Properties of quasitraces on real C*-algebras

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Abstract. Quasitraces play a significant role in the study of structural properties of C*-algebras, especially in connection with finiteness and classification problems. In this paper, quasitraces on real C*-algebras are studied and some of their main properties are established. The behavior of quasitraces under matrix amplifications is investigated, and the conditions ensuring their extension to 2-quasitraces are considered. In addition, the relationship between quasitraces on real C*-algebras and their complexifications is analyzed. The obtained results provide a basis for understanding tracial-type functionals in the real conditions and complement existing results known for complex C*-algebras.

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Introduction

As is known, traces play an important role in understanding the structure of C*-algebras, and moreover, they are very important in the theory of von Neumann algebra. For example, it is well known and very important that the II_1 factors admit a unique tracial state. But not every C*-algebra possesses a trace. There is a generalization of traces to quasitraces. In 1982, B.Blackadar and D.Handelman defined an analogue of the trace, called a quasitrace. Despite the fact that this concept does not completely replace a trace, they and some researchers managed to obtain an analogue of the results available for traces.

In this paper, a real analogue of a (center-valued) quasitrace is given, and its connection with the quasitrace of the enveloping C*-algebra is found. Similar to the complex case, some interesting properties of a quasitrace and the corresponding metric for real C*-algebras are obtained.

Preliminaries

A *W*-algebra* is a weakly closed *-subalgebra of the algebra of all bounded linear operators *B(H)* on a complex Hilbert space H, containing the identity operator. A *real W*-algebra* is a weakly closed *-subalgebra R with the identity operator, satisfying the condition $R \cap iR = \{0\}$. A *C*-algebra* is a Banach *-algebra over the complex numbers, in which the norm satisfies the equality $\|aa^*\| = \|a\|^2$ for all elements a. A *real C*-algebra* is a Banach *-algebra over the reals, where $\|xx^*\| = \|x\|^2$, and the element formed by adding the identity operator to the square of any element x is invertible. *Baer *-rings* are *-rings in which the right annihilator of any subset can be expressed as a principal right ideal generated by a projection. (Real) C*-algebras with a Baer *-ring are called *(real) AW*-algebras* (for more details see [1]). Every W*-algebra is an AW*-algebra, but not all AW*-algebras can be represented as W*-algebras. Factors are W*-algebras with trivial center and are classified into types I_n, I_∞, II_1, II_∞, and III (see, e.g., [2]). Analogous notions for AW*-algebras can be found in [1]. Any W*- or AW*-algebra can be uniquely decomposed along its center into these factors. For every element in an

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AW*-algebra, the right and left support projections exist, and they describe annihilation properties. Spectral projections for self-adjoint elements correspond to intervals such as (λ, ∞) and play a fundamental role in describing spectral properties within the algebra (see [Remark 1.5] in [3]).

Definition 1. [1] Let A be a unital C*-algebra. A *quasitrace* τ on A is a function $\tau : A \to \mathbb{C}$ that satisfies:

(i) $\tau(x^*x) = \tau(xx^*) \geq 0$, for $x \in A$;

(ii) $\tau(a + ib) = \tau(a) + i\tau(b)$, for $a, b \in A_h$, where $A_h = \{a = a^*, a \in A\}$;

(iii) τ is linear on any abelian C*-subalgebra B_c of A.

Furthermore, τ is called a n-quasitrace $(n \geq 2)$ if there exists a 1-quasitrace τ_n on $M_n(A) = A \otimes M_n(\mathbb{C})$ such that *(iv)* $\tau(x) = \tau_n(x \otimes e_{11}), \quad x \in A,$

We present the definition of quasitrace in the complex case given above in Real case. This is one of the main definitions of our article.

Definition 2. Let R be a unital real C*-algebra. A quasitrace τ on R is a function $\tau : R \to \mathbb{R}$ that satisfies:

(i′) $\tau(x^*x) = \tau(xx^*) \geq 0$, for $x \in R$;

(ii′) $\tau(a + b) = \tau(a)$, for $a \in R_h$, $b \in R_k$, where $R_k = \{b = -b^*, b \in R\}$;

(iii′) τ is linear on any abelian C*-subalgebra B of R.

The center $\mathcal{Z}(R)$ of a real AW* algebra R is the set of elements in R that commute with all other elements of R. Formally, it is defined as:

$$\mathcal{Z}(R) = \{a \in R \mid ab = ba \text{ for all } b \in R\}.$$

In other words, $a \in \mathcal{Z}(R)$ if a commutes with every element B in R.

A center-valued quasitrace on C*-algebras.

Definition 3. Let N be a unital complex (or real) C^*-algebra and $\mathcal{Z}(R)$ is the center of R. A center-valued quasitrace is the map $T : R \to \mathcal{Z}(R)$ that satisfies:

(a) $T(x^*x) = T(xx^*) \geq 0$;

(b) $T(a + b) = T(a)$, for $a \in R_h$ and $b \in R_k$;

(c) T is linear on commutative real C*-subalgebras of R;

The following theorem is a real analogue of Theorem 1.27 given in [3], the proof of which is given in [6].

Theorem 1. [[6], Theorem 4.] Let R be a finite real AW*-algebra. Then, there exists a unique center-valued quasitrace $T : R \to \mathcal{Z}(R)$ with the following properties:

1. $T(x^*x) = T(xx^*) \geq 0$, for all $x \in N$;
2. $T(a + b) = T(a)$, for $a \in N_h$ and $b \in N_k$;
3. T is linear on commutative real C*-subalgebras of N;
4. $T(x^*x) = 0$ if and only if $x = 0$;
5. $T|_{P(N)} = D$, where D is the center-valued dimension function from the previous theorem;
6. $T(hx) = hT(x)$, for all self-adjoint $h \in \mathcal{Z}(N)$ and for all $x \in N$;
7. $T|_{\mathcal{Z}(N)} = \mathrm{id}_{\mathcal{Z}(N)}$;
8. T is order-preserving on N_h;

9. T is continuous in norm, in particular, $\|T(x) - T(y)\| \leq 2\|x - y\|$, for all $x, y \in N$.

Theorem 2. Let R be a unital real AW*-algebra, and let $A = R + iR$ be its enveloping AW*-algebra. If $\overline{T}$ is a center-valued quasitrace on A, then the map $T : R \to \mathcal{Z}(R)$ defined by

$$T(a + b) = \overline{T}(a), \quad \text{where } a \in R_h,\, b \in R_k \tag{26}$$

is a center-valued quasitrace on R.

Proof. 1. Assume $x \in R$ and express it as $x = a + b$, where $a \in R_h$ and $b \in R_k$. From equation (1),

$$T(x^*x) = T(a^2 - b^2 + ab - ba) = \overline{T}(a^2 - b^2),$$

where $a^2 - b^2 \in R_h$ and $ab - ba \in R_k$. Similarly, for xx^*, we have

$$T(xx^*) = T(a^2 - b^2 + ba - ab) = \overline{T}(a^2 - b^2).$$

Thus, $T(x^*x) = T(xx^*)$. Since $a^2 - b^2 \geq 0$, it follows that $T(x^*x) = \overline{T}(a^2 - b^2) \geq 0$. 2. By equation (1), $T(a + b) = \overline{T}(a) = T(a + 0) = T(a)$. 3. Let B be an abelian real C*-subalgebra of R and let $B_c = B + iB$ be its complexification, which is an abelian C*-subalgebra of A. By Definition 1, the quasitrace $\overline{T}$ is linear on B_c. We now show that T is linear on B. Let $\lambda \in Z(R)$. Then, λ can be represented as $\lambda = \lambda_1 + \lambda_2$, where $\lambda_1 \in R_h, \lambda_2 \in R_k$. $x, y \in B$, with $x = a + b$ and $y = c + d$, where $a, c \in R_h$ and $b, d \in R_k$. Since $\lambda x + y = (\lambda_1 a + c + \lambda_2 b) + (\lambda_1 b + d + \lambda_2 a)$ where $(\lambda_1 a + c + \lambda_2 b) \in R_h; (\lambda_1 b + d + \lambda_2 a) \in R_k$, we compute

$$T(\lambda x + y) = T(\lambda_1 a + c + \lambda_2 b) + (\lambda_1 b + d + \lambda_2 a) = \overline{T}(\lambda_1 a + c + \lambda_2 b).$$

By $a, c \in R_h \subset B_c$ and the linearity of $\overline{T}$ on B_c,

$$\overline{T}(\lambda_1 a + c + \lambda_2 b) = \lambda_1 \overline{T}(a) + \lambda_2 \overline{T}(b) + \overline{T}(c) + \lambda_2 \overline{T}(a) - \lambda_2 \overline{T}(a) = (\lambda_1 + \lambda_2)\overline{T}(a) + 0 + \overline{T}(c) - 0 =$$

$$(\lambda_1 + \lambda_2)T(a + b) + T(c + d) = \lambda T(x) + T(y).$$

This confirms that T is linear on B. Consequently, T is a quasitrace on R. The theorem is proved.

Theorem 3. If T is a center-valued quasitrace on R, then the map $\overline{T} : A \to \mathcal{Z}(A)$ defined by

$$\overline{T}(x + iy) = T(x) + iT(y), \quad \text{where } x, y \in R, \tag{2}$$

is a center-valued quasitrace on A.

Proof. 1. Recall that $A = R + iR$, and that a is embedded into $M_2(A)$ as $x \mapsto e_{11} \otimes x$. The mapping $\pi : M_2(A) \to M_2(\mathbb{C}) \otimes A$ is defined by

$$\pi([a_{ij}]) = \sum_{i,j=1}^{2} e_{ij} \otimes a_{ij}$$

and is a *-isomorphism. Let $x = c + id$, where $c, d \in R$. Then,

$$\overline{T}(x^*x) = \overline{T}((c + id)^*(c + id)) = \overline{T}(c^*c + d^*d + i(c^*d - d^*c)).$$

Since $c^*d - d^*c \in R_k$, by applying the property $T(a + b) = T(a)$ (where $a \in R_h$ and $b \in R_k$), we get:

$$\overline{T}(c^*c + d^*d + i(c^*d - d^*c)) = T(c^*c + d^*d).$$

Since $T(x) = T(x \otimes e_{11})$, we have:

$$\overline{T}(x^*x) = T(c^*c + d^*d) = T((c^*c + d^*d) \otimes e_{11}) = T\begin{pmatrix} c^*c + d^*d & 0 \\ 0 & 0 \end{pmatrix}.$$

This can be rewritten as:

$$T\begin{pmatrix} c^* & d^* \\ 0 & 0 \end{pmatrix}\begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix} = T\begin{pmatrix} c & 0 \\ d & 0 \end{pmatrix}\begin{pmatrix} c^* & d^* \\ 0 & 0 \end{pmatrix}.$$

Finally, we have:

$$T\begin{pmatrix} cc^* & cd^* \\ dc^* & dd^* \end{pmatrix} = T(cc^* + dd^*) = \overline{T}((c+id)(c+id)^*) = \overline{T}(xx^*),$$

which is what we needed to prove.

2. Linearity of $\overline{T}$ on A_h: Let $x, y \in A_h$, and $x = a + ib$, $y = c + id$. Since $x = x^*$, it follows that $a = a^*$, $b^* = -b$, i.e., $a \in R_h$, $b \in R_k$. Similarly, $c \in R_h$, $d \in R_k$. Since $T(b) = T(d) = 0$, we obtain:

$$\overline{T}(x+iy) = \overline{T}(a-d+i(b+c)) = T(a-d) + iT(b+c) = T(a) + iT(c).$$

Therefore,

$$\overline{T}(x+iy) = \overline{T}(x) + i\overline{T}(y).$$

3. Abelian C*-subalgebra B_c and linearity of $\overline{T}$: Let B_c be an abelian C*-subalgebra of the AW*-algebra a. Since $A = R + iR$, for all $x \in B_c$, there exist $a, b \in R$ such that $x = a + ib$, and thus $B_c = B_1 + iB_2$, where $B_1, B_2 \subset R$.

a) Since $B_c \ni 0 = 0 + i0$, $B_c \ni \epsilon = \epsilon + i0$, and $B_c \ni i\epsilon$, we have $0, \epsilon \in B_i, i = 1, 2$. If $B_c = iB_2$, then for $\lambda \in \mathbb{C}$, $\lambda = \lambda_1 + i\lambda_2$, $x = ia, y = ib$, where $a, b \in B_2$, we have:

$$\overline{T}(\lambda x + y) = \overline{T}((\lambda_1 + i\lambda_2)ia + ib) = \overline{T}(-\lambda_2 a + i(\lambda_1 a + b)).$$

This simplifies to:

$$T(-\lambda_2 a) + iT(\lambda_1 a + b) = -\lambda_2 T(a) + i\lambda_1 T(a) + iT(b).$$

Factoring terms:

$$i(\lambda_1 + i\lambda_2)T(a) + iT(b) = \lambda(0 + iT(a)) + (0 + iT(b)).$$

Hence:

$$\overline{T}(\lambda x + y) = \lambda\overline{T}(x) + \overline{T}(y),$$

which proves that $\overline{T}$ is linear in this case.

b) Let $x = a + ic$, $y = b + id$, where $a, b \in B_1$ and $c, d \in B_2$. Then:

$$xy = ab - cd + i(cb + ad).$$

Consequently, $ab - cd \in B_1$ and $cb + ad \in B_2$. For $c = d = 0$, we have $ab \in B_1$, which shows B_1 is an algebra. Similarly, B_2 is also an algebra. When $d = 1$ and $b = 0$, we find $a \in B_2$, so $B_1 \subset B_2$. Conversely, it can also be shown that $B_2 \subset B_1$. Thus, $B_1 = B_2$, and $B_c = B + iB$. Since $x^* = a^* - ib^* \in B_c$, it follows that $a^*, b^* \in B$. Therefore, B is a real *-subalgebra.

c) For additivity:

$$\overline{T}(x+y) = \overline{T}(a + ib + c + id) = T(a+c) + iT(b+d),$$

which simplifies to:

$$T(a) + T(c) + iT(b) + iT(d) = \overline{T}(a+ib) + \overline{T}(c+id) = \overline{T}(x) + \overline{T}(y).$$

For homogeneity:

$$\overline{T}(\lambda x) = \overline{T}((\lambda_1 + i\lambda_2)(a+ib)) = \overline{T}(\lambda_1 a - \lambda_2 b + i(\lambda_1 b + \lambda_2 a)).$$

This simplifies to

$$T(\lambda_1 a - \lambda_2 b) + iT(\lambda_1 b + \lambda_2 a) = (\lambda_1 + i\lambda_2)(T(a) + iT(b)) = \lambda\overline{T}(x).$$

Thus, $\overline{T}$ is linear on B_c. The theorem is proved.

4. Properties of 1- and 2- quasitraces on real C*-algebras.

We have proven above the theorems about the relationship between T-center-valued quasitrace on R and $\overline{T}$-center-valued quasitrace on A. Similarly, we can introduce the dependence of the quasitrace τ in R and the quasitrace $\overline{\tau}$ in A as follows:

Theorem 7. [7] Let R be unital real C*-algebra and $A = R + iR$ be complexification of R.

1. If $\overline{\tau}$ is a quasi-trace on a C*-algebra $A = R + iR$, then its restriction to a real C*-algebra R is defined as
$$\tau(a+b) = \overline{\tau}(a) \tag{3}$$
$a \in R_h, b \in R_k$ is a quasi-trace on R.

2. If τ is a quasi-trace on a real C*-algebra R, then its extension $\overline{\tau}$ to $A = R + iR$, defined as
$$\overline{\tau}(x+iy) = \tau(x) + i\tau(y) \tag{4}$$
is a quasi-trace on A, where $x, y \in R$.

Corollary 6. [6] Let R be a real AW *-algebra. Then R is finite iff there is a family of faithful quasi-traces , i.e., a family $(\tau_i)_{i\in I}$ of quasi-traces with $\tau_i(x^*x) = 0$ for $i \in I$ if and only if $x = 0$.

Corollary is proven in [6] as Corollary 1.

Lemma 7. Let τ be a 1-quasitrace on an real AW*-algebra R. Then τ is order-preserving on R_{sa}. Furthermore, τ is continuous, in particular, $|\tau(x) - \tau(y)| \leq \|x - y\|$ for all $x, y \in M$.

Proof. The idea of the proof is the same as for the center-valued quasitrace in [[6], Theorem 4]. We use Lemma 1.24 in [3]: Let $a, b \in R_{sa}$ be self-adjoint elements with $a \leq b$ and $\lambda > 0$. Let $E_{(\lambda,\infty)}(a)$ be the spectral projection defined as $E_{(\lambda,\infty)}(a) := RP((a - \lambda 1)_+) = LP((a - \lambda 1)_+)$, where $RP(x)$ (resp. $LP(x)$) is the right (resp. left) support projection of x. Then $E_{(\lambda,\infty)}(a) \leq E_{(\lambda,\infty)}(b)$. So, there exists $v \in R$ such that $E_{(\lambda,\infty)}(a) = v^*v$ and $vv^* \leq E_{(\lambda,\infty)}(b)$, so we can compute
$$\tau(E_{(\lambda,\infty)}(a)) = \tau(v^*v) = \tau(vv^*) \leq \tau(E_{(\lambda,\infty)}(b)),$$
where we obtain the last inequality from the fact that vv^* and $E_{(\lambda,\infty)}(b)$ commute, so $0 \leq \tau(E_{(\lambda,\infty)}(b) - vv^*) = \tau(E_{(\lambda,\infty)}(b)) - \tau(vv^*)$. We again get the inequality $\tau(a) \leq \tau(b)$ by integrating over the spectral projections. The proof of the continuity of τ is then the same as for the center-valued quasitrace. The proof of Lemma 7 is completed.

Theorem 8. [[6], Theorem 6.] Let τ be a 1-quasitrace on a real finite AW*-algebra R, and let T be the center-valued quasitrace constructed in [6] Theorem 4. Then τ is uniquely expressible in the form $\tau = \varphi \circ T$ for a positive functional φ on $Z(R)$.

Corollary 9. [[6], Corollary 2.] Let τ be a 1-quasitrace on a finite AW*-algebra M, then τ is a n-quasitrace for every $n \in \mathbb{N}$.

Definition 10. [3] A rank function D is a map $D : A \to [0, 1]$ such that

(i) D is normalized, that is, $\sup_{a\in A} D(a) = 1$.

(ii) For all $a, b \in A$ with $a \perp b$, we have $D(a + b) = D(a) + D(b)$.

(iii) For all $a \in A$: $D(a) = D(a^*a) = D(aa^*) = D(a^*)$.

(iv) For all positive elements $0 \leq a \leq b$: $D(a) \leq D(b)$.

(v) For all $a, b \in A$ with $a \precsim b$, say that a is Cuntz sub-equivalent to b, which means that there exist sequences $(x_n)_{n\in\mathbb{N}}, (y_n)_{n\in\mathbb{N}}$ of elements in A such that $(x_n b y_n)_{n\in\mathbb{N}}$ converges in norm to a, we get $D(a) \leq D(b)$.

A dimension function is a map $D : \bigcup_{n\in\mathbb{N}} M_n(A) \to [0, \infty)$ that satisfies properties (i)-(v) above. A rank (dimension) function is called *subadditive* if $D(a + b) \leq D(a) + D(b)$ for all $a, b \in A$; *weakly subadditive* if $D(a + b) \leq D(a) + D(b)$ for all positive commuting $a, b \in A$.

Theorem 11. Let R be a real C^*-algebra. Denote by $QT(R)$ the family of all quasitraces on R and by $F(R)$ the collection of lower semi-continuous, weakly additive rank-type functions defined on R. There is a natural embedding of $QT(R)$ in the set $F(R)$. Furthermore, 2-quasitraces correspond exactly to the lower semi-continuous subadditive dimension functions.

Proof. Let $A = R+iR$ and $F(A)$ be weakly additive lower semi-continuous rank functions on a C* algebra A. By Theorems 1 and 2 [7], there is a one-to-one correspondence between sets $QT(R)$ and $QT(A)$, and by Theorem 2.8 [3], there is a one-to-one correspondence between sets $QT(A)$ and $F(A)$. Since for any $\overline{D} \in F(A)$ we have $D \in F(R)$, where $D(x) := \overline{D}(x)$, $x \in R$, the set $F(A)$ can be embedded in $F(R)$. The second part of the statement follows from Theorem 2.8 [3]. □

We need an induced quasitrace on quotient C*-algebras, so we need the following theorem. This is Proposition 3.3 in [8] and uses Theorem 1.1.17 in [1].

Theorem 12. Let τ be a 2-quasitrace on a C*-algebra A. We define the kernel of τ:

$$I_\tau := \{a \in A \mid \tau(a^*a) = 0\}.$$

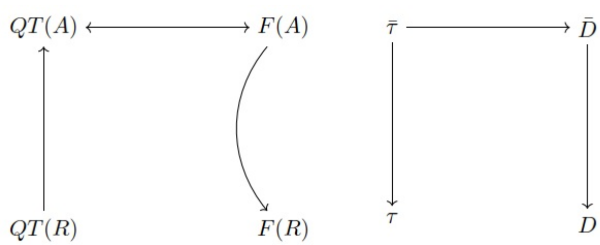
Then I_τ is a two-sided closed ideal of A, and there exists a quasitrace τ_1 on A/I_τ such that

$$\tau(a) = \tau_1(\pi(a))$$

for all $a \in A$ where $\pi : A \to A/I_\tau$ is the quotient map.

Proof. Let, by Theorem 11, D_τ be the corresponding dimension function to τ, and then define $\ker(D_\tau) := \{x \in R : D_\tau(x) = 0\}$. Theorem 1.1.17 in [BH82] shows that $\ker(\overline{D}_{\overline{\tau}}) = \{a \in A : \overline{D}_{\overline{\tau}}(x) = 0\}$ is a two-sided closed ideal in A and that $\overline{D}_{\overline{\tau}}$ induces a lower semi-continuous dimension function $\tilde{\overline{D}}_{\overline{\tau}}$ on $A/\ker(\overline{D}_{\overline{\tau}})$. It is easy to see that $\ker(D_\tau)$ is also a two-sided closed ideal in R. and that D_τ induces a lower semi-continuous dimension function $\tilde{D}_\tau$ on $R/\ker(D_\tau)$. We note that $I_\tau = \ker D_\tau$, and the quasitrace corresponding to $\tilde{D}_\tau$ is exactly the desired quasitrace τ_1.

The idea of the proof can be schematically illustrated as follows:



From Theorems 11 and 12 follows the following result

Theorem 13. Let τ be a 2-quasitrace on a C*-algebra A. Then there exists a finite AW*-algebra M, a 2-quasitrace τ' on M, and a unital *-homomorphism $\theta : A \to M$ such that $\tau = \theta \circ \tau'$.

Corollary 14. Let τ be a 2-quasitrace on a C*-algebra A.

1. τ is an n-quasitrace for every $n \in \mathbb{N}$.
2. τ is order-preserving on A_{sa}.
3. τ is continuous.
4. τ is bounded.

Proof. (i): The quasitrace τ is of the form $\tau = \theta \circ \tau'$ for a *-homomorphism $\theta : A \to M$ and an n-quasitrace τ' on M. Since θ is completely positive, it follows that τ is also an n-quasitrace. (ii): Again, τ and θ are both order-preserving, and so is τ. (iii): This again follows from (ii) or again from the fact that τ is the composition of continuous maps. (iv): From Remark I.1.19(b) in [1], we know that dimension functions are bounded, and then it follows from the fact that $\tau(a) \leq D_\tau(a)$ for all positive $a \in A$ with $\|a\| \leq 1$.

Remark From now on, when we write quasitrace, we will always mean 2-quasitraces on A. If A is a unital C*-algebra, we write $q(A)$ for the set of normalized quasitraces on A.

We want to state this last corollary without the proof. It is a combination of Theorem II.4.4 and Proposition II.4.5 in [1].

Corollary 15. If R is unital real C*-algebra, then $QT(R)$ is a compact convex set. Furthermore, $QT(R)$ is a simplex, and the set $T(R)$ of normalized traces is a closed face in $QT(R)$.

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