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The Fokas' method for heat transfer equation on symmetric graphs

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RESUME

In the present paper we obtained integral-representation of solutions in terms of given initial and boundary data for the initial-boundary value problems for heat transfer equation on symmetric metric graph via Fokas' unified transformation method.

Key words: Heat transfer equation, metric graphs, branched structures, Fokas' method, unified transformation, Fourier transformation, initial problem, boundary value problem.

1. Introduction

During the last years the branched thin structures and metric graphs are widely used as a model in theoretical investigation of many complex systems from physics, biology, ecology, sociology, economy and finance [1]-[3]. Despite to high interest for diffusive and dispersive wave transfer problems in such system, here only few papers on the initial-boundary value problems (IBVP) on metric graphs. Firstly IBVP for heat equation on graphs are investigated in [7]. Some numerical methods for solution of IBVP for heat equation can be found in [8]. In the paper [13] it is proved existence and uniqueness of the generalized (weak) solution of initial boundary value problem on general graphs in Sobolev spaces. In [9] and [10] thermal diffusion in branched structures based on linear and nonlinear heat equation and some other physical applications are discussed.

Usually, for investigation physical properties of quantum graphs used static Schrödinger equation [3]–[7]. In the references [16] was investigated nonlinear Schrödinger equation on two dimensional thin tabular branched domain and proved that the problem on metric graphs for one dimensional nonlinear Schrödinger equation on metric graph, with gluing (Kirchhoff) conditions on the vertex point, can be obtained when width of the branches tends to zero. Similar convergence result in the case of linear Schrödinger equation with different approaches can be found in [6]–[7].

Schrödinger equation can be also called to be heat equation with imaginary time. The heat equation on branched structures firstly used in the 50's of the nineteenth century. Thomson (Lord Kelvin) used heat equation (Thomson's cable equation) as mathematical models of signal decay in submarine (under water) telegraphic cables (Ch. IV in [9]). Later this method was widely used in neuroscience for theoretical analyzing data collected from intracellular micro electrode recordings and for analyzing the electrical properties of neuronal dendrites (see [10]). Initial and boundary value problems for some other types of PDE on metric graphs and their possible applications can be found in [11], [16]–[17].

In this paper we contruct the integral representation of solution of IBVP for heat equation on general star graph with both finite and semi infinite bonds [11]. For this purpose we generalize so called Fokas' method in the case of metric graph [12]-[15]. It should be noted that this method was used in [16], [17] for the IBVP for heat equation on star graphs with only finite or semi infinite bonds.

2. Formulation of the problem and main result

We consider symmetric metric graph which obtained by connecting n finite $B_1, ..., B_n$ and m semi infinite $B_{n+1}, B_{n+2}, ..., B_{n+m}$ bonds at one point, called to be vertex of the graph. We correspond the bonds B_j, $(j = \overline{1,n})$ to the intervals $(0,\ L_j)$ and the bonds B_r, $(r = \overline{n+1, n+m})$ to intervals $(0,\ \infty)$ to define coordinates in each bond. Here vertex of the graph corresponds to 0 on each bond (Figure 1).

In each bond of the graph consider the heat transfer equation

$$V_t^{(j)}(x,t) = V_{xx}^{(j)}(x,t) + f^{(j)}(x,t),\ j = \overline{1, n+m}. \tag{1}$$

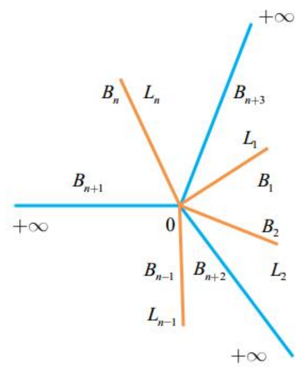

Figure 1. Symmetric metric graph

Define initial conditions

$$V^{(j)}(x,0) = V_0^{(j)}(x), \ \ x \in \overline{B_j}, \ j = \overline{1, n+m}. \tag{2}$$

Boundary and the asymptotic conditions

$$\left(\alpha_j V^{(j)}(x,t) + \beta_j V_x^{(j)}(x,t)\right)|_{x=L_j-0} = 0, \ \alpha_j^2 + \beta_j^2 \neq 0, \ j = \overline{1,n}, \tag{3}$$

$$\lim_{x\to\infty} V^{(r)}(x,t) = 0, \, t \geq 0, \, r = \overline{n+1, n+m} \tag{4}$$

on finite and semi infinite bonds, respectively.

Moreover, we need to define the following gluing conditions for connectivity of the graph

$$V^{(1)}(+0,t) = V^{(2)}(+0,t) = ... = V^{(n+m)}(+0,t), \tag{5}$$

$$\sum_{j=1}^{n+m} \delta_j^2 V_x^{(j)}(+0,t) = 0. \tag{6}$$

The last conditions usually called continuity and Kirchhoff conditions on branching point of the graphs. We suppose, that initial data are smooth enough functions and they satisfies the conditions (3)–(6).

Step 1. Write the PDE as a one-parameter "family" of equations, each of which is in divergence form [14]–[15].

For problem (1), a clear choice of an integrating factor to achieve this is $\widetilde{q}(x,t) = e^{-ikx+k^2t}$, leading to the form

$$\left[e^{-ikx+k^2t}V^{(j)}(x,t)\right]_t = \left[e^{-ikx+k^2t}\left(V_x^{(j)}(x,t) + ikV^{(j)}(x,t)\right)\right]_x + +e^{-ikx+k^2t}f^{(j)}(x,t), \ j = \overline{1,n+m}. \tag{7}$$

Step 2. Use the divergence form to determine a global relation coupling the various boundary values.

In the case of (), this takes the form

$$e^{wt}\widehat{V}^{(j)}(k,t) - \widehat{V}_0^{(j)}(k) = e^{-ikL_j}\left(h_1^{(j)}(w,t) + ikh_0^{(j)}(w,t)\right) -$$

$$-\left(g_j(w,t) + ikg_0(w,t)\right) + \widehat{F}^{(j)}(k,t), \ j = \overline{1,n}; \tag{8}$$

$$e^{wt}\widehat{V}^{(r)}(k,t) - \widehat{V}_0^{(r)}(k) = -g_r(w,t) - ikg_0(w,t) + \widehat{F}^{(r)}(k,t), \ r = \overline{n+1,n+m}. \tag{9}$$

where $w = k^2$, $\{k \in \mathbb{C}: \ Imk > 0\}$.

However dispersion relation $w = k^2$ invariant with respect to substitution $k \to -k$ then functions $g_0(w,t)$, $g_j(w,t), g_r(w,t)$, $h_0^{(j)}(w,t)$, $h_1^{(j)}(w,t)$, $j = \overline{1,n}$, $r = \overline{n+1, n+m}$ also be invariant. Hence from (8) and (9) we have:

$$e^{wt}\widehat{V}^{(j)}(-k,t) - \widehat{V}_0^{(j)}(-k) = e^{ikL_j}\left(h_1^{(j)}(w,t) - ikh_0^{(j)}(w,t)\right) - \\ - \left(g_j(w,t) - ikg_0(w,t)\right) + \widehat{F}^{(j)}(-k,t),\ j = \overline{1,n}; \tag{10}$$

$$e^{wt}\widehat{V}^{(r)}(-k,t) - \widehat{V}_0^{(r)}(-k) = -g_r(w,t) + ikg_0(w,t) + \widehat{F}^{(r)}(-k,t), \tag{11}$$

where $r = \overline{n+1, n+m}, \{k \in \mathbb{C}: \ Imk < 0\}$.

We can write solution in following form with using inverse Fourier transformation in global relation (8) and (9) [16]–[17].

$$V^{(j)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(\widehat{V}_0^{(j)}(k) + \widehat{F}^{(j)}(k,t)\right)dk + \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-ikL_j-wt}\left(h_1^{(j)}(w,t) + ikh_0^{(j)}(w,t)\right)dk \\ -\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(g_j(w,t) + ikg_0(w,t)\right)dk,\ j = \overline{1,n}, \tag{12}$$

$$V^{(r)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(\widehat{V}_0^{(r)}(k) + \widehat{F}^{(r)}(k,t)\right)dk \\ -\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(g_r(w,t) + ikg_0(w,t)\right)dk,\ r = \overline{n+1, n+m}. \tag{13}$$

The integrand of the second integral in (12) and (13) is entire and decays as $k \to \infty$ for $k \in \mathbb{C}^- \backslash D^-$. The third integral has an integrand that is entire and decays as $k \to \infty$ for $k \in \mathbb{C}^+ \backslash D^+$. Using the analyticity of the integrand and applying Jordan's Lemma we can replace the contour of integration of the second integral by - $\int_{\partial D^-}$ and of the third integral by - $\int_{\partial D^+}$ (see [12], [14]–[17]):

$$V^{(j)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(\widehat{V}_0^{(j)}(k) + \widehat{F}^{(j)}(k,t)\right)dk - \\ -\frac{1}{2\pi}\int_{\partial D^-} e^{ikx-ikL_j-wt}\left(h_1^{(j)}(w,t) + ikh_0^{(j)}(w,t)\right)dk - \\ -\frac{1}{2\pi}\int_{\partial D^+} e^{ikx-wt}\left(g_j(w,t) + ikg_0(w,t)\right)dk,\ j = \overline{1,n}, \tag{14}$$

$$V^{(r)}(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ikx-wt}\left(\widehat{V}_0^{(r)}(k) + \widehat{F}^{(r)}(k,t)\right)dk - \\ -\frac{1}{2\pi}\int_{\partial D^+} e^{ikx-wt}\left(g_r(w,t) + ikg_0(w,t)\right)dk,\ r = \overline{n+1, n+m}, \tag{15}$$

where $D^{\pm} = \left\{k \in \mathbb{C}^{\pm} : Rek^2 < 0\right\}$, $\mathbb{C}^+ = \{k \in \mathbb{C} : Imk > 0\}$,

$$\mathbb{C}^- = \{k \in \mathbb{C} : Imk < 0\}.$$

We need to find unknowns $g_0(w,t), g_j(w,t), g_r(w,t), h_0^{(j)}(w,t), h_1^{(j)}(w,t)$, $j = \overline{1,n}$, $r = \overline{n+1, n+m}$ representation of the solution.

Now, using vertex conditions we get

$$\begin{cases} e^{wt}\widehat{V}^{(j)}(k,t) - \widehat{V}_0^{(j)}(k) = e^{-ikL_j}h_1^{(j)}(w,t) + ike^{-ikL_j}h_0^{(j)}(w,t) - \\ -g_j(w,t) - ikg_0(w,t) + F^{(j)}(k,t); \\ e^{wt}\widehat{V}^{(j)}(-k,t) - \widehat{V}_0^{(j)}(-k) = e^{ikL_j}h_1^{(j)}(w,t) - ike^{ikL_j}h_0^{(j)}(w,t) - \\ -g_j(w,t) + ikg_0(w,t) + \widehat{F}^{(j)}(-k,t); \\ e^{wt}\widehat{V}^{(r)}(k,t) - \widehat{V}_0^{(r)}(k) = -g_r(w,t) - ikg_0(w,t) + \widehat{F}^{(r)}(k,t); \\ \sum_{j=1}^{6}\delta_j^2 g_j(w,t) = 0; \\ \alpha_j h_0^{(j)}(w,t) + \beta_j h_1^{(j)}(w,t) = 0. \end{cases} \tag{16}$$

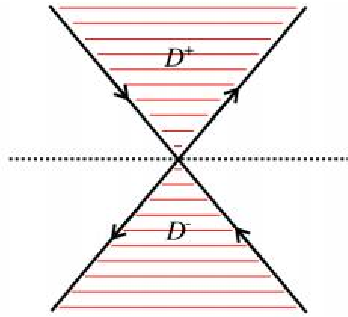

Figure 2. The domains D^+ and D^- for the heat transfer equation

Where $j=\overline{1,n}$, $r=\overline{n+1,n+m}$.

Solving this equations for $ikg_0(w,t)$, we have

$$ikg_0(w,t)=\frac{1}{\sum\limits_{j=1}^{n+m}\delta_j^2}\left[\sum_{j=1}^{n}\frac{\delta_j^2}{A_j+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\cdot B_j}\left[e^{ikL_j}\left(1+i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}_0^{(j)}(k)\ -\right.\right.$$

$$\left.-e^{-ikL_j}\left(1-i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}_0^{(j)}(-k)\right]+\sum_{r=n+1}^{n+m}\delta_r^2\widehat{V}_0^{(r)}(k)+$$

$$+\sum_{j=1}^{n}\frac{\delta_j^2}{A_j+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\cdot B_j}\left[e^{ikL_j}\left(1+i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{F}^{(j)}(k,t)-\right. \tag{17}$$

$$\left.-e^{-ikL_j}\left(1-i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{F}^{(j)}(-k,t)\right]+\sum_{r=n+1}^{n+m}\delta_r^2\widehat{F}^{(r)}(k,t)-$$

$$-\sum_{j=1}^{n}\frac{\delta_j^2e^{wt}}{A_j+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\cdot B_j}\left[e^{ikL_j}\left(1+i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}^{(j)}(k,t)-\right.$$

$$\left.\left.-e^{-ikL_j}\left(1-i\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}^{(j)}(-k,t)\right]-\sum_{r=n+1}^{n+m}\delta_r^2e^{wt}\widehat{V}^{(r)}(k,t)\right],$$

where $A_j=e^{ikL_j}-e^{-ikL_j}$, $B_j=e^{ikL_j}+e^{-ikL_j}$, $j=\overline{1,n}$, $\alpha_j\neq 0$.

Now putting $G^{(j)}(k,t)=\widehat{V}_0^{(j)}(k)-ikg_0(w,t)+\widehat{F}^{(j)}(k,t)$, $j=\overline{1,n}$ we can rewrite

$$\begin{cases} e^{wt}V^{(j)}(k,t)=G^{(j)}(k,t)+e^{-ikL_j}\left(1-i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)h_1^{(j)}(w,t)-g_j(w,t),\\ e^{wt}V^{(j)}(-k,t)=G^{(j)}(-k,t)+e^{ikL_j}\left(1+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)h_1^{(j)}(w,t)-g_j(w,t).\end{cases} \tag{18}$$

So, we have

$$h_1^{(j)}(w,t)=\frac{1}{A_j+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\cdot B_j}\left[G^{(j)}(k,t)-G^{(j)}(-k,t)-\right.$$

$$\left.-e^{wt}\left(\widehat{V}^{(j)}(k,t)-\widehat{V}^{(j)}(-k,t)\right)\right], \tag{19}$$

where $h_0^{(j)}(w,t)=-\frac{\beta_j}{\alpha_j}h_1^{(j)}(w,t),\ \alpha_j\neq 0,\ j=\overline{1,n}.$

$$g_j(w,t)=\frac{1}{A_j+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\cdot B_j}\left[e^{ikL_j}\left(1+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)G^{(j)}(k,t)-\right.$$

$$-e^{-ikL_j}\left(1-i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)G^{(j)}(-k,t)-e^{wt+ikL_j}\left(1+i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}^{(j)}(k,t)-$$

$$\left.-e^{wt-ikL_j}\left(1-i\cdot\frac{\beta_j}{\alpha_j}\cdot k\right)\widehat{V}^{(j)}(-k,t)\right], \tag{20}$$

this $\alpha_j\neq 0,\ j=\overline{1,n}$.

Solving (11) for $g_r(w,t)$ we find

$$g_r(w,t)=\widehat{V}_0^{(r)}(-k)+ikg_0(w,t)+\widehat{F}^{(r)}(-k,t)-e^{wt}\widehat{V}^{(r)}(-k,t). \tag{21}$$

Replacing in equation (15) $g_r(w,t)$ with the RHS of (21) we find [16]–[17]

$$V^{(r)}(x,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{ikx-wt}\left(\widehat{V}_0^{(r)}(k)+\widehat{F}^{(r)}(k,t)\right)dk- \tag{22}$$

$$-\frac{1}{2\pi}\int_{\partial D^+}e^{ikx-wt}\left(2ikg_0(w,t)+\widehat{V}_0^{(r)}(-k)+\widehat{F}^{(r)}(-k,t)\right)dk,r=\overline{n+1,n+m},$$

The term $e^{wt}\widehat{V}^{(r)}(-k,t)$ gives rise to the term [13]

$$-\frac{1}{2\pi}\int_{\partial D^+}e^{ikx}\widehat{V}^{(r)}(-k,t)dk,\ 0<x<\infty,\ t>0,$$

which vanishes, since both e^{wt} and $\widehat{V}^{(r)}(-k,t)$ are bounded and analytic in the upper half of the complex k plane, and furthermore $\widehat{V}^{(r)}(-k,t)$ is of $O\left(\frac{1}{k}\right)$ as $k\to\infty$:

$$\widehat{V}^{(r)}(-k,t)=\int_0^{\infty}e^{ikx}V^{(r)}(x,t)dx\sim-\frac{V^{(r)}(0,t)}{ik},\ k\to\infty.$$

Thus, Cauchy's theorem supplemented with Jordan Lemma in the domain D^+ [12].

We next substitute $g_1^{(j)}(w,t)$ and $h_1^{(j)}(w,t)$ in (14). We claim that the terms involving $\widehat{V}^{(j)}(\pm k,t)$ yield a zero contribution. Indeed, since this is a well-posed BVP, the relevant terms are bounded as $k\to\infty$. Let us verify this explicitly: the term in $g_1^{(j)}(w,t)$ involves the following contribution form $\widehat{V}^{(j)}(\pm k,t)$:

$$\frac{e^{ikL_j}\widehat{V}^{(j)}(k,t)-e^{-ikL_j}\widehat{V}^{(j)}(-k,t)}{e^{ikL_j}\left(1+i\frac{\beta_j}{\alpha_j}k\right)-e^{-ikL_j}\left(1-i\frac{\beta_j}{\alpha_j}k\right)}.$$

Since $Imk\geq 0$, e^{-ikL_j} grows, and then the above expression, as $k\to\infty$, becomes

$$\frac{-\widehat{V}^{(j)}(-k,t)+e^{ikL_j}\int_0^{L_j}e^{ik(L_j-x)}V^{(j)}(x,t)dx}{i\frac{\beta_j}{\alpha_j}k-1},$$

which is clearly bounded as $k\to\infty$ with $Imk\geq 0$. Similarly the term in $h_1^{(j)}(w,t)$ involves the following contribution from $\widehat{V}^{(j)}(\pm k,t)$:

$$\frac{\widehat{V}^{(j)}(k,t)-\widehat{V}^{(j)}(-k,t)}{e^{ikL_j}\left(1+i\frac{\beta_j}{\alpha_j}k\right)-e^{-ikL_j}\left(1-i\frac{\beta_j}{\alpha_j}k\right)},$$

which as $k\to\infty$, $Imk\leq 0$, simplifies to the expression

$$\frac{\int_0^{L_j}e^{-ik(L_j+x)}V^{(j)}(x,t)dx-e^{-ikL_j}\widehat{V}^{(j)}(-k,t)}{i\frac{\beta_j}{\alpha_j}k-1},$$

which is clearly bounded as $k \to \infty$, $Imk \leq 0$ [12].

3. Conclusion and results

In this paper, we presented a method for constructing solutions to initial-boundary value problems on certain metric graphs, such as star-shaped graphs. In general, this method is applicable to solving initial-boundary value problems on arbitrary metric graphs. In solving the problem, we use the so-called global relation and another relation obtained by replacing the complex parameter with its negative. These two relations are equivalent to mapping Dirichlet conditions to Neumann conditions at the vertices. Using these relations, the considered problem is reduced to a system of algebraic equations with respect to the unknown values of the solution at the branching points of the graph. The solution is expressed in the form of contour integrals of known functions. The contours are chosen so that the integrand functions decay exponentially at infinity along these contours. This property ensures good convergence of the integrals, which is very important, for example, for the numerical computation of the solution.

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REZYUME

Ushbu maqolada issiqlik uzatish tenglamasi uchun simmetrik metrik grafda boshlang'ich-chegaraviy masalalar yechimining integral ko'rinishdagi ifodasi Fokasning umumlashtirilgan almashtirish usuli yordamida, berilgan boshlang'ich va chegaraviy ma'lumotlar asosida topildi.

Kalit so'zlar: Issiqlik uzatish tenglamasi, metrik graflar, tarmoqlangan tuzilmalar, Fokas usuli, umumlashtirilgan almashtirish, Furye almashtirish, boshlang'ich masala, chegaraviy qiymat masalasi.

РЕЗЮМЕ

В настоящей статье получено интегральное представление решений начально-краевых задач для уравнения теплопроводности на симметричном метрическом графе в терминах заданных начальных и граничных данных с использованием унифицированного метода преобразования Фокаса.

Ключевые слова: Уравнение теплопередачи, метрические графы, разветвлённые структуры, метод Фокаса, унифицированное преобразование, преобразование Фурье, начальная задача, краевая задача.