A Simple Pursuit-Evasion Problem for Differential Games with Time Delay

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Abstract. In this article, pursuit-evasion differential games with simple motion and time delay are studied under geometric constraints on the controls of both players. Depending on the initial states of the players and the parametric values involved in the control constraints, the problem is analyzed accordingly. To solve the pursuit problem, a parallel pursuit strategy ($\Pi-$strategy) is proposed, which ensures the best possible convergence of the players, and its structure is examined with respect to the parameters. For the considered class of differential games, sufficient conditions for the solvability of both the pursuit and evasion problems are obtained.

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Key words: differential game, pursuit problem, evasion problem, $\Pi-$strategy, resolving function, control, time delay, convergence function, guaranteed time.

Introduction

The study of differential games was initiated by the American mathematician R. Isaacs. His research was published in the form of a monograph [1] in 1965, in which a large number of examples were considered, while theoretical issues were addressed only partially. Since the 1960s, differential games have become one of the main directions of scientific research, and their fundamental results were obtained by Isaacs [1], Pontryagin [2], Krasovskii [3], Petrosyan [4], and others.

In IsaacsвЂ™ monograph [1], a number of game-theoretic problems were considered, discussed in detail, and proposed for further investigation. A simplified analytical solution of this problem in the half-plane was presented by Isaacs in [1].

In the case where the controls of both players are subject to geometric constraints, this game was studied in considerable detail by Petrosyan [4], based on the approximation of measurable controls by the most effective piecewise-constant controls implementing a strategy of parallel approach. Later, this approach to control in differential pursuit games became known as the $\Pi-$strategy.

The strategy proposed in [4] for a simple pursuit game with geometric constraints served as a starting point for the development of pursuit methods in games with several pursuers [5]. Subsequently, for cases where the controls of both players are subject to integral, Gronwall-type, or mixed constraints, the game was investigated in the works of Samatov [5вЂ“6].

In [11], it is proved that under the stated conditions, as well as an additional condition imposed on the parameters of the game, the pursuit can be completed in an arbitrarily small neighborhood of the terminal set. To ensure the completion of the game, an ε-positional pursuit strategy is constructed.

Based on the fundamental approaches to the theory of differential games developed by Pontryagin [2] and Krasovskii [3], a differential game is viewed as a control problem from the standpoint of either the pursuer or the evader. According to this approach, the game is reduced either to a pursuit (approach) problem or to an evasion (escape) problem.

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The principal method for solving pursuit and evasion problems consists in constructing optimal strategies for the players and determining the value of the game. Papers [7-9] are devoted to the study of differential games of simple motion, in which the existence of the value of the game was established by means of optimal strategies of the players.

In [10], pursuit game problems described by a system of delay differential equations under integral constraints on the players controls are studied. The proposed scheme is based on the ideas of the resolving function method. Modifications of the methods (namely, the first and the so-called third methods) of pursuit are proposed for the case where integral constraints are imposed on the players' controls. Sufficient conditions are obtained for the possibility of completing the pursuit in finite time.

In [12], the pursuit problem for neutral-type differential-difference equations is investigated. A new sufficient condition for the solvability of this problem is obtained, and a model example illustrating the result is presented.

In [13], issues of strong and weak invariance of a constant multivalued mapping are studied for a boundary value problem of heat conduction in the presence of time delay. In this setting, the control parameter appears in the right-hand side of the equation, and its control action has an impulsive character, which is represented by the Dirac delta function. The obtained conditions differ from previously known results established for control problems with delay.

In the present paper, pursuitвЂ“evasion problems are considered under simple motion of the players with delay. Geometric constraints are imposed on the playersвЂ™ controls. To solve the pursuit problem, a $\Pi-$strategy of the pursuer is proposed. Furthermore, a sufficient condition for the realizability of pursuit is formulated, and the guaranteed pursuit time is determined.

Problem Formulation

Let an object P, called the *pursuer*, chase another object E, called the *evader*, in the space $\mathbb{R}^n$.

Denote the state vector of the pursuer by x, and the state vector of the evader by y. In the present work, we consider a pursuitвЂ“evasion problem whose dynamical properties are described by the equations [15]:

$$P: \qquad \dot{x}(t) = x(t-h) + u(t), \tag{1}$$

$$E: \qquad \dot{y}(t) = y(t-h) + v(t), \tag{2}$$

where $x(t), y(t), u(t), v(t) \in \mathbb{R}^n, t \geq 0, n \geq 1$ and h is the time delay.

The initial conditions for system (1)вЂ“(2) are given respectively by

$$x_0(t) = \varphi(t)x_0, \qquad y_0(t) = \varphi(t)y_0,$$

where $\varphi(t)$ is an absolutely continuous function satisfying

$$\varphi(t) > 0 \quad \text{for all } t \in [-h, 0],$$

and $x_0 \neq y_0$.

The set of all measurable functions $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}^n$ (the controls of player P) satisfying the condition

$$|u(t)| \leq \rho \quad \text{for } t \geq 0, \tag{3}$$

is denoted by U.

Similarly, the set of all measurable functions $v(\cdot) : \mathbb{R}_+ \to \mathbb{R}^n$ (the controls of player E) satisfying the condition

$$|v(t)| \leq \sigma \quad \text{for } \ t \geq 0, \tag{4}$$

is denoted by V.

In the theory of differential games, inequalities of the form (3) and (4) are called *geometric constraints* (briefly, $G-$constraints). In these definitions, ρ and σ are nonnegative numerical parameters.

Definition 1. A measurable function $u(\cdot)$ (respectively, $v(\cdot)$) satisfying condition (3) (respectively, condition (4)) is called an *admissible control* of the pursuer (respectively, the evader) belonging to class U (respectively, class V). The pair of admissible control classes (U, V) thus defines a differential game.

Introduce the notation [15]

$$\xi(t,\varphi(\cdot)) = \eta(t)\varphi(0) + \int_{-h}^{0} \eta(t-h-s)\varphi(s)ds,$$

where

$$\eta(t) = \begin{cases} 0 & \text{for } t<0, \\ \sum\limits_{j=0}^{\nu} \frac{(t-hj)^j}{j!} & \text{for } \nu h \le t \le (\nu+1)h,\ \nu = 0,1,2,\dots. \end{cases}$$

We present several necessary properties of the function $\eta(t)$ [16вЂ“18].

Property 1. *Integrability:*

$$\int_{t_1}^{t_2} \eta(s), ds = \eta(s+h)\Big|_{t_1}^{t_2}.$$

Property 2. *Differentiability:*

$$\eta'(t) = \eta(t-h).$$

By virtue of equations (1)-(2), each pair $(x_0(\cdot), u(\cdot))$, consisting of an initial position $x_0(\cdot)$ and a control function $u(\cdot) \in U$ (respectively, $(y_0(\cdot), v(\cdot))$, with $v(\cdot) \in V$), generates a trajectory given by the formulas

$$x(t) = \xi(t,\varphi(\cdot))x_0 + \int_0^t \eta(t-s)u(s)ds, \tag{5}$$

$$y(t) = \xi(t,\varphi(\cdot))y_0 + \int_0^t \eta(t-s)v(s)ds, \tag{6}$$

respectively.

The main objective of the pursuer P is to capture the evader E, that is, to achieve the equality

$$x(t^*) = y(t^*)$$

at some time $t^* > 0$. The objective of the evader is to ensure that the inequality

$$x(t) \neq y(t)$$

holds for all $t \geq 0$.

It is well known that control functions of the pursuer P alone are insufficient for solving the pursuit problem, since they depend only on the time parameter t, $t \geq 0$. Therefore, appropriate types of controls should be formulated in terms of strategies. There exist various approaches to defining this concept. For our purposes, the following formulation is sufficient.

First, we introduce the following notation:

$$z(t) = x(t) - y(t), \quad t \geq 0,$$

$$z_0(t) = x_0(t) - y_0(t) = \varphi(t)(x_0 - y_0) = \varphi(t)z_0, \quad t \in [-h, 0].$$

In what follows, it is assumed that the initial functions $x_0(\cdot)$ and $y_0(\cdot)$ are specified such that

$$x_0(t) \neq y_0(t), \quad t \in [-h, 0],$$

that is, $x_0 \neq y_0$.

Definition 2. A mapping $\mathbf{u} : \mathbb{R}^n \times V \to U$ is called a strategy of the pursuer if the following conditions are satisfied:

a) For every $v(\cdot) \in V$, the inclusion $\mathbf{u}(z_0(\cdot), v(\cdot)) \in U$ holds on some time interval $[0, T]$. The function $\mathbf{u}(z_0(\cdot), v(\cdot)), \quad t \geq 0$, is called the realization of the strategy corresponding to $v(\cdot) \in V$.

b) If for $v_1(\cdot), v_2(\cdot) \in V$ the equality $v_1(t) = v_2(t)$ holds almost everywhere on $[0, T]$, then $u_1(t) = u_2(t)$ holds almost everywhere on $[0, T]$, where $u_i(\cdot) = \mathbf{u}(z_0(\cdot), v_i(\cdot)), \quad i = 1, 2.$

Definition 3. A strategy $\mathbf{u} = \mathbf{u}(z_0(\cdot), v(\cdot))$ is called a parallel pursuit strategy (or P-strategy) if, for every $v(\cdot) \in V$, the solution of the Cauchy problem

$$\dot{z}(t) = z(t-h) + \mathbf{u}(z_0(\cdot), v(t)) - v(t),\ t \geq 0, \quad z(t) = z_0(t),\, t \in [-h, 0], \tag{7}$$

can be represented in the form

$$z(t) = \Lambda_G(t, v(\cdot))\xi(t, v(\cdot))z_0, \qquad \Lambda_G(0, v(\cdot)) = 1,$$

where $\Lambda_G(t, v(\cdot))$ is a scalar function continuous in t, $t \geq 0$. The function $\Lambda_G(t, v(\cdot))$ will hereafter be called the convergence function in the pursuit problem.

Definition 4. A $\Pi-$strategy is said to be winning for the pursuer on the time interval $[0, T]$ if, for any $v(\cdot) \in V$, the following conditions hold:

a) there exists a time $t^* \in [0, T]$ such that $z(t^*) = 0$;

b) $\mathbf{u}(z_0(\cdot), v(\cdot)) \in U$ on the interval $[0, T]$. The number T is called the guaranteed pursuit (or capture) time.

Let us now consider the game (U, V) from the viewpoint of the evader.

Definition 5. A control $\mathbf{v}^*(\cdot) \in V$ is called winning for the evader in the game (U, V) if, for every $u(\cdot) \in U$, the solution $z(t)$ of the Cauchy problem

$$\dot{z}(t) = z(t-h) + u(t) - \mathbf{v}^*(t),\, t \geq 0, \quad z(t) = z_0(t),\, t \in [-h, 0], \tag{8}$$

satisfies the inequality $z(t) \neq 0$ for all $t \geq 0$.

The present work is devoted to solving the following problems under the assumption that the players' controls satisfy constraints (3) and (4), respectively.

Problem 1. *Pursuit Problem:* Construct a $\Pi-$strategy for the pursuer and determine the guaranteed capture time in the game (U, V).

Problem 2. *Evasion Problem:* Construct a strategy for the evader and estimate the variation of the distance between the players.

Solution of the Pursuit Problem

In many parametric mathematical problems, analytical results explicitly depend on parameters that are regarded as constants. These parameters make it possible to formulate conditions for the solvability of the problem. In this section, necessary and sufficient conditions for the solvability of the pursuit problem in the game (U, V) are established.

If the pursuer and the evader choose admissible controls $u(\cdot) \in U$ and $v(\cdot) \in V$, respectively, then, according to equation (7), the solution takes the form

$$z(t) = \xi(t, \varphi(\cdot))z_0 + \int_0^t \eta(t-s)\big(u(s) - v(s)\big)ds. \tag{9}$$

For the pursuer, it is insufficient to use only open-loop (program) strategies depending solely on time t. Therefore, by analogy with [4], the pursuer's strategy in the considered problem may depend on the current value of the evader's control $v(t)$, as well as on the given parameters z_0 and ρ.

It is assumed that at time t, the pursuer knows the initial data $x_0, y_0, \varphi(\cdot)$, the constants ρ, σ, the current time t, and the current value of the evaderвЪ™s control $v(t)$.

Definition 6. Let $\rho \geq \sigma$. In the game (U, V), the function

$$\mathbf{u}(z_0(\cdot), v) = v - \lambda_G(z_0(\cdot), v)e_0, \tag{10}$$

is called the parallel pursuit strategy (briefly, the $\Pi-$strategy) of the pursuer, where

$$\lambda_G(z_0(\cdot), v) = \langle v, e_0\rangle + \sqrt{\langle v, e_0\rangle^2 + \rho^2 - |v|^2}, \qquad e_0 = \frac{z_0}{|z_0|}, \tag{11}$$

and $\langle v, e_0\rangle$ denotes the scalar product of the vectors v and e_0 in $\mathbb{R}^n$. The function $\lambda_G(z_0(\cdot), v)$ is commonly referred to as the *resolving function*.

The main properties of strategy (10) and the resolving function (11) are presented below.

Lemma 1. Strategy (10) is well-defined and continuous for all $v \in V$, and throughout the pursuit game the equality

$$|\mathbf{u}(z_0(\cdot), v)| = \rho$$

holds.

Lemma 2. The resolving function $\lambda_G(z_0(\cdot), v)$ is well-defined and nonnegative for all $v \in V$. Moreover, it satisfies the following bounds:

$$\rho - \sigma \le \lambda_G(z_0(\cdot), v) \le \rho + \sigma.$$

Proof. The minimum and maximum values of the resolving function $\lambda_G(z_0(\cdot), v)$ for arbitrary $v \in V$ are determined as follows:

$$\min_{v\in V} \lambda_G(z_0(\cdot), v) = \lambda_G(z_0(\cdot), v)\big|_{v=-\sigma e_0} = \langle -\sigma e_0, e_0\rangle + \sqrt{\langle -\sigma e_0, e_0\rangle^2 + \rho^2 - |-\sigma e_0|^2} = \rho - \sigma,$$

$$\max_{v\in V} \lambda_G(z_0(\cdot), v) = \lambda_G(z_0(\cdot), v)\big|_{v=\sigma e_0} = \langle \sigma e_0, e_0\rangle + \sqrt{\langle \sigma e_0, e_0\rangle^2 + \rho^2 - |\sigma e_0|^2} = \rho + \sigma.$$

Therefore,

$$\rho - \sigma \le \lambda_G(z_0(\cdot), v) \le \rho + \sigma.$$

□

Definition 7. If $\rho > \sigma$, then the scalar function

$$\Lambda_G(t, v(\cdot)) = 1 - \frac{1}{|z_0|\xi(t)} \int_0^t \eta(t-s)\lambda_G\big(z_0(\cdot), v(s)\big)ds \tag{12}$$

is called the players' approach (convergence) function in the game (U, V).

Lemma 3. Let $\rho > \sigma$. Then:

a) Function (12) is bounded for all $t \in [0, T]$ as follows:

$$\underline{\Lambda}_G(t, v(\cdot)) \le \Lambda_G(t, v(\cdot)) \le \overline{\Lambda}_G(t, v(\cdot)), \tag{13}$$

where

$$\underline{\Lambda}_G(t, v(\cdot)) = 1 - \frac{\rho+\sigma}{|z_0| \cdot \min\limits_{-h\le s\le 0} \varphi(s)} \left(1 - \frac{\eta(h)}{\eta(t+h)}\right),$$

$$\overline{\Lambda}_G(t, v(\cdot)) = 1 - \frac{\rho-\sigma}{|z_0| \cdot \max\limits_{-h\le s\le 0} \varphi(s)} \left(1 - \frac{\eta(h)}{\eta(t+h)}\right).$$

b) For all $v(\cdot) \in V$, function (12) is monotonically decreasing with respect to the variable t, $t \ge 0$.

Proof. Let $\rho > \sigma + |z_0| \cdot \max\limits_{-h\le s\le 0} \varphi(s)$.

a) From Property 1, Lemma 2, and formulas (5) and (12), we obtain the following inequality:

$$\Lambda_G(t, v(\cdot)) = 1 - \frac{1}{|z_0|\xi(t)} \int_0^t \eta(t-s)\lambda_G\big(z_0(\cdot)v(s)\big)ds \le$$

$$\le 1 - \frac{\rho-\sigma}{|z_0|\left(\eta(t)\varphi(0) + \int_{-h}^0 \eta(t-h-s)\varphi(s)ds\right)} \int_0^t \eta(t-s)ds \le$$

$$\le 1 - \frac{\rho-\sigma}{|z_0| \cdot \max\limits_{-h\le s\le 0} \varphi(s)} \cdot \frac{\int_0^t \eta(t-s)ds}{\eta(t) + \int_{-h}^0 \eta(t-h-s)ds} =$$

$$= 1 - \frac{\rho - \sigma}{|z_0| \cdot \max\limits_{-h \le s \le 0} \varphi(s)} \cdot \frac{\int_0^t \eta(s)ds}{\eta(t) + \int_{t-h}^t \eta(s)ds} =$$

$$= 1 - \frac{\rho - \sigma}{|z_0| \cdot \max\limits_{-h \le s \le 0} \varphi(s)} \cdot \frac{\eta(t+h) - \eta(h)}{\eta(t+h)} = \overline{\Lambda}_G(t, v(\cdot)).$$

The lower estimate of $\Lambda_G(t, v(\cdot))$ is obtained analogously to the upper estimate.

b) From Property 2 and formula (6), we obtain

$$\frac{d\overline{\Lambda}_G(t, v(\cdot))}{dt} = -\frac{\rho - \sigma}{|z_0| \cdot \max\limits_{-h \le s \le 0} \varphi(s)} \cdot \frac{\eta(h)\eta(t)}{\eta^2(t+h)} < 0.$$

Therefore, the function $\overline{\Lambda}_G(t, v(\cdot))$ is strictly decreasing. Consequently, by inequality (13), it follows that the function $\Lambda_G(t, v(\cdot))$ is also monotonically decreasing with respect to t. ☐

Theorem 1. Let

$$\rho > \sigma + |z_0| \cdot \max_{-h \le s \le 0} \varphi(s)$$

in the game (U, V). Then the $\Pi-$strategy (10) is winning for the pursuer, and the guaranteed time T is given by

$$T = \eta^{-1} \left(\frac{(\rho - \sigma)\eta(h)}{\rho - \sigma - |z_0| \cdot \max\limits_{-h \le s \le 0} \varphi(s)} \right) - h.$$

Proof. Consider the function

$$\overline{\Lambda}_G(t, v(\cdot)) = 1 - M \left(1 - \frac{\eta(h)}{\eta(t+h)} \right), \qquad M = \frac{\rho - \sigma}{|z_0| \cdot \max\limits_{-h \le s \le 0} \varphi(s)}.$$

From the assumption of the theorem, it follows that $M > 1$.

By the previously established properties of the function $\eta(t)$, the function $\overline{\Lambda}_G(t, v(\cdot))$ is continuous and monotonically decreasing with respect to t on the half-line $[0, +\infty)$. Moreover,

$$\overline{\Lambda}_G(0, v(\cdot)) = 1, \qquad \lim_{t \to +\infty} \overline{\Lambda}_G(t, v(\cdot)) = 1 - M < 0.$$

Therefore, there exists a unique time $T > 0$ such that

$$\overline{\Lambda}_G(T, v(\cdot)) = 0.$$

Substituting this condition, we obtain

$$1 - M \left(1 - \frac{\eta(h)}{\eta(T+h)} \right) = 0,$$

which implies

$$\frac{\eta(h)}{\eta(T+h)} = 1 - \frac{1}{M}.$$

Since $M > 1$, the right-hand side is positive, and therefore

$$\eta(T+h) = \frac{M\eta(h)}{M-1}.$$

Because the function $\eta(t)$ is strictly increasing on $[0, +\infty)$, it admits an inverse function η^{-1}. Hence,

$$T = \eta^{-1} \left(\frac{M\eta(h)}{M-1} \right) - h.$$

Substituting the expression for M, we finally obtain

$$T = \eta^{-1}\left(\frac{(\rho-\sigma)\eta(h)}{\rho-\sigma-|z_0|\cdot\max\limits_{-h\le s\le 0}\varphi(s)}\right)-h.$$

From formula (13) and Definition 4, it follows that there exists a time $t^* \in [0, T]$ such that $z(t^*) = 0$. This completes the proof of Theorem 1. □

Solution of the Evasion Problem

In this section, we propose an admissible strategy of the evader that guarantees evasion in the game.

Definition 8. The control function

$$v^*(t) = -\sigma e_0, \qquad t \ge 0, \tag{14}$$

is called the evader's strategy in the game (U, V).

Theorem 2. Let $\rho \le \sigma$. Then control (14) is winning for the evader in the game (U, V), and moreover, $|z(t)| \ge |z_0|\,\eta(h)\min\limits_{-h\le s\le 0}\varphi(s)$ for all $t \ge 0$.

Proof. Assume that $\rho \le \sigma$, and that the pursuer chooses a control $u(\cdot) \in U$, while the evader applies strategy (14). Then, from formula (9), we obtain

$$|z(t)| = \left|\xi(t,\varphi(\cdot))z_0 + \int_0^t \eta(t-s)\big(u(s)+\sigma e_0\big)ds\right| \ge$$

$$\ge \left|\xi(t,\varphi(\cdot))z_0 + \sigma e_0\int_0^t \eta(t-s)ds\right| - \left|\int_0^t \eta(t-s)u(s)ds\right| \ge$$

$$\ge |z_0|\xi(t,\varphi(\cdot))\left(1+\frac{\sigma}{|z_0|\xi(t,\varphi(\cdot))}\int_0^t \eta(t-s)ds\right) - \rho\int_0^t \eta(t-s)ds =$$

$$= |z_0|\xi(t,\varphi(\cdot))\left(1-\frac{\rho-\sigma}{|z_0|\xi(t,\varphi(\cdot))}\int_0^t \eta(t-s)ds\right) \ge$$

$$\ge |z_0|\xi(t,\varphi(\cdot)) \ge |z_0|\,\eta(t+h)\cdot\min_{-h\le s\le 0}\varphi(s) \ge |z_0|\,\eta(h)\cdot\min_{-h\le s\le 0}\varphi(s) > 0$$

for all $t \ge 0$.

Thus, the inequality

$$|z(t)| \ge |z_0|\,\eta(h)\cdot\min_{-h\le s\le 0}\varphi(s)$$

holds for all $t \ge 0$, which proves that control (14) is winning for the evader. Theorem 2 is proved. □

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