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INFLUENCE OF PLASMA ON DEFLECTION ANGLE BY A SCHWARZSCHILD BLACK HOLE IN PERFECT FLUID DARK MATTER

Annotation
We have studied the influence of uniform and non-uniform distributed plasma on gravitational weak lensing around a black hole in the presence of perfect fluid dark matter (PFDM). We have represented that the PFDM and plasma parameter has opposite effect on the change of the value of the deflection angle of the light rays in a weak field regime.

Introduction. One of the first fundamental signatures of the general relativity(GR) since its discovery by Einstein in 1916 is the influence of the observables black hole shadow and gravitational lensing(in strong and weak field regime). The effect is based on the light ray deflection near the gravitating object due to space-time curvature. The classical gravitational lensing in general relativity has been described in reviews with detail [1]. At the same time it is important to consider the photon motion(light rays) in curved space-time since it is relevant for further study of gravitational lensing. Moreover, the plasma medium around compact object may interact with the electromagnetic wave and consequently influence on trajectory of light rays. Particularly, in Refs. [2-11] scientists have studied the photon motion in different space-times metric and some of them devoted to investigate the effect of plasma together with gravity on photon motion. In addition, one of the hot topic is strong gravitational lensing in a vacuum; recently, the theoretical analysis of strong lensing is studied by several authors for various space-time geometries in Refs. [12-14].

There is some evidence for the existence of dark matter or dark energy, and according to the standard cosmological model [15] our Universe mainly contains 68% dark energy, about 27% dark matter and 5% baryonic matter [15]. Dark matter is nonbaryonic matter. Consequently one may ask the question whether dark energy or dark matter affect the astrophysical processes in the vicinity of compact objects. It is true that dark matter is invisible, so it is difficult to test it, but we can feel the effect of dark matter or dark energy from observations. From this point on, dark matter is extremely important for studying the properties of the physics of a black hole. It is well known that dark matter or dark energy also exists around a black hole, so their contribution is always very important and interesting, when we study the motion of particles around a black hole in the Universe. In the latest decay, a black hole surrounded by the quintessence of dark energy is examined in more detail [16]. Further properties of the spacetime around the black hole surrounded by quintessence have been explored in Refs. [17,18]. Alternative model for dark matter has been also proposed in [16] where quintessence scalar field is considered as PFDM. The solution of black hole surrounded by PFDM contains the logarithmic term $\lambda ln(r/|\lambda|)$ appearing in the metric functions, corresponding to the nonvanishing contribution of dark matter through parameter λ. Following the idea that dark matter halo is formed by weakly interacting massive particles in Ref. [19], it has been derived a similar black hole solution involving a logarithmic term. The properties of the spacetime around the black hole surrounded by PFDM have been investigated in [20].

The main motivation of the study performed in this paper is to test the PFDM model for dark matter using the photon motion around compact object in the presence of plasma medium. Observable properties of the light deflection angle due to gravitational lensing may help to get constraints on model and theory describing the dark matter around compact object. Consequently this will help us to explain the nature of the dark matter and the astrophysical processes related to the effects of dark matter. Particularly, in this paper we have considered the photon motion around black hole surrounded by PFDM. Performing the analysis of the effects of PFDM on light deflection we have found out that the presence of PFDM causes decrease the light deflection. This may be interpreted as negative gravitational effect of PFDM on photons since PFDM is surrounding the central object and photons. The gravitational effects on photons of PFDM and central object are opposite to each other.

Weak lensing in the presence of plasma. The metric describing a static and spherically symmetric black hole immersed in perfect fluid dark matter in Schwarzschild coordinates (t, r, θ, φ) is given by [19]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2 \qquad (1)$$

With

$$f(r) = 1 - \frac{2M}{r} + \frac{\lambda}{r} ln \frac{r}{|\lambda|} \qquad (2)$$

where *M* is the mass of the black hole and parameter λ is associated with the density and the pressure of dark matter [19]. We will consider a weak-field regime approximation defined as shown below:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \qquad (3)$$

where $\eta_{\alpha\beta}$ is expression for the flat geometry and $h_{\alpha\beta}$ is a perturbation of the flat spacetime geometry with the following general properties [5]:

$$\eta_{\alpha\beta} = diag(-1,1,1,1),$$
$$h_{\alpha\beta} \ll 1, \quad h_{\alpha\beta} \to 0 \ \ under \ \ x^i \to \infty$$
$$g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta}, \quad h^{\alpha\beta} = h_{\alpha\beta} \qquad (4)$$

Here we study the effect of the plasma on the gravitational deflection angle in the presence of the PFDM in the Schwarzschild spacetime geometry. Consider the basic expression for the deflection angle in the presence of a plasma medium as [5,8]

$$\hat{\alpha}_k = \frac{1}{2}\int_{-\infty}^{\infty}\left(h_{33} + \frac{h_{00}\omega^2 - K_e N(x^i)}{\omega^2 - \omega_e^2}\right)_k dz, \qquad (5)$$

where $N(x^i)$ is the concentration of charged particles in plasma, ω and ω_e are photon and plasma frequencies, respectively. $K_e = 4\pi e^2/m_e$ is constant value of plasma particle. On the other hand, one may rewrite Eq. (5) in the different form as [5]:

$$\hat{\alpha}_b = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{dh_{33}}{dr} + \frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr} - \frac{K_e}{\omega^2-\omega_e^2}\frac{dN}{dr}\right) dz. \qquad (6)$$

The value $\hat{\alpha}_b$ can be both negative (towards from the compact object) and positive (away from the compact object). In the weak field regime at large distances r from the compact object surrounded by PFDM the spacetime metric can be expressed as

$$ds^2 = ds_0^2 + \left(\frac{R_s}{r} - \frac{\lambda}{r}ln\frac{r}{|\lambda|}\right)dt^2 + \left(\frac{R_s}{r} - \frac{\lambda}{r}ln\frac{r}{|\lambda|}\right)dr^2, \qquad (7)$$

where $ds_0^2 = -dt^2 + dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)$ is the line element in the flat spacetime and we have used notation $R_s = 2M$.

In order to calculate Eq. (6) one can rewrite the components $h_{\alpha\beta}$ in Cartesian coordinates in the form

$$h_{00} = (\frac{R_s}{r} - \frac{\lambda}{r}ln\frac{r}{|\lambda|})$$
$$h_{ik} = (\frac{R_s}{r} - \frac{\lambda}{r}ln\frac{r}{|\lambda|})n_i n_k$$
$$h_{33} = \left(\frac{R_s}{r} - \frac{\lambda}{r}ln\frac{r}{|\lambda|}\right)cos^2\chi, \qquad (8)$$

where new notation $cos^2\chi = z^2/(b^2+z^2)$ and $r^2 = b^2 + z^2$ are introduced. One may easily calculate the derivative of h_{00} and h_{33} and get as

$$\frac{dh_{33}}{dr} = -\frac{3z^2R_s}{r^4} - \frac{\lambda z^2}{r^4} + \frac{3\lambda z^2 ln\left(\frac{r}{|\lambda|}\right)}{r^4}$$
$$\frac{dh_{00}}{dr} = -\frac{\lambda}{r^2} + \frac{\lambda ln\left(\frac{r}{|\lambda|}\right)}{r^2} - \frac{R_s}{r^2}. \qquad (9)$$

Now one may decompose the expression for the deflection angle in the following form [8]:

$$\hat{\alpha}_b = \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 \qquad (10)$$

With

$$\hat{\alpha}_1 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\frac{dh_{33}}{dr}dz,$$
$$\hat{\alpha}_2 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr}\right)dz,$$
$$\hat{\alpha}_3 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(-\frac{K_e}{\omega^2-\omega_e^2}\frac{dN}{dr}\right)dz, \qquad (11)$$

where the new designations $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\alpha}_3$ correspond to contribution of deflection angle due to gravity, homogeneous and inhomogeneous plasma medium, respectively. Throughout the paper we will use Eq. (10) in order to study the effects of plasma medium on the deflection angle.

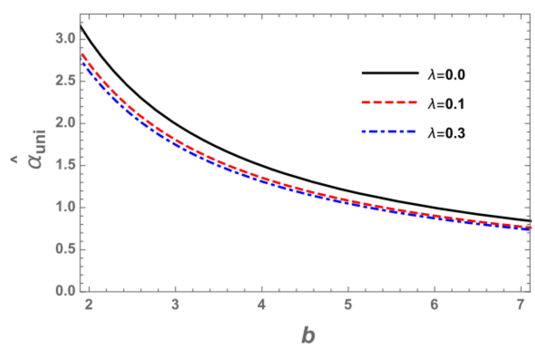

FIG. 1. The dependence of the deflection angle $\hat{\alpha}_{\text{uni}}$ uni on the impact parameter for different values of perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel).

A. Uniform plasma effect to the deflection angle

In this subsection, we will study the deflection angle in the presence of a uniform plasma medium using Eq. (10), which can be rewritten as

$$\hat{\alpha}_{\text{uni}} = \hat{\alpha}_{\text{uni1}} + \hat{\alpha}_{\text{uni2}} + \hat{\alpha}_{\text{uni3}}, \qquad (12)$$

where $\hat{\alpha}_{\text{uni1}}$ and $\hat{\alpha}_{\text{uni2}}$ can be interpreted as contribution to the deflection angle due to uniform plasma and $\hat{\alpha}_{\text{uni3}} = 0$ due to the uniform distribution of plasma. Using Eqs. (10), (9) and (12) one may easily obtain the deflection angle formula for photons around compact object surrounded by uniform plasma and PFDM as

$$\hat{\alpha}_{\text{uni}} = -\left(\frac{\lambda}{b} + \frac{\lambda ln\left(\frac{r}{|2\lambda|}\right)}{b} - \frac{R_s}{b}\right) - \left(\frac{\lambda ln\left[\frac{1}{2\lambda}\right]}{b} - \frac{R_s}{b}\right)\left(\frac{\omega^2}{\omega^2 - \omega_e^2}\right). \qquad (13)$$

Figure 1 shows the dependence of the deflection angle from the impact parameter of the photons for the different values of PFDM and plasma parameters. Figure 2 represents the dependence of the deflection angle form plasma and dark matter parameters for fixed values of impact parameter b. From Figs. 1 and 2 one may see that the deflection angle decreases with the increase of the PFDM parameter. On the other hand, the presence of plasma medium increase the gravitational deflection angle of photons. Note, that the deflection angle strive to zero with an increase of the impact parameter b.

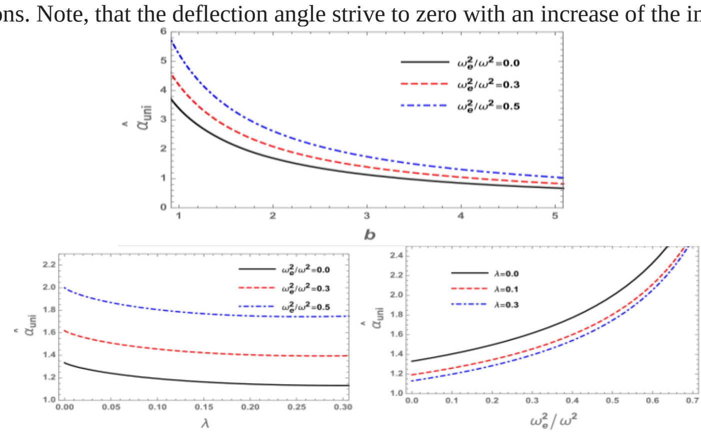

FIG. 2. The dependence of the deflection angle $\hat{\alpha}_{\text{uni}}$ on the perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel) for the fixed value of impact parameter $b = 3M$.

B. Non-uniform plasma effect to the deflection angle

Now we explore the deflection angle of photons around compact object in the presence of nonuniform plasma and perfect fluid dark matter. Here we use the singular isothermal sphere (SIS) medium for the nonuniform plasma distribution [5]. The plasma concentration of SIS medium has the following form [5,8]

$$N(r) = \frac{\rho(r)}{km_p} \qquad (14)$$

and

$$\rho(r) = \frac{\sigma_v^2}{2\pi r^2}, \qquad (15)$$

where σ_v is the dispersion velocity and $\rho(r)$ is the plasma density. Now Eq. (10) can be decomposed as

$$\hat{\alpha}_{SIS} = \hat{\alpha}_{\text{SIS1}} + \hat{\alpha}_{\text{SIS2}} + \hat{\alpha}_{\text{SIS3}}, \qquad (16)$$

where $\hat{\alpha}_{\text{SIS1}}$ and $\hat{\alpha}_{\text{SIS2}}$ correspond to the contribution to the deflection angle due to pure gravity and plasma effects and $\hat{\alpha}_{\text{SIS3}}$ corresponds to the contribution due to gradient of the density of the plasma medium. Using Eqs. (10), (9) and (16) one may easily obtain the deflection angle of photons around Schwarzschild black hole surrounded by nonuniform plasma in presence of PFDM in the following form:

$$\hat{\alpha}_{SIS} = -\frac{\lambda\omega_c^2 ln\left(\frac{1}{2\lambda}\right) R_s^2}{\pi b^2 \omega^2} + \frac{\omega_c^2 R_s^2}{\pi b^2 \omega^2} + \frac{\omega_c^2 R_s^2}{2b^2\omega^2} - \frac{\lambda}{b} + \frac{2\lambda ln\left(\frac{1}{2\lambda}\right)}{b} + \frac{2R_s}{b}, \qquad (17)$$

where we introduced new expression as

$$\omega_c^2 = \frac{\sigma_v^2 K_e}{2km_p R_s^2}. \qquad (18)$$

Figure 3 shows the influence of dark matter parameter and nonuniform plasma medium on the deflection angle of photons around gravitation object. Figure 4 shows the dependence of the deflection angle on plasma and ark matter parameter λ for the fixed values of impact parameter and nonuniform plasma case. From Figs. 3 and 4 one may see that the deflection angle is decreasing in the presence of PFDM parameter. Moreover, similarly to the case of uniform plasma case, due to the effect of nonuniform plasma medium the deflection angle increases. Now we make comparison the deflection angle for the photons around Schwarzschild black hole in the presence of perfect fluid dark matter for the cases uniform and nonuniform plasma medium. Figure 5 shows the difference of the deflection angle for the cases of uniform and nonuniform plasma medium. From this dependence one may easily speculate that the deflection angle of photon beam around black hole increases. From Fig. 5 one can easily compare both cases: the deflection angle is greater in a homogeneous case than an inhomogeneous plasma medium for the fixed value of impact parameter. Moreover, one can also see that with an increase of the PFDM parameter for uniform and nonuniform cases, the orbit of the photon comes closer to the central compact object. The decrease of the deflection angle with the increase of PFDM parameter for the fixed values of impact parameter can be interpreted as the negative effect of dark matter. Since the dark matter is distributed in the space around black hole the gravitational effect of PFDM on light rays from the outside of the photon trajectory has opposite effect with respect to the effect caused by central object. Thus gravity of the central object increases the deflection while PFDM causes to decrease the deflection angle of light rays.

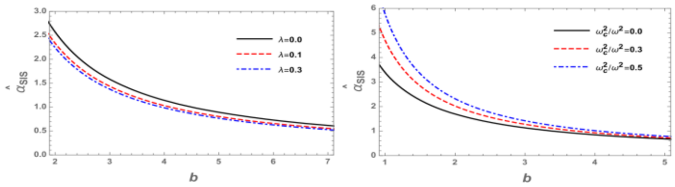

FIG. 3. The dependence of the deflection angle $\hat{\alpha}_{SIS}$ on the impact parameter for the different values of PFDM parameter λ (left panel) and plasma medium (right panel).

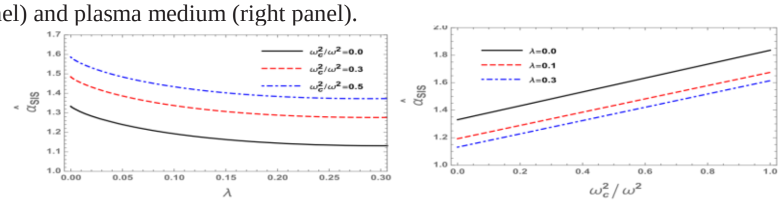

FIG. 4. The dependence of the deflection angle $\hat{\alpha}_{SIS}$ on the perfect fluid dark matter parameter λ (left panel) and plasma medium (right panel) for the fixed value of impact parameter $b = 3M$

Conclusions. In the present paper, the influence of plasma on weak gravitational lensing has been studied for the Schwarzschild black hole in the presence of perfect fluid dark matter(PFDM). Observable parameter such as the deflection angle is important for the gravitational lensing in general relativity. We have explored two cases: uniform nonuniform distributed plasma. Finally, the main results of the paper can be summarized as follows:

(i) The analysis showed that the deflection angle decreases with the increase of the PFDM parameter. At the same time the presence of plasma medium increase the gravitational deflection angle of photons.

(ii) We have obtained that the increase of parameter of nonuniform plasma medium causes the increase of deflection angle.

(iii) The comparison of the deflection angle for the photons around black hole in the presence of perfect fluid dark matter in the cases of uniform and non-uniform distributed plasma showed that the deflection angle is greater in a homogeneous case with compare to one in inhomogeneous plasma medium.

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