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Tasodifiy qo'zg'alishlarda o'yiq plastinkaning harakat differensial tenglamasini aniqlash masalasi

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ANNOTATSIYA:

Ushbu ishda tasodifiy qo'zg'alishlar ta'siridagi gisterezis tipidagi elastik-dissipativ xarakteristikaga ega o'yiq plastinkaning harakat differensial tenglamasini tuzish masalasi o'rganilgan. Plastinka materialining dissipativ xossalari Pisarenko-Boginich gipotezasi asosida tavsiflangan. Tadqiqotda to'g'ri to'rtburchak plastinka va undan ajratilgan to'g'ri to'rtburchak shakldagi o'yiq qaralgan. O'yiqning tomonlari plastinka tomonlariga parallel deb qabul qilingan. Shuningdek, o'yiqning plastinka tekisligidagi joylashuvi ixtiyoriy bo'lishi mumkinligi nazarda tutilgan. Ushbu geometrik va fizik model asosida plastinkaning harakat tenglamasi shakllantirilgan. Model tasodifiy tashqi ta?sirlar hamda materialning ichki dissipativ xususiyatlarini hisobga oladi. Olingan tenglama elastik dissipativ sistemalarning dinamikasini tadqiq etish uchun nazariy asos bo'lib xizmat qiladi.

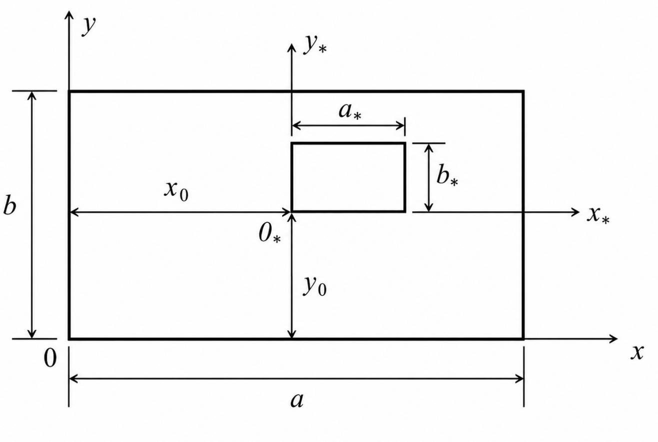
Kalit so'zlar: o'yiq plastinka, gisterezis, egilish, eguvchi moment, burovchi moment, differensial tenglama, xususiy tebranish formasi.

KIRISH

Bugungi kunda mexanik sistemalar murakkab elementlari materiallaridagi chiziqlimas elastik dissipativ xarakteristikalarini hisobga olgan holda ularni matematik modelini ishlab chiqish dolzarb masalalardan hisoblanadi. Bu borada ko'plab ilmiy tadqiqot ishlari olib borilgan. [1-9] – ishlarda o'yiq plastinkalarning xususiy va rezonans chastotalarini aniqlash, o'yiq parametrlarining dinamik xarakteristikalarga ta'siri hamda soddalashtirilgan hisoblash modellarini yaratish masalalari ko'rib chiqilgan. Plastinkalarning modal xususiyatlarini tahlil qilish uchun sonli usullar taklif etilgan. Ushbu usullar yordamida o'yiqlar mavjud bo'lgan plastinkalarning tebranish chastotasi aniqlangan. Bunda plastinkaning xususiy tebranish chastotasidagi o'zgarishlar qonuniyatlarini aniqlashda chekli elementlar usullari asosida modellashtirish va tajribalar natijalari hisobga olingan. Abacus dasturidan ham modellashtirishda foydalanilgan. Olingan modelning sonli natijalari PSV-500 vibrometrda olingan tajribalar natijalari bilan ustma-ust tushgan. O'yiq plastinkalarning xususiy tebranish chastotasi ularning parametrlariga bog'liqligi aniqlangan va tahlil qilingan. Plastinkalarning xususiy chastotasiga sezilarli ta'sir ko'rsatadigan parametrlar ajratib olingan. Bu olingan xususiy tebranish chastotalarining qonuniyatlari o'yiq plastinkalarda o'yiqlar orasida yoriqlar paydo bo'lishini oldindan bashorat qilish va ularning chidamliligini aniqlash imkonini berishi ko'rsatilgan. Gisterezis tipidagi elastik dissipativ xarakteristikali o'yiqga ega plastinkaning tebranishlarini o'rganish dolzarb hisoblanadi.

MATERIAL VA METODLAR

Ushbu ishda gisterezis tipidagi elastik dissipativ xarakteristikali o'yiqga ega plastinkaning energiya ifodalarini hamda harakat differensial tenglamasini aniqlash masalalari qaralgan (1-rasm). Bunda plastinka materialining dissipativlik xossalari Pisarenko-Boginich gipotezasiga asosan olingan.



1-rasm. O‘yiqga ega plastinka sxemasi

1-rasmda tasvirlangan gisterezis tipidagi elastik dissipativ xarakteristikali o‘yiqqa ega plastinkaning tomonlari a, b, o‘yiqning tomonlari a^*, b^* va bir uchi (x_0, y_0) nuqtada joylashgan. Bunda kiritilgan Oxy va $O^*x_*y_*$ koordinatalar orasida quyidagi bog‘lanishlar mavjud:

$$\begin{cases} x = x_0 + x_*, \\ y = y_0 + y_*. \end{cases} \tag{1}$$

Eguvchi va burovchi momentlar materiallardagi gisterezis tipidagi elastik dissipativ xossalarni hisobga olganda tasodifiy qo‘zg‘alishlar ta’siridagi mexanik sistemalar uchun quyidagicha aniqlanadi [10]:

$$M_{\tilde{x}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\tilde{x}} z\, dz; \qquad M_{\tilde{y}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\tilde{y}} z\, dz; \qquad M_{\tilde{x}\tilde{y}} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{\tilde{x}\tilde{y}} z\, dz, \tag{2}$$

bunda $\sigma_{\tilde{x}}, \sigma_{\tilde{y}}$ va $\sigma_{\tilde{x}\tilde{y}}$ kuchlanishlar deformatsiyalar orqali quyidagicha ifodalangan:

$$\sigma_{\tilde{x}} = -\frac{E_{\tilde{x}} z}{1-\mu}\, \tilde{V}_1, \qquad \sigma_{\tilde{y}} = -\frac{E_{\tilde{y}} z}{1-\mu}\, \tilde{V}_2, \qquad \sigma_{\tilde{x}\tilde{y}} = -\frac{G_{\tilde{x}\tilde{y}} z}{1-\mu}\, \tilde{V}_3, \tag{3}$$

$$E_{\tilde{x}} = E\left(1 - L_{1\tilde{x}} + jL_{2\tilde{x}}\right); \qquad E_{\tilde{y}} = E\left(1 - L_{1\tilde{y}} + jL_{2\tilde{y}}\right);$$

$$G_{\tilde{x}\tilde{y}} = G\left(1 - \tilde{L}_1^* + j\tilde{L}_2^*\right),$$

bu yerda E, G, μ, j lar ikkinchi bobda keltirilgan.

$$L_{1\tilde{x}} = \theta_1 f_{\tilde{x}}(z); \qquad L_{2\tilde{x}} = \theta_2 f_{\tilde{x}}^*(z),$$

$$L_{1\tilde{y}} = \theta_1 f_{\tilde{y}}(z); \qquad L_{2\tilde{y}} = \theta_2 f_{\tilde{y}}^*(z),$$

$$\tilde{L}_1^* = \tilde{\theta}_1 \tilde{g}(z); \qquad \tilde{L}_2^* = \tilde{\theta}_2 \tilde{g}(z).$$

$\theta_1, \theta_2, \tilde{\theta}_1, \tilde{\theta}_2$ — statistik chiziqlilashtirish koeffitsientlari; $f_{\tilde{x}}(z)$ va $f_{\tilde{y}}(z)$ — funksiyaning nisbiy deformatsiya maksimal qiymatidagi tebranish dekrementi; $\tilde{g}(z)$ — nisbiy deformatsiya amplitudasiga bog‘liq bo‘lgan tebranish dekrementi;

$$f_{\tilde{x}}(z) = \tilde{c}_0 + \tilde{c}_1 \left|\tilde{V}_1\right|_a |z| + \tilde{c}_2 \left|\tilde{V}_1\right|_a^2 |z|^2 + \cdots + \tilde{c}_r \left|\tilde{V}_1\right|_a^r |z|^r;$$

$$f_{\tilde{y}}(z) = \tilde{c}_0 + \tilde{c}_1 \left|\tilde{V}_2\right|_a |z| + \tilde{c}_2 \left|\tilde{V}_2\right|_a^2 |z|^2 + \dots + \tilde{c}_r \left|\tilde{V}_2\right|_a^r |z|^r;$$

$$\tilde{g}(z) = \tilde{k}_0 + \tilde{k}_1 \left|\tilde{V}_3\right|_a |z| + \tilde{k}_2 \left|\tilde{V}_3\right|_a^2 |z|^2 + \dots + \tilde{k}_r \left|\tilde{V}_3\right|_a^s |z|^s.$$

$\tilde{c}_0, \dots, \tilde{c}_r, \tilde{k}_0, \dots, \tilde{k}_s$ – eksperimentdan aniqlanadigan sonlar;

$$\tilde{V}_1 = \frac{\partial^2 w_t}{\partial x^2} + \mu \frac{\partial^2 w_t}{\partial y^2}; \qquad \tilde{V}_2 = \frac{\partial^2 w_t}{\partial y^2} + \mu \frac{\partial^2 w_t}{\partial x^2}; \qquad \tilde{V}_3 = \frac{\partial^2 w_t}{\partial x \partial y}. \tag{4}$$

$\sigma_{\tilde{x}}, \sigma_{\tilde{y}}$ va $\sigma_{\tilde{x}\tilde{y}}$ kuchlanishlarning (3) ifodalarini momentlarning (2) ifodalariga qo‘yamiz. Ba‘zi hisoblashlardan so‘ng,

$$M_{\tilde{x}} = -D\tilde{V}_1 \left[1 + 3\left(-\theta_1 + j\theta_{2^*}\right) \sum_{i_1}^{r} |\tilde{V}_1|_a^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)}\right]; \tag{5}$$

$$M_{\tilde{y}} = -D\tilde{V}_2 \left[1 + 3\left(-\theta_1 + j\theta_{2^*}\right) \sum_{i_1}^{r} |\tilde{V}_2|_a^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)}\right]; \tag{6}$$

$$M_{\tilde{x}\tilde{y}} = -D\tilde{V}_3 \left[1 + 3\left(-\tilde{\theta}_1 + j\tilde{\theta}_{2^*}\right) \sum_{i_2}^{s} |\tilde{V}_3|_a^{i_2} \frac{h^{i_2}}{2^{i_2}(i_2+3)}\right], \tag{7}$$

bunda $D = \frac{Eh^3}{12(1-\mu^2)}$.

O‘yiq ham to‘g‘ri to‘rtburchak shaklidagi plastinka ko‘rinishida bo‘lganligi sababli, uning ko‘ndalang egilishidagi kinetik va potensial energiya quyidagicha aniqlanadi[11]:

$$\tilde{T}_* = \frac{1}{2} \iint \rho h \left(\frac{\partial w_{t_*}}{\partial t}\right)^2 dx_* dy_*; \tag{8}$$

$$\tilde{V}_* = \frac{1}{2} \iint \left(M_{\tilde{x}_*} \frac{\partial^2 w_{t_*}}{\partial x_*^2} + M_{\tilde{y}_*} \frac{\partial^2 w_{t_*}}{\partial y_*^2} + 2M_{\tilde{x}_*\tilde{y}_*} \frac{\partial^2 w_{t_*}}{\partial x_* \partial y_*}\right) dx_* dy_*,$$

bunda $w_{t_*} = w_{t_*}(x_*, y_*, t)$ – o‘yiqqa mos keluvchi plastinkaning egilishi; $M_{\tilde{x}_*}$, $M_{\tilde{y}_*}$ va $M_{\tilde{x}_*\tilde{y}_*}$ lar mos ravishda o‘yiqqa mos keluvchi plastinkadagi eguvchi va burovchi momentlar.

Bunda ham eguvchi va burovchi momentlar materiallardagi gisterezis tipidagi elastik dissipativ xossalarni hisobga olganda quyidagicha aniqlanadi[10]:

$$M_{\tilde{x}_*} = -D\tilde{V}_{1*} \left[1 + 3\left(-\theta_1 + j\theta_{2*}\right) \sum_{i_1=1}^{r} |\tilde{V}_{1*}|_a^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)}\right]. \tag{9}$$

$$M_{\tilde{y}_*} = -D\tilde{V}_{2*} \left[1 + 3\left(-\theta_1 + j\theta_{2*}\right) \sum_{i_1=1}^{r} |\tilde{V}_{2*}|_a^{i_1} \frac{h^{i_1}}{2^{i_1}(i_1+3)}\right]. \tag{10}$$

$$M_{\tilde{x}_*\tilde{y}_*} = -D\tilde{V}_{3*} \left[1 + 3\left(-\tilde{\theta}_1 + j\tilde{\theta}_{2*}\right) \sum_{i_2=1}^{s} |\tilde{V}_{3*}|_a^{i_2} \frac{h^{i_2}}{2^{i_2}(i_2+3)}\right]. \tag{11}$$

bunda

$$\begin{cases} \tilde{V}_{1*} = \dfrac{\partial^2 w_{t*}}{\partial x_*^2} + \mu \dfrac{\partial^2 w_{t*}}{\partial y_*^2}, \\ \tilde{V}_{2*} = \dfrac{\partial^2 w_{t*}}{\partial y_*^2} + \mu \dfrac{\partial^2 w_{t*}}{\partial x_*^2}, \\ \tilde{V}_{3*} = \dfrac{\partial^2 w_{t*}}{\partial x_* \partial y_*}. \end{cases} \tag{12}$$

Hosil qilingan momentlar ifodalari plastinkaning gisterezis tipidagi elastik dissipativ xarakteristikalarini hisobga olish imkonini beradi.

NATIJALAR VA ULARNING MUHOKAMASI

Tasodifiy qo‘zg‘alishlarda gisterezis tipidagi elastik dissipativ xarakteristikali o‘yiq plastinkaning kinetik va potensial energiyalari quyidagicha bo‘ladi:

$$\tilde{T} = \frac{1}{2}\iint \rho h \left(\frac{\partial w_t}{\partial t}\right)^2 dx\,dy - \frac{1}{2}\iint \rho h \left(\frac{\partial w_{t*}}{\partial t}\right)^2 dx_*\,dy_*; \tag{13}$$

$$\begin{aligned} \tilde{V} = \frac{1}{2}\iint &\left(M_{\tilde{x}}\frac{\partial^2 w_t}{\partial x^2} + M_{\tilde{y}}\frac{\partial^2 w_t}{\partial y^2} + 2M_{\tilde{x}\tilde{y}}\frac{\partial^2 w_t}{\partial x \partial y}\right) dx\,dy \\ - \frac{1}{2}\iint &\left(M_{\tilde{x}_*}\frac{\partial^2 w_{t*}}{\partial x_*^2} + M_{\tilde{y}_*}\frac{\partial^2 w_{t*}}{\partial y_*^2} + 2M_{\tilde{x}_*\tilde{y}_*}\frac{\partial^2 w_{t*}}{\partial x_* \partial y_*}\right) dx_*\,dy_*. \end{aligned} \tag{14}$$

Aniqlangan momentlar va (13-14) ifodalar gisterezis tipidagi elastik dissipativ xarakteristikali o‘yiq plastinkaning kinetik va potensial energiyalarini aniqlash imkonini beradi.

Lagranj II-tur tenglamasiga asosan yozamiz,

$$\begin{aligned} &\rho h\left(\ddot{w}_t - \ddot{w}_{t*}\right) + D\left(\frac{\partial^4 w_t}{\partial x^4} + 2\frac{\partial^4 w_t}{\partial x^2 \partial y^2} + \frac{\partial^4 w_t}{\partial y^4}\right) - D\left(\frac{\partial^4 w_{t*}}{\partial x_*^4} + 2\frac{\partial^4 w_{t*}}{\partial x_*^2 \partial y_*^2} + \frac{\partial^4 w_{t*}}{\partial y_*^4}\right) \\ &\quad + 3D(-\theta_1 + j\theta_{2*})\sum_{i_1=0}^{r_1} \tilde{c}_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}\left(\frac{\partial^2}{\partial x^2}\left(\tilde{V}_1|\tilde{V}_1|_a^{i_1}\right) - \frac{\partial^2}{\partial x_*^2}\left(\tilde{V}_{1*}|\tilde{V}_{1*}|_a^{i_1}\right)\right) \\ &\quad + 3D(-\theta_1 + j\theta_{2*})\sum_{i_1=0}^{r_1} \tilde{c}_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}\left(\frac{\partial^2}{\partial y^2}\left(\tilde{V}_2|\tilde{V}_2|_a^{i_1}\right) - \frac{\partial^2}{\partial y_*^2}\left(\tilde{V}_{2*}|\tilde{V}_{2*}|_a^{i_1}\right)\right) \\ &\quad + 6D(1-\mu)(-\tilde{\theta}_1 + j\tilde{\theta}_{2*})\sum_{i_2=0}^{r_2} \tilde{k}_{i_2}\frac{h^{i_2}}{2^{i_2}(i_2+3)}\left(\frac{\partial^2}{\partial x \partial y}\left(\tilde{V}_3|\tilde{V}_3|_a^{i_2}\right) - \frac{\partial^2}{\partial x_* \partial y_*}\left(\tilde{V}_{3*}|\tilde{V}_{3*}|_a^{i_2}\right)\right) \\ &\quad = -\rho h\,\ddot{w}_0. \end{aligned} \tag{15}$$

bunda $\ddot{w}_0$ – asos tezlanishi.

Bu olingan differensial tenglama tasodifiy qo‘zg‘alishlar ta’siridagi gisterezis tipidagi elastik dissipativ xarakteristikali o‘yiq plastinkaning harakat differensial tenglamasi hisoblanadi.

XULOSA

Tasodifiy qo‘zg‘alishlar ta’siridagi gisterezis tipidagi elastik dissipativ xarakteristikali o‘yiq plastinkaning harakat differensial tenglamasi Lagranj II-tur tenglamasiga asosan olindi. Bu olingan harakat differensial tenglamasi tasodifiy qo‘zg‘alishlar ta’siridagi gisterezis tipidagi elastik dissipativ xarakteristikali, o‘yiqqa ega plastinkaning dinamikasini o‘rganish va ustuvorligini parametrlarning turli qiymatlarida tekshirish imkonini beradi.

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Rezyume

This paper investigates the problem of deriving the differential equation of motion for a perforated plate with hysteretic elastic-dissipative characteristics subjected to random excitations. The dissipative properties of the plate material are described based on the Pisarenko-Boginich hypothesis. A rectangular plate containing a rectangular cutout is considered. It is assumed that the sides of the cutout are parallel to the sides of the plate. Furthermore, the cutout may be located arbitrarily within the plane of the plate. Based on this geometrical and physical model, the equation of motion of the plate is derived. The proposed model takes into account both random external excitations and the internal dissipative properties of the material. The obtained equation provides a theoretical foundation for studying the dynamics of elastic-dissipative systems.

Key words: perforated plate, hysteresis, bending, bending moment, twisting moment, differential equation, natural mode shape.

Rezyume

В данной работе исследуется задача построения дифференциального уравнения движения пластинки с отверстием, обладающей упруго-диссипативной характеристикой гистерезисного типа, при воздействии случайных возбуждений. Диссипативные свойства материала пластинки описаны на основе гипотезы Писаренко-Богинича. В исследовании рассматривается прямоугольная пластинка с прямоугольным отверстием. Предполагается, что стороны отверстия параллельны сторонам пластинки. Кроме того, допускается произвольное расположение отверстия в плоскости пластинки. На основе данной геометрической и физической модели сформулировано уравнение движения пластинки. Построенная модель учитывает случайные внешние воздействия, а также внутренние диссипативные свойства материала. Полученное уравнение служит теоретической основой для исследования динамики упруго-диссипативных систем.

Ключевые слова: пластинка с отверстием, гистерезис, изгиб, изгибающий момент, крутящий момент, дифференциальное уравнение, форма собственных колебаний.