployed individuals, the number of officially registered unemployed individuals, and the number of employed individuals. The conducted research has shown that the proposed model adequately describes the internal mechanisms of the labor market while accounting for the rate of unemployed individuals seeking assistance and the impact of migration. Stability analysis has revealed an equilibrium point that remains stable under certain conditions. This implies that, under specific conditions, the unemployment rate stabilizes.

The practical significance of the proposed model lies in its application for analyzing dynamics and forecasting the effectiveness of various strategies to combat unemployment. As a particular case, we can illustrate that an increase in the employment rate and a rise in the number of registered unemployed individuals can significantly reduce the level of hidden unemployment.

Future research on this topic will continue, taking into account external economic factors and workforce qualifications. In addition to the methods presented, numerical analysis will be employed in further studies, allowing for a better understanding of unemployment dynamics.

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REZYUME

Ushbu maqolada ishsizlik muammosining chiziqsiz matematik modeli taklif qilinadi va tahlil qilinadi. Ushbu model uchta asosiy o'zgaruvchini hisobga oladi: ishsizlar soni, bandlar va norasmiy ishlayotgan shaxslar. Model o'tish iqtisodiyotiga ega ba'zi davlatlarda yoki qishloq xo'jaligi sektori rivojlangan mamlakatlarda kuzatiladigan holatni aks ettiradi. Bunday mamlakatlarda ishsizlar ishga joylashish uchun bandlik organlariga kamdan-kam murojaat qiladilar, bu esa yashirin ishsizlik darajasining oshishiga olib keladi. Matematik model uchun natijalar chiziqsiz differensial tenglamalar turg'unligi nazariyasi yordamida olingan.

Kalit so'zlar: Yashirin ishsizlik, chiziqsiz differensial tenglamalar tizimi, muvozanat nuqtasi, turg'unlik tahlili.

РЕЗЮМЕ

В данной статье предлагается и анализируется нелинейная математическая модель проблемы безработицы, основанная на рассмотрении трёх основных переменных: количества безработных, занятых и неофициально трудоустроенных лиц. Модель отражает ситуацию, наблюдаемую в некоторых странах с переходной экономикой или в странах с развитым сельскохозяйственным сектором, где безработные редко обращаются в органы занятости для трудоустройства, что, в свою очередь, приводит к увеличению уровня скрытой безработицы. Результаты для математической модели были получены с использованием теории устойчивости нелинейных дифференциальных уравнений.

Ключевые слова: Скрытая безработица, система нелинейных дифференциальных уравнений, точка равновесия, анализ устойчивости.