UDC 15A.80

On the solution of a system max-plus linear equations in three variables

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ABSTRACT

This research work is devoted to study solutions of a max-plus linear equation system. The study outlines the fundamental concepts and properties of max-plus algebra and highlights the theoretical and practical aspects of solving systems of equations within this algebraic structure. In addition, the article analyzes the conditions for the existence of solutions and presents methods for determining them using max-plus techniques. The results obtained are of significant importance for applications of idempotent algebra and tropical mathematics, particularly in optimization and modeling of discrete event systems. Then, we provided examples of finding roots of a max-plus system and plotted the graphs of these max-plus equations in the Cartesian coordinate system.

Key words: Max-plus algebra; idempotent semi-ring; max-plus linear equation system.

1. Introduction

Max-Plus Linear (MPL) systems are a class of discrete-event dynamic systems (DEDS) based on the Max-Plus algebra, an algebraic structure that uses maximisation and addition as its binary operations. MPL systems are employed to model processes with features of synchronization but without concurrency, and as such are widely used for applications in transportation networks [1, 2], biological systems [3, 4] and manufacturing [7, 8]. In MPL models, the states correspond to time instances related to discrete events.

Max-plus algebra has been studied in research papers and books from the early 1960's. Perhaps the first paper was that of R. A. Cuninghame-Green [5] in 1960, followed by [10, 11, 12, 13] and numerous other articles. Independently, a number of pioneering articles were published, e. g. by B. Giffler, M. Gondran and M. Minoux. Intensive development of max-algebra has followed since 1985 in the works of M. Akian, R. Bapat, R. E. Burkard, G. Cohen, B. De Schutter, P. van den Driessche, S. Gaubert, M. Gavalec, R. Goverde, J. Gunawardena, B. Heidergott, M. Joswig, R. Katz, G. Litvinov, J. -J. Loiseau, G. -J. Olsder, J. Plávka, J. -P. Quadrat, I. Singer, S. Sergeev, E. Wagneur, K. Zimmermann, U. Zimmermann and many others. Note that idempotency of addition makes max-algebra part of idempotent mathematics [9, 17, 18].

2. Preliminary notions

The max-plus algebra is an algebraic structure consisting of real numbers where the standard operations of addition and multiplication are replaced by the operation of taking a maximum and the operation of standard addition, respectively. More precisely, let $\mathbb{R}_{\max}$ denote the set $\mathbb{R}\cup\{-\infty\}$, let $\oplus$ be a binary operator on $\mathbb{R}_{\max}$ with $x\oplus y=\max\{x,\,y\}$, and let $\odot$ be the binary operator on $\mathbb{R}_{\max}$ with $x\odot y=x+y$. The resulting algebraic structure $\mathbb{R}_{\max}=(\mathbb{R}_{\max},\oplus,\odot)$ is called a *max-plus semi-field* or simply a *max-plus algebra*. $\mathbb{R}_{\max}$ is also called a commutative idempotent semiring or a dioid. In max-plus algebra $\varepsilon=-\infty$ is the additive identity: $x\oplus\varepsilon=\varepsilon\oplus x=x$, for each $x\in\mathbb{R}_{\max}$ and the multiplicative identity is $e=0$: $x\odot e=e\odot x=x$, for all $x\in\mathbb{R}_{\max}$ [14], [15].

Let $n \in \mathbb{R}$. The n^{th} max-plus algebraic power of $x \in \mathbb{R}$ is denoted by $x^{\odot^n}$ and corresponds to nx in conventional algebra. If $x \in \mathbb{R}$ then $x^{\odot^0} = 0$ and the inverse element of x written $\odot$ is $x^{\odot^{-1}} = -x$. There is no inverse element for ε written $\odot$ since ε is absorbing for $\odot$. If $n > 0$ then $\varepsilon^{\odot^n} = \varepsilon$. If $n < 0$ then $\varepsilon^{\odot^n}$ is not defined [15].

The max-plus algebraic division operation is defined as follows:

$$\text{if } x,\, y \in \mathbb{R}_{\max} \text{ and } y \neq \varepsilon, \text{ then } \tfrac{x}{y} = x \odot y^{\odot^{-1}} = x \odot (-y).$$

If y is equal to ε then the max-plus algebraic division is not defined [16].

In [6], we studied how to determine the solution set of the system

$$\begin{cases} a_{11} \odot x \oplus a_{12} \odot y = b_1, \\ a_{21} \odot x \oplus a_{22} \odot y = b_2. \end{cases} \tag{1}$$

by applying the Theorem 1 stated below.

We denote:

$$\triangle = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \quad \triangle_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}, \quad \triangle_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}.$$

Theorem 1. [6] Let $\triangle \neq \varepsilon$.

1. If $\frac{b_1}{b_2} = \frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$, then the solution system () consists of the pairs $(x,\, v)$ here
$$x = \tfrac{\triangle_x}{\triangle} \text{ and } v \in \{s \in \mathbb{R}_{\max} \colon s \leq \tfrac{\triangle_y}{\triangle}\};$$
2. If $\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$, then the solution system () consists of the pairs $(u,\, y)$ here
$$u \in \{t \in \mathbb{R}_{\max} \colon t \leq \tfrac{\triangle_x}{\triangle}\} \text{ and } y = \tfrac{\triangle_y}{\triangle};$$
3. If $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$, then the solution system () consists of the pairs $(u,\, v)$ here
$$x = \tfrac{\triangle_x}{\triangle},\ v \in \{s \in \mathbb{R}_{\max} \colon s \leq \tfrac{\triangle_y}{\triangle}\} \text{ and } u \in \{t \in \mathbb{R}_{\max} \colon t \leq \tfrac{\triangle_x}{\triangle}\},\ y = \tfrac{\triangle_y}{\triangle};$$
4. If $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$, then the system () has no solution;
5. If $\frac{a_{12}}{a_{22}} < \frac{b_1}{b_2} < \frac{a_{11}}{a_{21}}$ or $\frac{a_{11}}{a_{21}} < \frac{b_1}{b_2} < \frac{a_{12}}{a_{22}}$, then the system () consists of a unique root $(x,\, y)$ with
$$x = \tfrac{\triangle_x}{\triangle} \text{ and } y = \tfrac{\triangle_y}{\triangle};$$
6. If $\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$ and both of inequalities in 5) do not hold, then the system () has no solution.

3. Main part

Let us be given the following max-plus linear system:

$$\begin{cases} a_{11} \odot x \oplus a_{12} \odot y \oplus a_{13} \odot z = b_1, \\ a_{21} \odot x \oplus a_{22} \odot y \oplus a_{23} \odot z = b_2, \\ a_{31} \odot x \oplus a_{32} \odot y \oplus a_{33} \odot z = b_3. \end{cases} \tag{2}$$

Assume that $a_{11} \neq \varepsilon$. In this case, we divide the first equation of the system by a_{11} and denote the resulting new coefficients by a'_{12}, a'_{13}. Similarly, if $a_{21} \neq \varepsilon$ and $a_{31} \neq \varepsilon$, then we divide the second and third equations of the system by a_{21} and a_{31}, respectively, and denote the resulting new coefficients by a'_{22}, a'_{23} and a'_{32}, a'_{33}, respectively. As a result, we obtain system (3), which is equivalent to system (2):

$$\begin{cases} x \oplus a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ x \oplus a'_{22} \odot y \oplus a'_{23} \odot z = b'_2, \\ x \oplus a'_{32} \odot y \oplus a'_{33} \odot z = b'_3. \end{cases} \quad (3)$$

We now construct the following systems based on system (3):

$$\begin{cases} a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ a'_{22} \odot y \oplus a'_{23} \odot z = b'_2, \end{cases} \quad (4)$$

$$\begin{cases} a'_{22} \odot y \oplus a'_{23} \odot z = b'_2, \\ a'_{32} \odot y \oplus a'_{33} \odot z = b'_3. \end{cases} \quad (5)$$

It is known that the following system consists of the common solution of systems (4) and (5)

$$\begin{cases} a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ a'_{32} \odot y \oplus a'_{33} \odot z = b'_3. \end{cases}$$

The solution of systems (4) and (5) can be found according to Theorem 1.

We denote the solution sets of systems (4) and (5) by W_1 and W_2, respectively.

Theorem 2. If systems (4) and (5) have solutions, then system (3) also has a solution and the solution set of system (3) is given as follows:

$$W = \{(x,\, y,\, z)\colon x \le \min\{b'_1,\, b'_2,\, b'_3\},\, (y,\, z) \in W_1 \cap W_2\}.$$

Proof. We divide system (3) into the following cases and find its solution.

1-case. Let $x < \min\{b'_1,\, b'_2,\, b'_3\}$. Then we obtain the systems (4) and (5). Consequently, the solution of system (3)

$$W = \{(x,\, y,\, z)\colon x < \min\{b'_1,\, b'_2,\, b'_3\},\, (y,\, z) \in W_1 \cap W_2\}.$$

2-case. Let $x = \min\{b'_1,\, b'_2,\, b'_3\} = b'_1$. Then we have the following system:

$$\begin{cases} a'_{12} \odot y \oplus a'_{13} \odot z \le b'_1, \\ a'_{22} \odot y \oplus a'_{23} \odot z = b'_2, \\ a'_{32} \odot y \oplus a'_{33} \odot z = b'_3. \end{cases}$$

This case, the system () of solution:

$$W = \{(x,\, y,\, z)\colon x = \min\{b'_1,\, b'_2,\, b'_3\} = b'_1,\, (y,\, z) \in W_2,\, y \le \tfrac{b'_1}{a'_{12}} \wedge z \le \tfrac{b'_1}{a'_{13}}\}.$$

3-case. Let $x = \min\{b'_1,\, b'_2,\, b'_3\} = b'_2$. Then we have the following system:

$$\begin{cases} a'_{22} \odot y \oplus a'_{23} \odot z \le b'_2, \\ a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ a'_{32} \odot y \oplus a'_{33} \odot z = b'_3. \end{cases}$$

This case, the system (3) of solution:

$$W = \{(x,\, y,\, z)\colon x = \min\{b'_1,\, b'_2,\, b'_3\} = b'_2,\, (y,\, z) \in W_1 \cap W_2,\, y \le \tfrac{b'_2}{a'_{22}} \wedge z \le \tfrac{b'_2}{a'_{23}}\}.$$

4-case. Let $x = \min\{b'_1,\, b'_2,\, b'_3\} = b'_3$. Then we have the following system:

$$\begin{cases} a'_{32} \odot y \oplus a'_{33} \odot z \le b'_3, \\ a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ a'_{22} \odot y \oplus a'_{23} \odot z = b'_2. \end{cases}$$

This case, the system (3) of solution:

$$W = \{(x,\, y,\, z)\colon x = \min\{b_1',\, b_2',\, b_3'\} = b_3',\, (y,\, z) \in W_1,\, y \le \tfrac{b_3'}{a_{32}'} \wedge z \le \tfrac{b_3'}{a_{33}'}\}.$$

5-case. Let $x = b_1' = b_2' = b_3'$. Then we have the following system:

$$\begin{cases} a_{12}' \odot y \oplus a_{13}' \odot z \le b_1', \\ a_{22}' \odot y \oplus a_{23}' \odot z \le b_2', \\ a_{32}' \odot y \oplus a_{33}' \odot z \le b_3'. \end{cases}$$

This case, the system (3) of solution:

$$W = \{(x,\, y,\, z)\colon x = b_1',\, y \le \min\{\tfrac{b_1'}{a_{12}'},\, \tfrac{b_2'}{a_{22}'},\, \tfrac{b_3'}{a_{32}'}\} \wedge z \le \min\{\tfrac{b_1'}{a_{13}'},\, \tfrac{b_2'}{a_{23}'},\, \tfrac{b_3'}{a_{33}'}\}\}.$$

6-case. It is clear that in the remaining cases, system (3) has no solution.

Thus, the general solution set of the system (3) is

$$W = \{(x,\, y,\, z)\colon x \le \min\{b_1',\, b_2',\, b_3'\},\, (y,\, z) \in W_1 \cap W_2\}.$$

□

Example 1. Solve the system:

$$\begin{cases} 2 \odot x \oplus 3 \odot y \oplus z = -3, \\ 2 \odot x \oplus 5 \odot y \oplus 6 \odot z = -1, \\ 2 \odot x \oplus y \oplus 9 \odot z = 0. \end{cases}$$

Solution. We rewrite the given system as follows:

$$\begin{cases} x \oplus 1 \odot y \oplus (-2) \odot z = -5, \\ x \oplus 3 \odot y \oplus 4 \odot z = -3, \\ x \oplus (-2) \odot y \oplus 7 \odot z = -2. \end{cases}$$

1-case. $x < \min\{-5,\, -3,\, -2\} = -5$. Then we obtain the following systems:

$$\begin{cases} 1 \odot y \oplus (-2) \odot z = -5, \\ 3 \odot y \oplus 4 \odot z = -3, \end{cases}$$

and

$$\begin{cases} 3 \odot y \oplus 4 \odot z = -3, \\ -2 \odot y \oplus 7 \odot z = -2. \end{cases}$$

Now according to Theorem 1, we find the solution of these systems. In the first system, since $\frac{-2}{4} < \frac{-5}{-3} = \frac{1}{3}$, its solution is $y = -6,\, z \le -7$ and in the second system, since $\frac{4}{7} < \frac{-3}{-2} < \frac{3}{-2}$, its solution is $y = -6,\, z = -9$.

Thus, in this case, the solution set of the system is $\{(x,\, y,\, z)\colon x < -5,\, y = -6,\, z = -9\}$.

2-case. $x = \min\{-5,\, -3,\, -2\} = -5$. Then we get the following system:

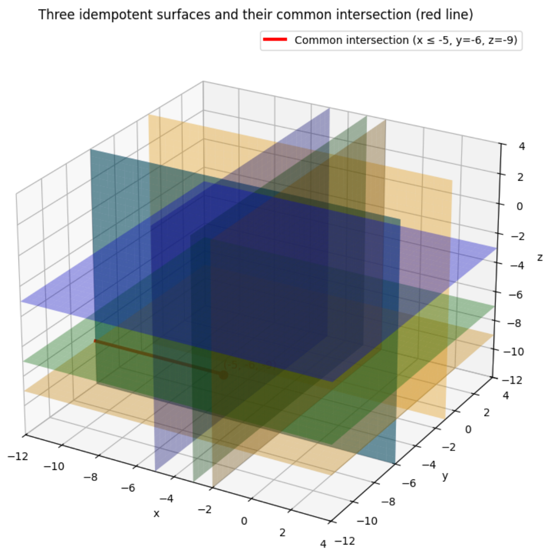
$$\begin{cases} 1 \odot y \oplus (-2) \odot z \le -5, \\ 3 \odot y \oplus 4 \odot z = -3, \\ -2 \odot y \oplus 7 \odot z = -2. \end{cases}$$

According to Theorem 1, since $\frac{4}{7} < \frac{-3}{-2} < \frac{3}{-2}$, its solution is $y = -6,\, z = -9$. By the first inequality of the system $y \le -6$ and $z \le -3$.

Therefore, in this case, the solution set of the system is $\{(x,\, y,\, z)\colon x = -5,\, y = -6,\, z = -9\}$.
So, the general solution set of the system is

$$W = \{(x,\, y,\, z)\colon x \leq -5,\, y = -6,\, z = -9\}.$$

The graphical representation of these planes in the coordinate system is as follows:



Example 2. Solve the system:

$$\begin{cases} 2 \odot x \oplus 3 \odot y \oplus z = 3, \\ -1 \odot x \oplus 2 \odot y \oplus 3 \odot z = -4, \\ -2 \odot x \oplus (-4) \odot y \oplus 5 \odot z = -4. \end{cases}$$

Solution. We rewrite the given system as follows:

$$\begin{cases} x \oplus 1 \odot y \oplus (-2) \odot z = 1, \\ x \oplus 3 \odot y \oplus 4 \odot z = -3, \\ x \oplus (-2) \odot y \oplus 7 \odot z = -2. \end{cases}$$

1-case. $x < \min\{1,\, -3,\, -2\} = -3$. Then we obtain the following systems:

$$\begin{cases} 1 \odot y \oplus (-2) \odot z = 1, \\ 3 \odot y \oplus 4 \odot z = -3, \end{cases}$$

and

$$\begin{cases} 3 \odot y \oplus 4 \odot z = -3, \\ -2 \odot y \oplus 7 \odot z = -2. \end{cases}$$

Now according to Theorem 1, we find the solution of these systems. In the first system, since $\frac{-2}{4} < \frac{1}{3} < \frac{1}{-3}$, its solution is empty set, i. e. the first system has no solution and in the second system, since $\frac{4}{7} < \frac{-3}{-2} < \frac{3}{-2}$, its solution is $y = -6$, $z = -9$.

Thus, in this case, the solution set of the system is empty set.

2-case. $x = \min\{1,\, -3,\, -2\} = -3$. Then we get the following system:

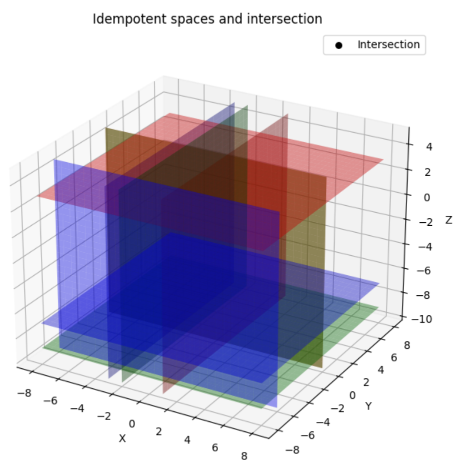
$$\begin{cases} 3 \odot y \oplus 4 \odot z \leq -3, \\ 1 \odot y \oplus (-2) \odot z = 1, \\ -2 \odot y \oplus 7 \odot z = -2. \end{cases}$$

According to Theorem 1, since $\frac{-2}{7} < \frac{1}{-2} = \frac{1}{-2}$, its solution is $y = 0$, $z \leq -9$. By the first inequality of the system $y \leq -6$ and $z \leq -7$, but $0 > -6$.

Therefore, in this case, the solution set of the system is empty set.

So, the given system has no solution.

The graphical representation of these planes in the coordinate system is as follows:



Let us be given the following max-plus linear system:

$$\begin{cases} a_{11} \odot x \oplus a_{12} \odot y \oplus a_{13} \odot z = b_1, \\ a_{21} \odot x \oplus a_{22} \odot y \oplus a_{23} \odot z = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ a_{m1} \odot x \oplus a_{m2} \odot y \oplus a_{m3} \odot z = b_m. \end{cases} \tag{6}$$

Hence we obtain the following system (7), which is equivalent to system (6):

$$\begin{cases} x \oplus a'_{12} \odot y \oplus a'_{13} \odot z = b'_1, \\ x \oplus a'_{22} \odot y \oplus a'_{23} \odot z = b'_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots, \\ x \oplus a'_{m2} \odot y \oplus a'_{m3} \odot z = b'_m, \end{cases} \tag{7}$$

We now construct the following system based on system (7):

$$\begin{cases} a'_{i2} \odot y \oplus a'_{i3} \odot z = b'_i, \\ a'_{(i+1)2} \odot y \oplus a'_{(i+1)3} \odot z = b'_{i+1}, \end{cases} \tag{8}$$

where $i = 1, 2, \dots, (m-1)$.

Corollary 1. If the systems (8) has a solution and its solution sets is W_i, then system (7) also has a solution and the solution set of system (7) is given as follows:

$$W = \{(x, y, z)\colon x \leq \min_{1 \leq i \leq m} \{b'_i\}, (y, z) \in \bigcap_{i=1}^{m-1} W_i\}.$$

Example 3. Solve the system:

$$\begin{cases} 2\odot x\oplus 3\odot y\oplus z=-3,\\ 1\odot x\oplus 4\odot y\oplus 5\odot z=-2,\\ -2\odot x\oplus 3\odot y\oplus 7\odot z=1,\\ -3\odot x\oplus 4\odot y\oplus 1\odot z=0. \end{cases}$$

Solution. We rewrite the given system as follows:

$$\begin{cases} x\oplus 1\odot y\oplus (-2)\odot z=-5,\\ x\oplus 3\odot y\oplus 4\odot z=-3,\\ x\oplus 5\odot y\oplus 9\odot z=3,\\ x\oplus 7\odot y\oplus 4\odot z=3. \end{cases}$$

1-case. $x<\min\{-5,\,-3,\,3,\,3\}=-5$. Then we obtain the following systems:

$$\begin{cases} 1\odot y\oplus (-2)\odot z=-5,\\ 3\odot y\oplus 4\odot z=-3, \end{cases}$$

$$\begin{cases} 3\odot y\oplus 4\odot z=-3,\\ 5\odot y\oplus 9\odot z=3 \end{cases}$$

and

$$\begin{cases} 5\odot y\oplus 9\odot z=3,\\ 7\odot y\oplus 4\odot z=3. \end{cases}$$

Now according to Theorem 1, we find the solution of these systems. In the first system, since $\frac{-2}{4}<\frac{-5}{-3}=\frac{1}{3}$, its solution is $y=-6$, $z\le -7$ and in the second system, since $\frac{-3}{3}<\frac{4}{9}<\frac{3}{5}$, it has no solution and in the third system, since $\frac{5}{7}<\frac{3}{3}<\frac{9}{4}$, its solution is $y=-4$, $z=-6$

Thus, in this case, the solution set of the system is empty set.

2-case. $x=\min\{-5,\,-3,\,3,\,3\}=-5$. Then we get the following system:

$$\begin{cases} 1\odot y\oplus (-2)\odot z\le -5,\\ 3\odot y\oplus 4\odot z=-3,\\ 5\odot y\oplus 9\odot z=3,\\ 7\odot y\oplus 4\odot z=3. \end{cases}$$

Obviously, the system

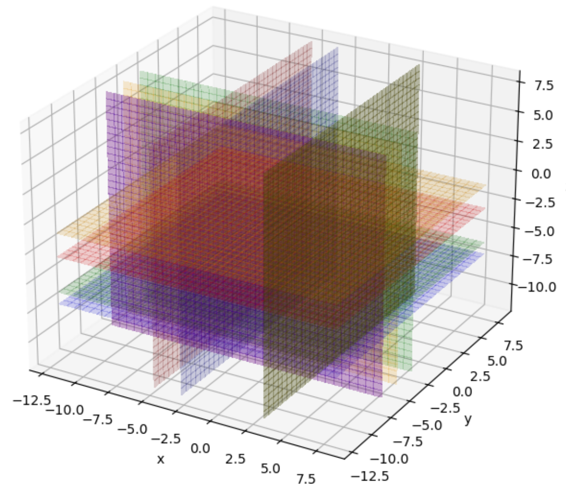
$$\begin{cases} 3\odot y\oplus 4\odot z=-3,\\ 5\odot y\oplus 9\odot z=3,\\ 7\odot y\oplus 4\odot z=3. \end{cases}$$

has no solution.

So, the given system has no solution.

The graphical representation of these planes in the coordinate system is as follows:

12 planes — NO common line (inconsistent)



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ANNOTATSIYA

Ushbu tadqiqot ishi max-plus chiziqli tenglamalar sistemasining yechimlarini o'rganishga bag'ishlangan. Tadqiqotda max-plus algebra asosiy tushunchalari va xossalari eslatib o'tilgan hamda ushbu algebraik tuzilma doirasida tenglamalar sistemasini yechishning nazariy va amaliy jihatlari yoritilgan. Shu bilan birga, maqolada yechimlarning mavjudlik shartlari tahlil qilingan va max-plus metodlari yordamida ularni aniqlash usullari ko'rsatib berilgan. Olingan natijalar idempotent algebra va tropik matematikaning qo'llanilish sohalari, jumladan, optimallashtirish va diskret hodisalar sistemalarini modellashtirishda muhim ahamiyat kasb etadi. So'ngra max-plus sistemaning ildizlarini topishga doir misollar keltirilgan va Dekart koordinatalar sistemasida bu max-plus tenglamalarning grafiklari yasalgan.

Kalit so'zlar: Max-plus algebra; idempotent yarim halqa; max-plus chiziqli sistema.

АННОТАЦИЯ

Данная исследовательская работа посвящена изучению решений систем max-plus линейных уравнений. В исследовании упомянуты основные понятия и свойства max-plus алгебры, а также освещены теоретические и практические аспекты решения систем уравнений в рамках данной алгебраической структуры. Кроме того, в статье проанализированы условия существования решений и показаны методы их определения с использованием max-plus подходов. Полученные результаты имеют важное значение для применения идемпотентной алгебры и тропической математики, в частности, в оптимизации и моделировании дискретных систем событий. Затем приведены примеры нахождения корней max-plus линейной системы, и построены графики этих max-plus уравнений в декартовой системе координат.

Ключевые слова: max-plus алгебра; идемпотентное полукольцо; max-plus линейная система.