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Uchinchi tur klassik soha yordamida aniqlangan matritsaviy poliedrda Bishop integral formulasi

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ANNOTATSIYA:
Bu maqolada uchinchi tur klassik soha yordamida matritsaviy poliedir aniqlangan. Bu aniqlangan poliedrda sikllarning gamologik bo‘lishi ko‘rsatilgan hamda Bishop integral formulasi olingan.

Kalit so‘zlar: Kososimmetrik matritsalar, uchinchi tur klassik soha, matritsaviy poliedir, matritsa argumentli golomorf fuksiyalar va akislantirishlar, gomologik sikllar, integral formula, Bishop integral formulasi.

Elementlari kompleks sonlardan iborat, $[n \times n]-$ tartibli kososimmetrik matritsalar bo‘lgan fazoni $\tilde{\mathbb{C}}\,[n \times n\,]$ kabi belgilaymiz. Ushbu

$$D_3 = \left\{ Z \in \tilde{\mathbb{C}}\,[n \times n\,] : I^{(n)} + Z\bar{Z} > 0 \right\}$$

sohaga uchinchi tur klassik soha deb aytiladi. Bunda $I^{(n)} - n$ tartibli birlik matritsa, $\bar{Z}-$ matritsa esa Z matritsaning kompleks qo‘shma matritsasi hisoblanadi (eslatma, yuqoridagi sohada H $>$ 0 Ermit matritsasining musbat aniqlanganligini bildiradi, ya‘ni, barcha xos sonlari musbat aniqlangan) ([1-3]).

Uchunchi tur klassik sohaning chegarasi va ostovi mos ravishda quyidagicha aniqlanadi [2]:

$$\partial D_3 = \left\{ Z \in \tilde{\mathbb{C}}\,[n \times n] : det\left(I^{(\,n)} + Z\overline{Z}\,\right) \; = \; 0, \; I^{(\,n)} + Z\overline{Z} \geq 0 \right\},$$

$$\Gamma = \left\{ Z \in \tilde{\mathbb{C}}\,[n \times n] : \; I^{(\,n)} + Z\overline{Z} = 0 \right\}.$$

Bu D_3 sohada golomorf va uning yopig‘ida uzluksiz har qanday $h\,(Z)$, ya‘ni, $h\,(Z) \in \mathcal{O}\,(D_3) \bigcap C\left(\overline{D_3}\right)$ funksiya uchun (juft n larda) ushbu

$$h(Z) = c_n \int\limits_{\Gamma} \frac{h\,(X)\,dX}{\det^{\frac{n-1}{2}}\,(X - Z)} \; (1)$$

Xua Lo-ken integral formulasi o‘rinli bo‘ladi ([2], 95-sah.). Bu yerda $dX = \bigwedge\limits_{\substack{i=1,j=2\\ i<j}}^{n} dx_{ij}$, integrallash tartibini va c_n o‘zgarmas

$$c_n \int\limits_{I^{(n)} + X\overline{X} = 0} \frac{dX}{\det^{\frac{n-1}{2}}\,(X)} = 1$$

shart bo‘yicha aniqlanadi.

Biror $G \subset \mathbb{C}^{\frac{n(n-1)}{2}}$ sohada golomorf bo‘lgan $f = \left(f_1, \ldots, f_{\frac{n(n-1)}{2}}\right) : G \to \mathbb{C}^{\frac{n(n-1)}{2}}$ akslantirishni qaraymiz. Yuqoridagi $f = \left(f_1, \ldots, f_{\frac{n(n-1)}{2}}\right) : G \to \mathbb{C}^{\frac{n(n-1)}{2}}$ akslantirishni quyidagicha $[n \times n]-$tartibli kososimmetrik matritsa ko‘rinishida tushunish kerak:

$$f(Z) = \begin{pmatrix} 0 & f_{12}(Z) & \ldots & f_{1n}(Z) \\ -f_{12}(Z) & 0 & \ldots & f_{2n}(Z) \\ \vdots & \vdots & \ddots & \vdots \\ -f_{1n}(Z) & -f_{2n}(Z) & \ldots & 0 \end{pmatrix} : G \to \tilde{\mathbb{C}}\,[n \times n]\,.$$

Endi matritsaviy poliedr tushunchasini keltiramiz.

1-ta’rif. Agar $f : G \to \tilde{\mathbb{C}}\left[n \times n\right]$ golomorf akslantirish yordamida aniqlangan

$$f^{-1}\left(D_{3,\,r}\right) = \left\{Z \in G :\, r^2 I^{(n)} + f(Z)\overline{f(Z)} > 0,\, r > 0\right\}$$

to‘plam G sohada kompakt yotsa, ya‘ni, $f^{-1}\left(D_{3,\,r}\right) \Subset G$, u holda $f^{-1}\left(D_{3,\,r}\right)$ to‘plamga matritsaviy poliedrik to‘plam deyiladi.

Shuni alohida ta‘kidlash lozimki, bu to‘plam har doim ham bog‘lamli bo‘lavermaydi.

2-ta’rif. Matritsaviy poliedrik to‘plamning bog‘lamli komponentasini matritsaviy poliedrik soha yoki qisqacha matritsaviy poliedr deb atamiz va uni $\Omega_{f,r}$ kabi belgilaymiz.

Matritsaviy poliedrning ostovi

$$\Gamma_{f,r} = \left\{Z \in G :\, r^2 I^{(n)} + f(Z)\overline{f(Z)} = 0,\, r > 0\right\},$$

ko‘rinishda aniqlanadi.

Bu ishda matritsaviy poliedrik sohada Bishop integral formulasi olingan.

Aytaylik $f = \left(f_1, \ldots, f_{\frac{n(n-1)}{2}}\right) : D \to G$ akslantirish $D \subset \mathbb{C}_Z^{\frac{n(n-1)}{2}}$ sohani $G \subset \mathbb{C}_W^{\frac{n(n-1)}{2}}$ sohaga akslantiruvchi golomorf akslantirish bo‘lsin. Agar har bir $W \in G$ nuqtalar uchun $f(Z) = W$ tenglama D sohada karrali ildizlarining tartibi bilan qo‘shib hisoblaganda bir xil sondagi yechimlarga ega bo‘lsa, u holda $f\left(Z\right)$ akslantirish chekli turdagi akslantirish deb ataladi (bu yerda nollar soni karrasi bilan olingan).

Aytaylik, $f : D \to G$ akslantirish $D \subset \mathbb{C}_Z^{\frac{n(n-1)}{2}}$ sohani $G \subset \mathbb{C}_W^{\frac{n(n-1)}{2}}$ sohaga akslantiruvchi chekli turdagi golomorf akslantirish bo‘lsin va

$$H(Z) = \frac{\varphi(Z)}{\psi(Z)}$$

funksiya D sohada meromorf bo‘lsin.

3-ta’rif: f akslantirishga nisbatan $H(Z)$ meromorf funksiyaning izi deb ushbu

$$[\mathrm{Tr}\, H](W) = \sum_{\nu} H(Z^{(\nu)}(W)), W \in G \backslash f(\psi = 0),$$

funksiyaga aytiladi, bu yerda yig‘indi $f(Z) = W$ tenglamaning ildizlari (karralisining tartibi ham e‘tiborga olingan) bo‘yicha olinadi. Bu ishda matritsaviy poliedrda maxsus ko‘rinishdagi meromorf funksitalar uchun Bishop integral formulasi olingan.

Teoremani isbotlashga o‘tishdan avvalo quyidagi muhim lemmani isbotlaymiz.

1-lemma. Aytaylik $X - [n \times n]$ simmetrik matritsa va $\|X\|_s < \varepsilon$ bo‘lsin. U holda $G_* = G \backslash \{\, Z : det\left(f(Z) - X\right) = 0\}$ to‘plamda $0 < \delta < \varepsilon - \|X\|_s$ tengsizlikni qanoatlantiruvchi barcha δ uchun quyidagi sikllar gomologik sikllar bo‘ladi:

$$\Gamma_{f-X,\delta} \sim \Gamma_{f,\varepsilon}.$$

Bu yerda $\|.\|_s$−spektral norma hamda

$$\Gamma_{f-X,\delta} = \left\{\, Z \in G :\ \delta^2 I + (f(Z) - X)\overline{(f(Z) - X)} = 0\right\}$$

ko‘rinishga ega bo‘lgan sikil.

Isbot. Aytaylik, ushbu

$$C = \left\{\, Z \in G :\, (\varepsilon - t\|X\|_s)^2 I + (f(Z) - tX)\overline{(f(Z) - tX)} = 0, 0 \le t \le 1\right\}$$

$-$ $(\frac{n(n+1)}{2} + 1)$-o‘lchamli zanjir berilgan bo‘lsin. Shunindek, $Z \in C$ uchun $\|f(Z) - tX\|_s = \varepsilon - t\|X\|_s$ tenglik o‘rinli.

Ravshanki, $C \in \Omega_{f,r}$ bo‘ladi. Haqiqatan ham, agar $Z^0 \in G \backslash \overline{\Omega}_{f,\varepsilon}$ bo‘lsa, u holda $\left\|f(Z^0)\right\|_s > \varepsilon$ bo‘ladi. Bu esa $\Omega_{f,r}$ ta‘rifiga zid.

Shunday qilib, ushbu

$$\left\|f(Z^0) - tX\right\|_s \geq \left\|f(Z^0)\right\|_s - t\|X\|_s > \varepsilon - t\|X\|_s$$

tengsizlikka egamiz, bundan esa $C-$ komakt zanjir ekanini kelib chiqadi.

Biz endi, agar Z nuqta C zanjirni $|C|$ tashuvchisidan bo‘lsa, u holda $det(f(Z) - X) \neq 0$ bo‘lishini ko‘rsatamiz. Faraz qilaylik, $Z \in |C|$ uchun $det(f(Z) - X) = 0$ tenglik o‘rinli bo‘lsin. U holda shunday nolmas z‘ vektor mavjudki, z‘$(f(Z) - X) = 0$ bo‘ladi. Demak, quyidagi tengliklarni topamiz:

$$\overline{(f(Z) - tX)}(z‘)^* + (1-t)\overline{X}(z‘)^* = 0, \qquad z‘(f(Z) - tX) + (1-t)z‘X = 0.$$

Bulardan esa ushbu

$$z‘((\varepsilon - t\|X\|_s)^2 I + (1-t)^2 X\overline{X})(z‘)^* = 0$$

tenglik o‘rinli bo’lishini topamiz, ya‘ni

$$(1-t)\|X\|_s = \varepsilon - t\|X\|_s$$

yoki

$$\|X\|_s = \varepsilon$$

ekan.

Bu esa lemma shatiga zid! Shunday qilib, $|C|$ tashuvchi G_* da yotibdi. Shuning uchun barcha $\delta < \varepsilon - \|X\|_s$ larda $\Gamma_{f-X,\delta}$ sikllar G_* da yotadi va gomologiya sikllar uchun ushbu

$$\Gamma_{f-X,\delta} \sim \Gamma_{f-X,\varepsilon-\|X\|_s} = \partial C - \Gamma_{f,\varepsilon}$$

munosabat o‘rinli ekanini ko‘ramiz, u holda bu oxirgi tenglikdan lemma o‘rinli ekanini topamiz. Lemma isbot bo‘ldi.

Aytaylik $f : D \to G$ akslantirish $D \subset \tilde{\mathbb{C}}_Z\,[n \times n]$ sohani $G \subset \tilde{\mathbb{C}}_W\,[n \times n]$ sohaga golomorf akslantirish bo‘lib, $W^0 \in G$ ixtiyoriy nuqta bo‘lsin. Markazi W^0 nuqtada bo‘lib, G sohada joylashgan, quyidagi $\Omega_r\left(W^0\right)$ uchinchi tur klassik sohani qaraymiz:

$$\Omega_r\left(W^0\right) = \left\{\ W\ :\ r^2 I + \ \left(W - W^0\right)\ \overline{\left(W - W^0\right)}\ >\ 0\ \right\} \Subset G.$$

1-teorema. Agar $H\left(Z\right)$ funksiya D sohada golomorf bo‘lsa, u holda $\Omega_r\left(W^0\right)$ sohada ushbu

$$[Tr\,H]\left(W\right) = c_n \int\limits_{f\,,\,r} \frac{H df}{\det^{\frac{n-1}{2}}\left(f\left(Z\right) - W\right)} (2)$$

integral formula o‘rinli bo‘ladi(juft n larda).

Bunda $\Gamma_{f\,,\,r} = \left\{\ Z \in D\ :\ r^2 I + \left(f\left(Z\right) - W\right)\ \overline{\left(f\left(Z\right) - W\right)} = 0\ \right\}$ va c_n o‘zgarmas bilan differensiallash tartibi quyidagi tenglik bilan aniqlanadi:

$$c_n \int_{\Gamma_{f,r}} \frac{df\left(Z\right)}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)\right)} = 1.$$

Isbot. Teoremani $W^0 = 0$ bo‘lgan hol uchun isbotlaymiz. ([4], 26-sahifa) ishda keltirilgan 1-natijaga ko‘ra deyarli barcha $W \in \Omega_r\left(W^0\right)$ nuqtalarda $f\left(Z\right) - W = 0$ tenglamalar sistemasi oddiy ildizga ega bo‘ladi, ularni $Z^{(1)}(W), \ldots, Z^{(\mu)}(W)$ kabi belgilab olamiz. Aytaylik, $U_\nu \subset D$ atroflar $Z^{(\nu)}(W)$ nuqtalarni keshishmaydigan atroflar oilasi bo‘lsin va U_ν da $\Gamma_\nu = \Gamma_{f(Z)-W,\delta} = \left\{Z \in D : \delta^2 I + \left(f\left(Z\right) - W\right)\overline{\left(f\left(Z\right) - W\right)} = 0\right\}$ –sikl bo‘lsin. U hola, izning ta‘rifi va (1) Xua Lo-ken integral formulasiga ko‘ra ushbu

$$[Tr\,H]\ \left(W\right) = \sum_{\nu=1}^{\mu} c_n \int\limits_{\Gamma_{Z^{(\nu)}(W),\delta}} \frac{H\,df\left(Z\right)}{\det^{\frac{n-1}{2}}\left(f\left(Z\right) - W\right)}.$$

formulaga egamiz.

Bundan tashqari 1-lemmaga ko‘rsa (2) integral ostidagi formaning regulyarlik sohasida $\sum_{\nu=1}^{\mu}\Gamma_\nu$ yig‘indi $\Gamma_{f,r}$ siklda gomologik bo‘ladi. Shuning uchun Stoks formulasiga asosan, quyidagi

$$\sum_{\nu=1}^{n} c_n \int\limits_{\Gamma_{Z^{(\nu)}(W),\delta}} \frac{Hdf}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)-W\right)} = c_n \int\limits_{\Gamma_{f,r}} \frac{Hdf}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)-W\right)}.$$

tenglikni olamiz. Teorema isbot bo‘ldi.

Isbotlangan teoremadan foydalanib, maxsus ko‘rinishdagi meromorf funksiyaning izi uchun integral formulani olamiz.

1-natija. $h(Z)$ funksiya D sohaning yopigida golomorf bo‘lib, $J_f\left(Z\right)-$ chekli turdagi akslantirishning yakobiani bo‘lsin. Unda $\Omega_r\left(W^0\right)$ sohada meromorf bo‘lgan $H=h/J_f$ funksiyaning izi ushbu

$$\left[Tr\, h/J_f\right]\left(W\right) = c_n \int\limits_{\Gamma_{f\,,\,r}} \frac{h\left(Z\right) \bigwedge\limits_{\substack{i=1,j=2\\ i<j}}^{n} dz_{ij}}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)-W\right)} (3)$$

integral formula orqali ifodalanadi(juft n larda).

Isbot. Haqiqatan ham, (2) formuladan $W\in\Omega_r\left(W^0\right)$ uchun ushbu

$$\left[Tr\, h/J_f\right]\left(W\right) = \sum_{\nu} h/J_f\left(Z^{(\nu)}\left(W\right)\right) =$$

$$= \sum_{\nu} c_n \int\limits_{\Gamma_{Z^{(\nu)}(W)\,,\,\delta}} \frac{h \bigwedge\limits_{\substack{i=1,j=2\\ i<j}}^{n} dz_{ij}}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)-W\right)} = c_n \int\limits_{\Gamma_{f\,,\,r}} \frac{h \bigwedge\limits_{\substack{i=1,j=2\\ i<j}}^{n} dz_{ij}}{\det^{\frac{n-1}{2}}\left(f\left(Z\right)-W\right)}.$$

formulani olamiz. Bu esa natijani isbotlaydi.

Hisoblashlarga ko‘ra, (3) integral formulaning yadrosi Z bo‘yicha golomorf, u holda ushbu $\left[\mathrm{Tr}\, h/J_f\right]\left(W\right)$ iz $\Omega_r\left(W^0\right)$ da golomorf funnksiya bo‘ladi. Bu oxirgi natija $\Omega_{f,r}$ matritsaviy poliedrda h/J_f meromorf funskiya uchun Bishop integral formulasni olish imkonini beradi.

2-teorema. Agar $h\left(Z\right)\in\mathcal{O}\left(\Omega_{f,r}\right)\bigcap C\left(\overline{\Omega}_{f,r}\right)$ bo‘lib, $Z\in\Omega_{f,r}$ nuqtada $-J_f\left(Z\right)\neq 0$ bo‘lsa, u holda h/J_f meromorf funksiya uchun

$$\frac{h\left(Z\right)}{J_f\left(Z\right)} = c_n \int\limits_{\Gamma_{f,r}} \frac{h\left(X\right)\Psi\left(Z,\,X\right) \bigwedge\limits_{\substack{i=1,j=2\\ i<j}}^{n} dx_{ij}}{\Psi\left(\,Z\,,Z\,\right)\det^{\frac{n-1}{2}}\left(f\left(X\right)-f\left(Z\right)\right)}$$

integral formula o‘rinli bo‘ladi.

Isbot. Izning ta‘rifiga asosan, (3) integral $W=f(X)$ bo‘lgandagi qiymati h/J_f funksiyaning $Z=X$ nuqtadagi qiymati qo‘shilgan $X^\nu\left(Z\right),\ \nu=2,...,\mu$ nuqtalardagi qiymatlarining yig’indisiga teng. Bu yerda $X^\nu\left(Z\right),\ \nu=2,...,\mu$ nuqtalar $W=f(X)$ nuqtadagi proobrazlardir. Endi, quyidagi shartni qanoatlantiruvchi vaznli $\Psi(X,Z)\neq 0$ funksiyani tuzamiz: ushbu

$$\Omega_{f,r} = \left\{\, Z\in D\ :\ r^2 I^{(n)} + f\left(Z\right)\overline{f\left(Z\right)} > 0 \right\} \Subset D$$

matritsaviy poliedrdan olingan barcha fiksirlangan X lar uchun $\Psi(X,Z)$ funkisya X nuqtadan boshqa $Z=X^{(\nu)}\left(f\left(X\right)\right)$ nuqtalarda nolga teng.

Bunday funksiya mavjud. Faraz qilaylik, W^0- f akslantirishning kritik bo‘lmagan qiymati va $g(z)-$chiziqli funksiya bo‘lsin, ya‘ni $g(X^\nu(W^0))-$turlicha bolsin. U holda quyidagicha determinant funksiyani aniqlashumiz mumkin

$$\Psi(\xi,z) = \prod_{\nu=2}^{\mu}\left[g(z^{(\nu)}) - g(\xi)\right] (4)$$

bu yerda tartib raqami $X^{(\nu)} = X^{(\nu)}(Z)$ asllar shunday tartiblanganki bunda $X^{(1)}(Z) = Z$ bo‘ladi. Ravshanki, $[g(X^{(2)}(Z))]^k + \ldots + ... + [g(X^{(\mu)}(Z))]^k$ yig‘indi, $[g(Z)]^k$golomorf bo‘lgani uchun golomorf bo‘ladi. Shundya qilib, (4) munosabatning birinchi ko‘paytuvchisi koeffitsiyentlari Z ga bog‘liq $g(X)$ ga bog‘liq ko‘phad bo‘ladi. Bu ifodaning ikkinchi ko‘paytuvchisi esa birnchi ko‘paytuvchidan simmetrik qilib $X = Z$ bo‘lganda $g(Z^{(\nu)}(f(Z))), \nu = 2, ..., \mu$ bo‘lganda birinchi ko’paytuvchidan olinadi. Bundan esa uning Z bo‘yicha golomorfligi kelib chiqadi. Endi X va Z o‘zgaruvchilarni o‘rnini almashtirib, ushbu

$$\Psi(Z, X) = \sum_{k=1}^{\mu-1} c_k(X) g^k(Z),$$

munosabatni olamiz, bu yerda $c_k(X)$ koeffsitsiyentlar $\overline{\Omega}_{f,r}$ poliedr yopig‘ida golomorf funksiyalardir. Tuzilishiga ko‘ra $X^{(\nu)}(Z) \neq Z$ nuqtalar uchun $\Psi(Z, X^{(\nu)}(Z)) = 0$ bo‘ladi.

Yuqoridagi natija va $\Psi(Z, X)$ vaznli funksiayning qurilishi $\Omega_{f,r}$ sohadagi h/J_f meromorf funksiyalar uchun Bishop integral formulasini olish imkonini beradi. Haqiqatdan ham,

$$\sum_{v} \frac{h\left(Z^{(v)}(X)\right) \Psi\left(Z, Z^{(v)}(X)\right)}{J_f\left(Z^{(v)}(X)\right)} =$$

$$= \frac{h(Z)\Psi(Z, Z)}{J_f(Z)} + \frac{h\left(Z^{(2)}(X)\right) \Psi\left(Z, Z^{(2)}(X)\right)}{J_f\left(Z^{(2)}(X)\right)} + \ldots =$$

$$= \frac{h(Z)\Psi(Z, Z)}{J_f(Z)} = \int_{\Gamma_{f,r}} \frac{h(X)\Psi(Z, X) \bigwedge\limits_{\substack{i=1,j=2 \\ i<j}}^{n} dx_{ij}}{\det^{\frac{n-1}{2}}(f(Z) - f(X))}.$$

Teorema isbot bo‘ldi.

ADABIYOTLAR RO‘YXATI:

1. Хуа Ло-кен. Гармонический анализ функций многих комплексных переменных в классических областях, М., изд. инст. лит., 1959.
2. Айзенберг Л.А. Формула Карлемана в комплексном анализе. Первые приложения, “Наука“, Новосибирск, 1990, 248 с.
3. Худайберганов Г., Кытманов А.М., Шаимкулов Б.А. Анализ в матричных областях, Монография. Красноярск, Ташкент, 2017, с. 293
4. Цих А.К., Многомерные вычеты и их применения, Новосибирск: Наука, 1988. -240 с.

Rezyume

In this paper, a matrix polyhedron is defined using the classical domain of the third type. In the defined polyhedron, the homological closedness of cycles and the Bishop integral formula are obtained.

Key words: Skew-symmetric matrices, classical domain of the third type, matrix polyhedron, holomorphic functions and mappings of matrix argument, homological cycles, integral formula, Bishop integral formula.

Rezyume

В данной статье с помощью классической области третьего типа определен матричный полиэдр. В определённом полиэдре получена гомологическая замкнутость циклов, а также выведена интегральная формула Бишопа.

Ключевые слова: кососимметрические матрицы, классическая область третьего типа, матричный полиэдр, голоморфные функции и отображения матричного аргумента, гомологические циклы, интегральная формула, интегральная формула Бишопа.