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# IKKINCHI TIP KLASSIK SOHALARDA LAPLAS ALMASHTIRISHI, TESKARI LAPLAS ALMASHTIRISH FORMULASI VA TASVIR FUNKSIYANING GOLOMORFLIGI HAQIDAGI TEOREMANING ANALOGI

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**ANNOTATSIYA:**

Ushbu maqolada biz Operatsion hisobning asosiy tushunchalari tasvir va original funksiyalar orasidagi bog‘lanishlarni, xususan ikkinchi tip klassik sohalar uchun Laplas almashtirishi, teskari Laplas almashtirishi hamda muhim teoremalaridan biri bo‘lgan, ya’ni *matritsaviy tasvir funksiyaning golomorfligi haqidagi* teoremaning analogini quramiz. Buning uchun eng avvalo asosiy ta’rif va tushunchalarni kiritib olamiz. Ma’lumki klassik sohalar bir-biri bilan o‘zaro bigolomorf ekvivalent munosabatga ega emas, shu sababli ularning har biri uchun kompleks analiz alohida-alohida quriladi. Shuning uchun bu maqolada faqat simmetrik Ermit matritsalar sinfiga tegishli bo‘lgan ikkinchi tip klassik sohalarda Laplas almashtirishining analoglarini olish bilan shug‘ullanamiz. Keyingi ilmiy izlanishlarimizda birinchi tip to‘g‘ri to‘rtburchak matritsalar sinfiga tegishli va uchinchi tip antisimmetrik matritsalar sinfiga tegishli matritsa-funksiyalar uchun Laplas almashtirishining analoglarini olishga harakat qilamiz.

***Kalit so’zlar:*** bir jinsli soha, simmetrik soha, keltirilmaydigan soha, klassik soha, ikkinchi tip klassik soha, matritsaviy original, matritsaviy tasvir, matritsa-funksiya, matritsa izi, Laplas almashtirishi, tasvirning golomorfligi haqidagi teorema, teskari Laplas almashtirish formulasi, matritsaviy o‘ng yarim tekislik, Koshi integral formulasi, golomorf funksiya, originalning yagonaligi haqidagi teorema.

Fransuz matematigi E. Kartan tomonidan 1935-yilda keltirilmaydigan, chegaralangan, simmetrik sohalar sinflarining oltita tipi borligi ko‘rsatilgan [1]. Shu sinflardan dastlabki to‘rttasiga tegishli bo‘lgan sohalar klassik sohalar deyiladi [2].

**1-ta’rif.** Agar $D \subset \mathbb{C}^n$ sohaning avtomorfizmlar gruppasi tranzitiv, ya’ni ixtiyoriy $z_1,\ z_2 \in D$ uchun $\varphi(z_1) = z_2$ shartni qanoatlantiruvchi shunday $\varphi \in Aut(D)$ avtomorfizm mavjud bo‘lsa, u holda $D \subset \mathbb{C}^n$ soha **bir jinsli** soha deyiladi.

**2-ta’rif.** Agar bir jinsli $D \subset \mathbb{C}^n$ sohadagi ixtiyoriy $\zeta \in D$ nuqtasi uchun quyidagi shartlarni qanoatlantiruvchi shunday $\varphi \in Aut(D)$ avtomorfizm mavjud bo‘lsaki:
1) $\varphi(\zeta) = \zeta$ tenglik o‘rinli bo‘lsin, biroq $\zeta$ nuqtadan farqli $z \in D$ nuqtalar uchun $\varphi(z) \neq z$ bo‘lsin;
2) $\varphi \circ \varphi = e$, bu yerda $e \in Aut(D)$ ayniy akslantirish; u holda $D \subset \mathbb{C}^n$ **simmetrik soha** deyiladi.

**3-ta’rif.** Agar $D \subset \mathbb{C}^n$ sohani o‘lchami undan kichik chegaralangan simmetrik sohalarning to‘g‘ri kopaytmasi ko‘rinishida tasvirlash mumkin bo‘lmasa, u holda $D \subset \mathbb{C}^n$ **keltirilmaydigan soha** deyiladi.

**4-ta’rif.** Agar chegaralangan $D \subset \mathbb{C}^n$ sohaning avtomorfizmlar gruppasi tranzitiv bo‘lsa va Li gruppasi tashkil qilsa u holda $D \subset \mathbb{C}^n$ sohaga **klassik soha** deyiladi.

Quyida E. Kartan tasniflagan klassik sohalarni keltiramiz [2]:

$$\Re_I(m,k) = \left\{ Z \in \mathbb{C}[m,k] : \ I^{(m)} - ZZ^* > 0 \right\},$$

$$\Re_{II}(m) = \left\{ Z \in \mathbb{C}[m,m] : \ I^{(m)} - Z\bar{Z} > 0,\ \forall Z' = Z \right\},$$

$$\Re_{III}(m) = \left\{ Z \in \mathbb{C}[m,m] : \ I^{(m)} + Z\bar{Z} > 0,\ \forall Z' = -Z \right\},$$

$$\Re_{IV}(n) = \left\{ Z \in \mathbb{C}^n : \ |\langle z,z\rangle|^2 - 2|z|^2 + 1 > 0,\ |\langle z,z\rangle| < 1 \right\},$$

bu yerda $I^{(m)}$ $m$-tartibli birlik matritsa, $Z^*$ matritsa esa matritsaning qo‘shmasi va transponirlanganidir ($H$ Ermit matritsasi uchun: $H > 0$ belgi uning musbat aniqlangan matritsa ekanligini bildiradi, ya’ni uning barcha xos sonlari musbat: $\det|\lambda I - H| = 0 \Rightarrow \forall\lambda_i > 0$).

Bu sohalarning har biri markazi $O$ ($m$-tartibli nol matritsa) nuqtada bo‘lgan bir jinsli, simmetrik, keltirilmaydigan, to‘la doiraviy qavariq sohalardir. Bu sohalar bir-birlari bilan o‘zaro bigolomorf ekvivalent munosabat o‘ranatmaydi, shu sababli ularning har biri uchun kompleks analiz alohida quriladi.

Biz ushbu tadqiqot ishimizni $\Re_{II}(m) = \left\{Z \in \mathbb{C}[m,m] : I^{(m)} - Z\bar{Z} > 0, \forall Z' = Z\right\}$ ikkinchi tip klassik sohalarda olib boramiz.

Bizga $f : S_m \to S_m$ $(A \in S_m \subset \mathbb{R}[m \times m] \,|\, f(A) = f(UAU') \in S_m)$ (bu yerda, $S_m$-haqiqiy simmetrik matritsalar sinfi, $UU' = I$-ortogonal matritsalar) matritsa argumentli simmetrik matritsa-funksiya berilgan bo‘lsin [4].

**5-ta’rif.** Quyidagi shartlarni qanoatlantiruvchi $f(A)$ funksiyaga **matritsaviy original** deyiladi:

I. $A < 0$ da $f(A) \equiv 0$ (bu yerda, $A < 0$ munosabatni $A$ matritsaning har bir elementi 0 dan kichik deb tushuniladi);

II. $\forall A \in S_m$ uchun $f(A)$ matritsa-funksiya haqiqiy matritsa argumentli matritsaviy o‘ng yarim tekislikdan olingan $\Upsilon = \{A = (a_{ij}) \in \mathbb{R}[m \times m] : \forall a_{ij} \geq 0\}$ sohada uzluksiz yoki bo‘lakli uzluksiz (ya’ni $\forall X_0 \in \Upsilon$ : $\lim\limits_{X \to X_0} f(X) = f(X_0)$ yoki $n_0$ $(n_0 \leq m)$ −chekli sondagi I tur uzilish nuqtalariga ega);

III. $\forall A \in S_m$ uchun $\exists M > 0$ va $\alpha \geq 0$ sonlar topiladiki $|f(A)| \leq M \cdot e^{\alpha A}$ tengsizlik o‘rinli bo‘ladi (bu yerda, $|f(A)|$ deb har bir elementi $f(A)$ simmetrik matritsa-funksiya elementining moduliga teng deb tushuniladi).

Endi yuqorida berilgan 5-ta’rif asosida matritsaviy tasvirning ta’rifini keltiramiz.

**6-ta’rif.** $f(A)$ matritsaviy originalning ikkinchi tip klassik sohada aniqlangan **matritsaviy tasviri** deb, $Z = X + iY$ $(Z \in \Re_{II}(m))$ o‘zgaruvchining

$$F(Z) = \mathcal{L}_Z\{f(A)\} = \int\limits_{A>0} e^{-Sp(ZA)} f(A)\, dA \tag{1}$$

integrali bilan aniqlanadigan matritsa argumentli funksiyaga aytiladi. Bu yerda, $A = (a_{ij})$ uchun $dA = \prod\limits_{i \leq j} da_{ij}$ kabi aniqlanadi, $Sp(ZA) = \sum\limits_{i \geq j} z_{ij} \cdot a_{ij}-$ matritsa izi.

**1-izoh.** (1) munosabatda $Z = (\eta_{ij} z_{ij})$ kabi aniqlanadigan kompleks parametrik matritsa bo‘lib, matritsa elementlarining old koeffitsientlari ($\eta_{ij}$ o‘zgarmas koeffitsientlar) quyidagicha aniqlanadi [5]:

$$\eta_{ij} = \begin{cases} \frac{1}{2}, & i \neq j \\ 1, & i = j, \end{cases} \tag{2}$$

**7-ta’rif.** (1) formula orqali matritsaviy originaldan matritsaviy tasvirga o‘tishga ikkinchi tip klassik sohada $\Re_{II}(m)$ aniqlangan **Laplas almashtirishi** deyiladi.

**2-izoh.** $f(A)$ matritsaviy original bilan $F(Z)$ matritsaviy tasvir orasidagi moslik $F(Z) \dot{\to} f(A)$ yoki $f(A) \dot{\leftarrow} F(Z)$ kabi belgilanadi [10-12]. Bu yerda, “ $\dot{\to}$ ” belgining yo‘nalishi umumiylikka zid kelmagan holda har doim matritsaviy tasvirdan matritsaviy originalga tomon yo‘nalgan bo‘ladi. Shuningdek, $\mathcal{L}_Z\{f(A)\} = F(Z)$ kabi belgilash munosabatidan ham foydalaniladi [4-5].

**1-teorema** (*tasvirning golomorfligi haqidagi teorema*). Agar $f(A)$ matritsaviy original uchun (1) Laplas almashtirish munosabati bilan aniqlangan $F(Z)$ matritsaviy tasvir funksiya mavjud bo‘lsa: $\mathcal{L}_Z\{f(A)\} = F(Z)$, u holda $F(Z)$ funksiya matritsaviy o‘ng yarim tekislikdan $\Upsilon = \{Z \in \Re_{II}(m) : \operatorname{Re} Z = X > X_0 > 0\}$ olingan $Z$ matritsa o‘zgaruvchining golomorf funksiyasi bo‘ladi.

**Isbot.** Faraz qilaylik $\forall Z \in \Upsilon = \{Z \in \Re_{II}(m) : \operatorname{Re} Z = X > X_0 > 0\}$ nuqtada $F(Z)$ funksiya (1) Laplas almashtirishi yordamida aniqlangan $f(A)$ matritsaviy originalning tasvir funksiyasi berilgan bo‘lsin. Ya’ni $\mathcal{L}_Z\{f(A)\} = F(Z)$ bo‘lsin.

Ma’lumki matritsaviy-eksponentsial $e^{Sp(-ZA)}$ ham o‘z navbatida original funksiya tashkil qilib, u uchun quyidagi baholash o‘rinli bo‘ladi:

Fikserlangan $Z \in \Re_{II}(m)$ hamda $\forall A \in S_m \subset \mathbb{R}[m \times m]$ uchun $\exists C > 0$ va $\frac{\alpha}{2} \geq 0$ haqiqiy sonlar topiladiki, $\sigma := \min\limits_i \sigma_i > \frac{\alpha}{2}$ (bu yerda $\sigma_i = \lambda_i(Z) - Z$ matritsaning xos sonlari) son uchun

$$\left|e^{-Sp(ZA)}\right| \leq C \cdot e^{-\left(\sigma - \frac{\alpha}{2}\right) \cdot Sp(A)}. \tag{3}$$

Ikkinchi tomondan $f(A)$ matritsaviy original haqidagi 5-ta’rif III shartga ko‘ra $\forall A \in S_m \subset \mathbb{R}[m \times m]$ uchun $\exists M > 0$ va $\frac{\alpha}{2} \geq 0$ sonlar topiladiki

$$|f(A)| \leq M \cdot e^{\frac{\alpha}{2} \cdot Sp(A)} \tag{4}$$

baholash o‘rinli bo‘ladi. (3) va (4) baholashlarga ko‘ra:

$$\begin{gathered}\left|f(A) \cdot e^{-Sp(ZA)}\right| \leq |f(A)| \cdot C \cdot e^{-\left(\sigma - \frac{\alpha}{2}\right) \cdot Sp(A)} \leq \\ M \cdot e^{\frac{\alpha}{2} \cdot Sp(A)} \cdot C \cdot e^{-\left(\sigma - \frac{\alpha}{2}\right) \cdot Sp(A)} = K \cdot e^{-(\sigma - \alpha) \cdot Sp(A)},\end{gathered} \tag{5}$$

bu yerda $M \cdot C = K$.

Oxirgi hosil qilingan $e^{-(\sigma - \alpha) \cdot Sp(A)}$ majorant funksiya integrali:

$$\int\limits_{A>0} e^{-(\sigma - \alpha) Sp(A)}\, dA < +\infty$$

yaqinlashuvchi, bundan kelib chiqadiki (1) integral absolyut va tekis yaqinlashuvchi ekan. Bu esa bizga (1) integralni differensiallasak integral ostiga differensialni kiritish imkonini beradi:

$$\begin{gathered}\frac{\partial}{\partial Z} F(Z) = \frac{\partial}{\partial Z}\left[\int\limits_{A>0} e^{-Sp(ZA)} f(A)\, dA\right] = \int\limits_{A>0} \frac{\partial}{\partial Z} e^{-Sp(ZA)} f(A)\, dA = \\ = \left[\frac{\partial}{\partial Z} Sp(ZA) = A\right] = -\int\limits_{A>0} A \cdot e^{-Sp(ZA)} f(A)\, dA.\end{gathered}$$

Demak, $F(Z)$ tasvir funksiyadan $\forall Z \in \Upsilon = \{Z \in \Re_{II}(m) : \operatorname{Re} Z = X > X_0 > 0\}$ nuqtada kompleks analiz ma’nosida differensial mavjud. $Z$ ixtiyoriy ekanligidan $F(Z)$ matritsaviy tasvir funksiya $\Upsilon = \{Z \in \Re_{II}(m) : \operatorname{Re} Z = X > X_0 > 0\}$ sohada golomorf bo‘ladi. Teorema isbotlandi.

**2-teorema** (*teskari Laplas almashtirish formulasi*). Agar ikkinchi tip klassik sohadan $\Re_{II}(m) = \left\{Z \in \mathbb{C}[m \times m] : I^{(m)} - Z\overline{Z} > 0, \ \ \forall Z' = Z\right\}$ olingan $F(Z)$ matritsaviy tasvir funksiya

$$\int\limits_{-\infty}^{+\infty} |F(X + iY)|\, dY < +\infty \tag{6}$$

ixtiyoriy $X > X_0 > 0$ uchun

$$\lim_{X \to 0} \int\limits_{-\infty}^{+\infty} |F(X + iY)|\, dY = 0 \tag{7}$$

bo‘lib, $\mathcal{L}_Z\{f(A)\} = F(Z)$ bo‘lsa, u holda $f(A)$ differensiallanuvchi bo‘lgan har bir nuqtada

$$f(A) = \frac{2^{\frac{1}{2}m(m-1)}}{(2\pi i)^{\frac{1}{2}m(m+1)}} \int\limits_{\operatorname{Re} Z>0} e^{Sp(ZA)} F(Z) dZ \tag{8}$$

munosabat o‘rinli bo‘ladi. Quyidagicha belgilanadi: $\mathcal{L}^{-1}(F(Z)) = f(A)$.

**3-izoh.** Shuni ta’kidlashimiz lozimki, (8) integral munosabatda $Z = (\eta_{ij}\, z_{ij})\,, \ \ \eta_{ij} = 1$ deb qaralmoqda.

**Isbot.** Teorema shartiga ko‘ra $\mathcal{L}_Z\{f(A)\} = F(Z)$, bundan kelib chiqadiki teskari Laplas almashtirish munosabati uchun quyidagi munosabat o‘rinli bo‘lishi kerak:

$$\mathcal{L}^{-1}(\mathcal{L}_Z\{f(A)\}) = \mathcal{L}^{-1}(F(Z)) \Rightarrow f(A) = \mathcal{L}^{-1}(F(Z))\,. \tag{9}$$

(9) munosabatga ko‘ra:

$$f\left(A\right)=\mathcal{L}^{-1}\left(\int\limits_{A>0} e^{-Sp(ZA)}f\left(A\right)dA\right). \qquad (10)$$

Yuqorida isbot qilingan 1-teoremadan kelib chiqadiki $F\left(Z\right)$ matritsaviy tasvir funksiya ikkinchi tip klassik sohada aniqlangan matritsaviy o‘ng yarim tekislikda $\Upsilon=\{Z\in\Re_{II}\left(m\right):\ \mathrm{Re}\,Z=X>X_0>0\}$ golomorf, ikkinchi tomondan teorema shartlari (6) va (7) ga ko‘ra sohaning yopig‘ida $\overline{\Upsilon}$ uzluksiz bo‘ladi. Ya’ni $F\left(Z\right)\in\mathcal{O}\left(\Upsilon\right)\cap C\left(\overline{\Upsilon}\right)$. Bu esa bizga matritsa argumentli funksiyalar uchun Koshi integral formulasini qo‘llash imkonini beradi:

$$\frac{1}{\left(2\pi i\right)^{\frac{1}{2}m(m+1)}}\int\limits_{\mathrm{Re}\,Z>0} e^{Sp(ZA)}F(Z)dZ=\begin{cases} f\left(A\right), & agar\ A>0\\ O, & agar\ A\leq 0.\end{cases} \qquad (11)$$

Demak, (10) va (11) munosabatlardan kelib chiqadiki:

$$f(A)=\frac{1}{\left(2\pi i\right)^{\frac{1}{2}m(m+1)}}\int\limits_{\mathrm{Re}\,Z>0} e^{Sp(ZA)}F(Z)dZ. \qquad (12)$$

Agar (2) formulada $Z=\left(\eta_{ij}\,z_{ij}\right)$ uchun $\eta_{ij}=1$ deb olsak, u holda (15) integralda $2^{\frac{m(m+1)}{2}}$ o‘zgarmas koeffitsient hosil bo‘ladi. U holda quyidagi integral munosabatga ega bo‘lamiz:

$$f(A)=\frac{2^{\frac{1}{2}m(m-1)}}{\left(2\pi i\right)^{\frac{1}{2}m(m+1)}}\int\limits_{\mathrm{Re}\,Z>0} e^{Sp(ZA)}F(Z)dZ.$$

Teorema isbotlandi.

**4-izoh.** Shuni ham ta’kidlash lozimki, $\mathcal{L}^{-1}\left(F(Z)\right)=f\left(A\right)$ munosabat bilan $\mathcal{L}_Z\left\{f\left(A\right)\right\}=F(Z)$ munosabatlar o‘zaro teng kuchli munosabatlardir: $\mathcal{L}_Z\left\{f\left(A\right)\right\}=F(Z)\Leftrightarrow\mathcal{L}^{-1}\left(F(Z)\right)=f\left(A\right)$.

**1-lemma.** $F(Z)\in\Re_{II}\left(m\right)$ matritsaviy tasvir uchun quyidagi chiziqlilik munosabatlari o‘rinli bo‘ladi:

I. agar $c=const$ va $\mathcal{L}_Z\left\{f\left(A\right)\right\}=F(Z)$ bo‘lsa, u holda

$$\mathcal{L}_Z\left\{\,f\left(A\right)\right\}=c\,F(Z), \qquad (13)$$

II. agar $\mathcal{L}_Z\left\{f_1\left(A\right)\right\}=F_1(Z)$ va $\mathcal{L}_Z\left\{f_2\left(A\right)\right\}=F_2(Z)$ bo‘lsa, u holda

$$\mathcal{L}_Z\left\{f_1\left(A\right)+f_2\left(A\right)\right\}=F_1(Z)+F_2(Z). \qquad (14)$$

**Isbot.** Lemmaning isboti bevosita integrallashda o‘zgarmasni integral belgisidan tashqariga chiqarish va chiziqlilik xossalaridan kelib chiqadi.

**3-teorema** (*originalning yagonaligi haqidagi teorema*). Agar $F(Z)\in\Re_{II}\left(m\right)$ matritsaviy tasvir funksiya $f_1(A)$ va $f_2(A)$ matritsaviy orginallarning tasviri bo‘lsa, u holda bu originallar o‘zlarining barcha uzluksiz nuqtalarida ustma-ust tushadi. Ya’ni $f_1(A)\equiv f_2(A)$ tenglik o‘rinli bo‘ladi.

**Isbot.** Teorema shartiga ko‘ra: $\mathcal{L}_Z\left\{f_1\left(A\right)\right\}=F(Z)$ va $\mathcal{L}_Z\left\{f_2\left(A\right)\right\}=F(Z)$. Matritsaviy tasvir funksiyaning chiziqlilik xossasidan foydalanib quyidagi munosabatni hosil qilamiz: $\mathcal{L}_Z\left\{f_1\left(A\right)-f_2\left(A\right)\right\}=F(Z)-F(Z)=0$. Bundan kelib chiqadiki $f_1(A)\equiv f_2(A)$. Teorema isbotlandi.

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**Rezyume**

In this article, we will consider the basic concepts of operational calculus, the connections between images and original functions, in particular, the Laplace transform, the inverse Laplace transform, and one of its important theorems, namely *the theorem on the holomorphism of a matrix image function*, for classical domains of the second type. To do this, we first introduce the basic definitions and concepts. It is known that classical domains do not have a biholomorphic equivalence relation with each other, therefore, a complex analysis is constructed separately for each of them. Therefore, in this article, we will only deal with obtaining analogues of the Laplace transform in classical domains of the second type, which belong to the class of symmetric Hermitian matrices. In our further scientific research, we will try to obtain analogues of the Laplace transform for matrix-functions belonging to the class of rectangular matrices of the first type and the class of antisymmetric matrices of the third type.

***Key words:*** homogeneous domain, symmetric domain, irreducible domain, classical domain, classical domains of the second type, matrix original, matrix image, matrix function, matrix trace, Laplace transform, holomorphic image theorem, inverse Laplace transform formula, right half-plane of a matrix, Cauchy integral formula, holomorphic function, theorem on the uniqueness of the original image.

**Резюме**

В данной статье мы рассмотрим основные понятия операционального исчисления, связи между образами и исходными функциями, в частности, преобразование Лапласа, обратное преобразование Лапласа и одну из его важных теорем, а именно *теорему о голоморфизме матричной функции-образа*, для классических областях второго типа. Для этого сначала введем основные определения и понятия. Известно, что классические области не имеют биголоморфного отношения эквивалентности друг с другом, поэтому комплексный анализ строится отдельно для каждого из них. Следовательно, в данной статье мы будем рассматривать только получение аналогов преобразования Лапласа в классических областях второго типа, принадлежащих классу симметричных Эрмитовых матриц. В наших дальнейших научных исследованиях мы попытаемся получить аналоги преобразования Лапласа для матричных функций, принадлежащих классу прямоугольных матриц первого типа и классу антисимметричных матриц третьего типа.

***Ключевые слова:*** однородная область, симметричная область, неприводимая область, классическая область, классических областях второго типа, матричный оригинал, матричный образ, матричная функция, след матрицы, преобразование Лапласа, теорема о голоморфном образе, формула обратного преобразования Лапласа, правая полуплоскость матрицы, формула интеграла Коши, голоморфная функция, теорема о единственности исходного изображения.