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### COGNITIVE PEDAGOGICAL APPROACHES TO STATISTICAL STABILITY WITH RANDOM SAMPLE SIZES

Annotation

The study is devoted to the stability of terms of the variational series formed a random number of observations. The independence of the number of observations from the observed quantiles themselves is not required.

**Key words:** Random variables, Variation series, Sample size, Asymptotic distribution.

### TASODIFIY TANLANMA HAJMI SHAROITIDA STATISTIK BARQARORLIKNI O'QITISHDA KOGNITIV YONDASHUVLAR

Annotatsiya

Maqola tasodifiy sondagi kuzatishlardan tashkil topgan variatsion qatorlar hadrlarining turg'unligiga bag'ishlangan. Bunda kuzatishlar sonining kuzatilayotgan kvantillarning o'zidan mustaqil bo'lishi talab etilmaydi.

**Kalit so'zlar:** Tasodifiy miqdorlar, variatsion qator, tanlanma hajmi, asimptotik taqsimot.

### КОГНИТИВНЫЕ ПОДХОДЫ К ОБУЧЕНИЮ СТАТИСТИЧЕСКОЙ УСТОЙЧИВОСТИ В УСЛОВИЯХ СЛУЧАЙНОГО ОБЪЕМА ВЫБОРКИ

Аннотация

Статья посвящена устойчивости членов вариационного ряда, состоящего из случайного числа наблюдений. При этом не требуется независимость количества наблюдений от самих наблюдаемых квантилей.

**Ключевые слова:** случайные величины, вариационный ряд, объем выборки, асимптотическое распределение.

**Introduction.** The introduction contains the necessary definitions and some well-known result that will be used in the study.

Let  $X_1, X_2, \dots, X_n$  be a sequence of independent identically distributed random variables (r.v.) with the general distribution function (d.f.)  $F(x)$  and

$\xi_1^{(n)} \leq \xi_2^{(n)} \leq \dots \leq \xi_k^{(n)}$  be a variational series (v.s.) constructed according to r. v.  $X_1, X_2, \dots, X_n$ .

The relation  $\frac{k}{n}$  is called the rank of term  $\xi_k^{(n)}$ . If as  $n \rightarrow \infty, \frac{k}{n} \rightarrow \lambda, 0 \leq \lambda \leq 1$ , then  $\lambda$  is called the limit rank of sequence  $\xi_k^{(n)}$ . Terms  $\xi_k^{(n)}$ , for which  $\lambda$  is other than zero or unit, are called central terms of the v.s., and terms  $\xi_k^{(n)}$  for which the limit rank is  $\lambda = 0$  or  $\lambda = 1$  are called the extreme terms of the v.s.

Definition 1.1. The sequence of terms of the v.s. with limit rank  $\lambda$  is called "stable" if there exists a sequence of constants  $A_k^{(n)}$  such that for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\left|\xi_k^{(n)} - A_k^{(n)}\right| < \varepsilon\right\} \rightarrow 1$$

The following Theorem 1.1 was proved in [1].

Theorem 1.1. If the rank number  $k$  of terms  $\xi_k^{(n)}$  remains constant with increasing  $n$ , then for stability of  $\xi_k^{(n)}$ , it is necessary and that, properly chosen constants  $A_k^{(n)}$  and any  $\varepsilon > 0$ , the following relations be fulfilled

$$nF(A_k^{(n)} + \varepsilon) \rightarrow +\infty, \quad (1)$$

$$nF(A_k^{(n)} - \varepsilon) \rightarrow 0, \text{ as } n \rightarrow \infty. \quad (2)$$

For the case  $F(x) > 0$  for any  $x$ , broader definition of stability is considered.

Definition 1.2. Random sequence  $\xi_k^{(n)}$  with constant rank number  $k$  is said to be relatively stable if there exist negative constants  $B_k^{(n)}$ , such that for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{B_k^{(n)}(1 + \varepsilon) \leq \xi_k^{(n)} \leq B_k^{(n)}(1 - \varepsilon)\right\} \rightarrow 1. \quad (3)$$

In [1], arguments are given from which it follows that relation (3) is equivalent to two following relations:

$$nF(B_k^{(n)}(1 - \varepsilon)) \rightarrow +\infty, \quad (4)$$

$$nF(B_k^{(n)}(1 + \varepsilon)) \rightarrow 0. \quad (5)$$

Let for  $0 < \lambda < 1$ , values of  $\bar{a}_\lambda$  and  $\underline{a}_\lambda$  be defined as:

$$\begin{cases} \bar{a}_\lambda = \inf\{x: F(x) > \lambda\} \\ \underline{a}_\lambda = \sup\{x: F(x) < \lambda\} \end{cases} \quad (6)$$

It is obvious that  $\bar{a}_\lambda \geq \underline{a}_\lambda$ .

The following theorem from paper [1] answers the question of the stability conditions for the central terms of the v.s.

Theorem 1.2. If as  $n \rightarrow \infty, \frac{k}{n} \rightarrow \lambda, 0 < \lambda < 1$  and  $\bar{a}_\lambda = \underline{a}_\lambda = a_\lambda$ , then for any  $\varepsilon > 0$

$$\lim_{n \rightarrow \infty} P\left\{\left|\xi_k^{(n)} - a_\lambda\right| < \varepsilon\right\} = 1.$$

Moreover,

$$P\left\{w: \xi_k^{(n)} \rightarrow a_\lambda, \text{ as } n \rightarrow \infty\right\} = 1.$$

If  $\underline{a}_\lambda < \bar{a}_\lambda$  then  $F(\underline{a}_\lambda + 0) = F(\bar{a}_\lambda - 0)$  and hence, interval  $(\underline{a}_\lambda, \bar{a}_\lambda)$  is the interval of constancy of the distribution function  $F(x)$ . In this case, the sequence of central terms is, generally speaking, not stable (see [1]).

Theorem 1.3. If  $\underline{a}_\lambda < \bar{a}_\lambda$  and  $\lambda$  is the limit rank of sequence  $\xi_k^{(n)}$ , then for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\underline{a}_\lambda - \varepsilon < \xi_k^{(n)} \leq \underline{a}_\lambda\right\} + P\left\{\bar{a}_\lambda \leq \xi_k^{(n)} < \bar{a}_\lambda + \varepsilon\right\} \rightarrow 1.$$

Denote the frequency of positive outcomes in  $n$  independent tests regarding events  $(X_1 < x), (X_2 < x), \dots, (X_n < x)$  by  $S_n(x)$ . Then

$$S_n(x) = I(X_1 < x) + I(X_2 < x) + \dots + I(X_n < x),$$

where  $I(A)$  is the indicator of event  $A$ .

In what follows, we will use the following simple but very important relation:

$$\{\xi_k^{(n)} < x\} = \{S_n(x) \geq k\}, \tag{7}$$

For the realization of inequality, it is necessary and sufficient that at least k terms in the original sample were less than x. Likewise, the following equality is true:

$$\{\xi_k^{(n)} \leq x\} = \{S_n(x+0) \geq k\}. \tag{8}$$

where  $S_n(x+0)$  denotes the frequency of terms of the variational not exceeding x.

Note that if  $\Phi_{kn}(x) = P(\xi_k^{(n)} < x)$ ,  $\bar{\Phi}_{kn}(x)$  is the distribution function of the k-th term of the v.s. of random variable  $-X_1, -X_2, \dots, -X_n$  with the distribution function  $\bar{F}(x) = 1 - F(-x)$ , then the following relation is true

$$\Phi_{n-k+1,n}(x) = 1 - \bar{\Phi}_{kn}(x),$$

Which allows us to transfer the results obtained for  $\xi_k^{(n)}$  to the distribution of the  $(n - k + 1)$ -th term  $\xi_{n-k+1}^{(n)}$  and vice versa.

Main results. In this section, we present our main results on the stability of sequences of the variational series

$$\xi_1^{(v_n)} \leq \xi_2^{(v_n)} \leq \dots \leq \xi_{v_n}^{(v_n)},$$

constructed from a random sample size  $X_1, X_2, \dots, X_{v_n}$  from the general population the distribution function  $F(x)$ . Here and below  $v_n$  is a sequence of positive integer r.v. We note that in all the results presented in this section, the independence of sequences of r.v.  $\{X_n\}$  and  $\{v_n\}$  is not assumed.

We assume that relatively to  $\{v_n\}$ , there is the distribution function  $G(x)$  such that for any x, which is a point of continuity  $G(x)$ , as  $n \rightarrow \infty$

$$P\left\{\frac{v_n}{n} < x\right\} \rightarrow G(x), G(+0) = 0, \tag{9}$$

For the extreme terms of the v.s.  $\xi_k^{(n)}$  with constant rank number k and for the central terms, the following theorems are proven.

Theorem 2.1. Let (9) hold and for with constant k there is a sequence of numbers  $A_k^{(n)}$  such that as  $n \rightarrow \infty$  for any  $\varepsilon > 0$

$$P\left\{\left|\xi_k^{(n)} - A_k^{(n)}\right| < \varepsilon\right\} \rightarrow 1.$$

Then for for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\left|\xi_k^{(v_n)} - A_k^{(n)}\right| < \varepsilon\right\} \rightarrow 1.$$

Theorem 2.2. Let (9) hold and for  $\xi_k^{(n)}$  with constant k there is a sequence of numbers  $B_k^{(n)}$  such that as  $n \rightarrow \infty$  for any  $\varepsilon > 0$

$$P\left\{\left|\frac{\xi_k^{(n)}}{B_k^{(n)}} - 1\right| < \varepsilon\right\} \rightarrow 1.$$

Then for for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\left|\frac{\xi_k^{(v_n)}}{B_k^{(n)}} - 1\right| < \varepsilon\right\} \rightarrow 1.$$

Remark. Theorems similar to Theorem 2.1 and Theorem 2.2 can also be formulated for right-hand terms  $\xi_{v_n-k+1}^{(v_n)}$  with constant rank number k.

Theorem 2.3. Let condition (9) be satisfied for  $\{v_n\}$  and sequence  $\xi_{k(n)}^{(n)}$  as  $n \rightarrow \infty, \frac{k(n)}{n} \rightarrow \lambda, 0 < \lambda < 1, \bar{a}_\lambda = \underline{a}_\lambda = a_\lambda$ . Then for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\left|\xi_{k(n)}^{(v_n)} - a_\lambda\right| < \varepsilon\right\} = 1.$$

The following peculiar result, which shows the influence of the distribution function F(x), the random samle size, and the limit rank  $\lambda$  on the asymptotic distribution of the terms of the v.s., is also of interest.

Theorem 2.4. Let (9) hold. If  $k(n) \rightarrow \infty$ , and  $\frac{k(n)}{n} \rightarrow \lambda, 0 < \lambda < 1$  as  $n \rightarrow \infty$ , then as  $n \rightarrow \infty$

$$P\left\{\xi_{k(n)}^{(v_n)} < x/v_n \geq k(n)\right\} \rightarrow \Psi(x) = \frac{1-G\left(\frac{\lambda}{F(x)}\right)}{1-G(\lambda)},$$

where for  $0 \leq \lambda \leq 1, F(x) = 0$  we assume that  $G\left(\frac{\lambda}{F(x)}\right) = 1$  and  $G(\lambda) = 1$  we assume that  $\Psi(x) = 0$ . The following corollary follows this theorem.

Corollary. If  $P(v_n = n) = 1$ , then as  $n \rightarrow \infty$  if

$$P\left\{\xi_{k(n)}^{(v_n)} < x/v_n \geq k(n)\right\} \rightarrow 1 - G\left(\frac{\lambda}{F(x)}\right) = \begin{cases} 0, & \text{if } x \in \{x: F(x) < \lambda\} \\ 1, & \text{if } x \in \{x: F(x) \geq \lambda\} \end{cases}$$

Other corollaries of Theorem 2.1 are Theorem 2.2 and 2.3., if we assume that  $P(v_n = n) = 1$ . Let us pass to the proof of Theorems 2.1-2.4.

Proof of the Theorem 2.1. Since

$$P\left\{\left|\xi_k^{(v_n)} - A_k^{(n)}\right| < \varepsilon\right\} = P\left\{\xi_k^{(v_n)} < A_k^{(n)} + \varepsilon\right\} -$$

$$P\left\{\xi_k^{(v_n)} \leq A_k^{(n)} - \varepsilon\right\}$$

Let us show that for any  $\varepsilon > 0$  as  $n \rightarrow \infty$

$$P\left\{\xi_k^{(v_n)} < A_k^{(n)} + \varepsilon\right\} \rightarrow 1 \text{ and } P\left\{\xi_k^{(v_n)} \leq A_k^{(n)} - \varepsilon\right\} \rightarrow 0.$$

For this purpose, we set

$$\eta_{nj} = \frac{I(X_j < A_k^{(n)} + \varepsilon)}{nF(A_k^{(n)} + \varepsilon)}, j = 1, 2, \dots, n \text{ and}$$

$$\tau_{nj} = \frac{I(X_j \leq A_k^{(n)} - \varepsilon)}{n}, j = 1, 2, \dots, n.$$

Then we have two sequences of series of independst equally distribution in each series r.v.  $\{\eta_{nj}\}_{j=1}^n$  and  $\{\tau_{nj}\}_{j=1}^n, n = 1, 2, \dots$

By applying the Chebyshev inequality, and considering relations (1) and (2), it is easy to see that as  $n \rightarrow \infty$

$$\eta_{n1} + \eta_{n2} + \dots + \eta_{nn} \xrightarrow{p} 1 \text{ and } \tau_{n1} + \tau_{n2} + \dots + \tau_{nn} \xrightarrow{p} 0,$$

where symbol  $\xrightarrow{p}$  means convergence in probability.

Using relation (7), we have

$$P\left\{\xi_k^{(v_n)} < A_k^{(n)} + \varepsilon\right\} = P\left\{S_{v_n}(A_k^{(n)} + \varepsilon) \geq k\right\} =$$

(10)

$$= P\left\{\frac{\sum_{i=1}^{v_n} I(X_i < A_k^{(n)} + \varepsilon)}{nF(A_k^{(n)} + \varepsilon)} \geq \frac{k}{nF(A_k^{(n)} + \varepsilon)}\right\} = P\left\{\eta_{n1} + \eta_{n2} + \dots + \eta_{nv_n} \geq \frac{k}{nF(A_k^{(n)} + \varepsilon)}\right\}.$$

According to (1) as  $n \rightarrow \infty$

$$\frac{k}{nF(A_k^{(n)} + \varepsilon)} \rightarrow 0, \quad (k = const). \tag{11}$$

Sequences of r.v.  $\eta_{nj}, v_n$  satisfy the conditions of the theorem given in [3] with  $k_n = n$  and  $A(x) = G(x)$ . According to these theorems, taking into account (10), we obtain the following as  $n \rightarrow \infty$

$$P\left\{\eta_{n1} + \eta_{n2} + \dots + \eta_{nv_n} \geq \frac{k}{nF(A_k^{(n)} + \varepsilon)}\right\} \rightarrow 1 - G(0) = 1$$

and hence with (10) as  $n \rightarrow \infty$

$$P\left\{\xi_k^{(v_n)} < A_k^{(n)} + \varepsilon\right\} \rightarrow 1, \quad (k = const) \tag{12}$$

(12)

On the other hand, using (2), we have

$$P\left\{\xi_k^{(v_n)} \leq A_k^{(n)} - \varepsilon\right\} = P\left\{S_{v_n}(A_k^{(n)} - \varepsilon + 0) \geq k\right\} =$$

$$= P\left\{S_{v_n}(A_k^{(n)} - \varepsilon + 0) > k - 1\right\} = P\left\{\tau_{n1} + \tau_{n2} + \dots +$$

$$\tau_{nv_n} > \frac{k-1}{n}\right\}.$$

By applying the results of the segunce of r.v.  $\{\tau_{nj}\}_{j=1}^n, n = 1, 2, \dots$ , we conclude that as  $n \rightarrow \infty$

$$P\left\{\xi_k^{(v_n)} \leq A_k^{(n)} - \varepsilon\right\} \rightarrow 0. \tag{13}$$

Relations (12) and (13) according to equality

$$P\left\{\left|\xi_k^{(v_n)} - A_k^{(n)}\right| < \varepsilon\right\} = P\left\{\xi_k^{(v_n)} < A_k^{(n)} + \varepsilon\right\} -$$

$$P\left\{\xi_k^{(v_n)} \leq A_k^{(n)} - \varepsilon\right\}$$

complete the proof of Theorem 2.1.

Similar arguments are used in the proofs of theorems 2.2 and 2.3.

Proof of Theorem 2.4.

Let  $x \in \{x: 0 < F(x) \leq 1\}$  and  $G(\lambda) < 1$ . Using relation (7), we have:

$$P\left\{\xi_{k(n)}^{(v_n)} < x/v_k \geq k(n)\right\} = \frac{P\left\{\xi_{k(n)}^{(v_n)} < x, v_k \geq k(n)\right\}}{P\{v_k \geq k(n)\}} = \frac{P\{Sv_n(x) \geq k(n), v_k \geq k(n)\}}{P\{v_k \geq k(n)\}} = \frac{1}{P\{v_k \geq k(n)\}} \cdot P\left\{\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \cdot \frac{v_n}{n} \geq \frac{k(n)}{n}, \frac{v_n}{n} \geq \frac{k(n)}{n}\right\} \tag{14}$$

Since inequality  $\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \leq 1$  is true with probability 1, then for any  $n = 1, 2, \dots$

$$\left\{\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \cdot \frac{v_n}{n} \geq \frac{k(n)}{n}\right\} \subset \left\{\frac{v_n}{n} \geq \frac{k(n)}{n}\right\}$$

Hence it follows that

$$P\left\{\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \cdot \frac{v_n}{n} \geq \frac{k(n)}{n}, \frac{v_n}{n} \geq \frac{k(n)}{n}\right\} = P\left\{\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \cdot \frac{v_n}{n} \geq \frac{k(n)}{n}\right\}$$

From the strong law of large numbers, by virtue of condition (9), it follows that as  $n \rightarrow \infty$

$$\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \xrightarrow{p} F(x).$$

Then, by virtue of condition (9) and the well-known Cramer theorem [4] on the convergence of distributions, and also taking into account that  $\frac{k(n)}{n} \rightarrow \lambda, n \rightarrow \infty$ , we obtain

$$P\left\{\frac{\sum_{j=1}^{v_n} I(X_j < x)}{v_n} \cdot \frac{v_n}{n} \geq \frac{k(n)}{n}\right\} \rightarrow 1 - G\left(\frac{\lambda}{F(x)}\right) \quad \text{as } n \rightarrow \infty$$

Under the assumptions of the theorem, the following relation is clear

$$P\{v_n \geq k(n)\} = P\left\{\frac{v_n}{n} \geq \frac{k(n)}{n}\right\} \rightarrow 1 - G(\lambda) \quad \text{as } n \rightarrow \infty.$$

The last two relations together with (14) show that for  $x \in \{x: 0 < F(x) \leq 1\}$  as  $n \rightarrow \infty$

$$P\left\{\xi_{k(n)}^{(v_n)} < x/v_k \geq k(n)\right\} \rightarrow \Psi(x) = \frac{1 - G\left(\frac{\lambda}{F(x)}\right)}{1 - G(\lambda)} \tag{15}$$

Now let  $x \in \{x: F(x) = 0\}$ . Then it is clear that  $P\left\{\xi_{k(n)}^{(v_n)} < x/v_k \geq k(n)\right\} = 0$  for any  $n = 1, 2, \dots$

Therefore for  $x \in \{x: F(x) = 0\}$  we assume that  $G\left(\frac{\lambda}{F(x)}\right) = 1$ , (then

$$\Psi(x) = 0)$$
 and again we obtain relation (15).

The theorem is proven for the case  $G(\lambda) < 1$ . And, if  $G(\lambda) = 1$ , then, by the definition of conditional probability, we assume that  $\Psi(x) = 0, -\infty < x < \infty$ .

The proof of the theorem is complete.

Consider, as an illustration of Theorem 2.4 the following examples.

Example 1. Let the sequence of r.v.  $v_n$  obey the geometric distribution law with parameter  $\frac{1}{n}$ , i.e.

$$P\{v_n = k\} = \frac{1}{n} \left(1 - \frac{1}{n}\right)^{k-1}, \quad k=1, 2, \dots$$

It is well known that the geometric distribution law is often used in many problems of theory and reliability theory, and for this case, relation (9) is satisfied with the standart exponential distribution function, i. e.

$$B, \lim_{n \rightarrow \infty} P\left\{\frac{v_n}{n} < x\right\} = E(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1 - e^{-x}, & \text{if } x > 0 \end{cases}$$

Under these assumptions, if  $F(x) = U(x, a, b)$  where

$$U(x, a, b) = \begin{cases} 0, & \text{if } x \leq a, \\ \frac{x - a}{b - a}, & \text{if } a < x \leq b, \\ 1, & \text{if } x > b \end{cases}$$

Then in Theorem 2.4, we have

$$\Psi(x) = \begin{cases} 0, & \text{if } x \leq a, \\ e^{-\lambda \left(\frac{b-a}{x-a}\right)}, & \text{if } a < x \leq b, \\ 1, & \text{if } x > b. \end{cases}$$

$$\text{If } F(x) = E(x) \text{ then } \Psi(x) = \begin{cases} 0, & \text{if } x \leq 0, \\ e^{-\frac{\lambda e^{-x}}{1 - e^{-x}}}, & \text{if } x > 0. \end{cases}$$

Example 2. Let

$$F(x) = \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy \quad \text{and} \quad G(x) =$$

$$\begin{cases} 0, & \text{if } x \leq 0, \\ x, & \text{if } 0 < x \leq 1, \\ 1, & \text{if } x > 1. \end{cases}$$

Then

$$\Psi(x) = \begin{cases} 0, & \text{if } x \leq a_\lambda \\ \frac{1}{1 - \lambda} \left(1 - \frac{1}{\Phi(x)}\right), & \text{if } x > a_\lambda, \end{cases}$$

where  $a_\lambda$  is such that  $\Phi(a_\lambda) = \lambda, 0 < \lambda < 1$ .

**Conclusion.** From [2], it follows that theorems that assume the existence of a limit distribution for a deterministic sequence and, under appropriate additional conditions, assert the existence of a limit distribution for sequence with arandom index, are called transfer theorems. As indicated in the preface to monograph [5], the research on transfer theorems is important and necessary, in particular, for sums of a random number of terms. To date, a large number of results on random summation have been accumulated, and we can speak with complete certainty about the existence of a relevant theory. The available results can be conditionally divided into two groups. One of them includes the results of studies in which the indispensable condition is the independence of terms from the number of terms in the sum (“independent scheme”). The other group consists of results in which, such an assumption is not made (“dependent scheme”). Theorems 2.1-2.3 proved by us refer to the “dependent scheme” and extend the previously known results on the stability of terms of the variational series to the case of a random sample size. [6] – [10]. Asymptotic distributions of the terms of the v.s. with a random sample size and similar problems were also studied in.

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