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## COLLISION OF CHARGED PARTICLES IN THE VICINITY OF SCHWARZSCHILD-LIKE BLACK HOLE

Annotation

This work is devoted to the study of the nature of the collision of charged test particles moving around a Schwarzschild-like black hole surrounded by a uniform magnetic field. The equations of motion are derived in an immeasurable form. To study the effect of a black hole on the acceleration of particle in the case of colliding particles around ISCO with different charges was obtained.

**Key words:** black holes, stable orbit, horizon, ISCO, collision, center of mass-energy.

## СТОЛКНОВЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ В БЛИЗОСТИ ЧЕРНОЙ ДЫРЫ ШВАРЦШИЛЬДА-ЛАЙК

Аннотация

Работа посвящена изучению характера столкновения заряженных пробных частиц, движущихся вокруг черной дыры типа Шварцшильда, окруженной однородным магнитным полем. Уравнения движения выведены в неизмеримой форме. Для изучения влияния черной дыры на энергию центра масс системы сталкивающихся частиц рассмотрен случай столкновения: столкновение частиц, вращающихся вокруг ISCO с разными зарядами.

**Ключевые слова:** чёрные дыры, стабильная орбита, горизонт, ISCO, столкновение, центр масс-энергии.

## SCHWARZSCHILD-LIKE QORA TUYNUK YAQINIDA ZARYADLANGAN ZARRALARNING TO‘QNASHISHI

Annotatsiya

Ushbu tadqiqot bir jinsli magnit maydon bilan o‘ralgan Schwarzschild-like qora tuynuk atrofida harakatlanuvchi zaryadlangan sinov zarralarining to‘qnashuvi tabiatini o‘rganishga bag‘ishlangan. Harakat tenglamalari o‘lchovsiz shaklda olinadi. Qora tuynukning to‘qnashuvchi zarralar tizimining massa-energiya markaziga ta’sirini o‘rganish uchun to‘qnashuv holati ko‘rib chiqiladi: ISCO атрофида турли зарядли айланувчи заррачаларнинг to‘qnashuvi.

**Kalit so‘zlar**: qora tuynuklar, stabil orbita, gorizont, ISCO, to‘qnashuv, massa-energiyalar markazi.

**Introduction.** There is a belief that a black hole is the source of the most energetic objects in the universe, such as active galactic nuclei, ultra-luminous X-ray binaries, gamma-ray bursts, etc. The leading dynamo of the energetics of these objects is the gravity surrounding the black holes. Some of the energy extraction mechanisms from black holes are based on the matter accretion onto the central object. Another way is to extract the thermodynamical energy of the black hole (which is also defined through the gravitational energy of the compact object). In Refs. [1-5] were observed cases of thermodynamics of the spacetime in different gravities.

Black hole also can be as a particle accelerator. Banados, Silk, and West have studied the center of mass energy of the particles falling into rotating black holes and observed that the latter may reach ultra-high values [6]. Moreover, it has been shown that for the fine-tuned values of the angular momentum of the particles, the center of mass energy of colliding particles at the horizon of an extremely rotating black hole may diverge [6]. The center of mass energy and energetic processes of the particles around black holes in different gravity models have been extensively studied in Refs. [7-10].

Additionally, we will examine the interactions between these particles and the surrounding field and the collisions that occur. A novel solution for the metric tensor describing a regular black hole of Bardeen type has been proposed in [11]. The outline of the paper is as follows: we present the dynamics of a particle in a dimensionless form in order to study the characteristic features of its motion. After that we discuss the case of particle collisions in the vicinity of a black hole. Finally, we summarize the main conclusions and results obtained during the research.

**Dimensionless form of dynamical equations.** We are interested in the case of collision of charged particles in the electromagnetic field in the vicinity of Schwarzschild-like black hole. In spherical symmetric Boyer-Linquist coordinate system $(t, r, \theta, \phi)$ described by the metric [12, 13]:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

where:

$$f(r) = 1 - \frac{2Me^{-a/r}}{r} \tag{2}$$

Corresponding Eq. (1), metric has of two Killing vectors:

$$\xi_{(t)}=\xi^{\mu}_{(t)}=\partial_t, \xi_{(\phi)}=\xi^{\mu}_{(\phi)}=\partial_\phi, \tag{4}$$

where $\xi^{\mu}_{(t)}=(1,0,0,0)$ and $\xi^{\mu}_{(\phi)}=(0,0,0,1)$. In this section, we will derive the basic equations of motion for a charged particle around a Schwarzschild-like black hole in dimensionless form. For clarity and to facilitate further investigation, we will assume a value of $M$ equal to 1. Since our metric resembles that of a Schwarzschild-like solution, as described in Eq. (1), and due to the presence of two Killing vectors in Eq. (4), we can express the constants of motion using the 4-momentum in the following form [14]:

$$E=-\xi^{\mu}_{(t)}p_\mu=m\frac{dt}{d\tau}f \tag{5}$$

$$L=\xi^{\mu}_{\phi}p_\mu=\left[m\frac{d\phi}{d\tau}r^2+\frac{1}{2}qB(r^2-2a)\right]\sin^2\theta \tag{6}$$

where $p_\mu$ is the generalization of 4-momentum. For more convenient use, we use the following dimensionless versions of $,\mathcal{L},\tau,b$ :

$$\rho=\frac{r}{r_h e^a},\sigma=\frac{\tau}{r_h e^a},l=\frac{L}{m r_h e^a},$$
$$\mathcal{E}=\frac{E}{m},b=\frac{qBr_he^a}{2m},f=1-\frac{e^{-a}}{\rho},$$

Where $r_h$ is the radius of the horizon of our metric from Eq. (3). We use the expression for 4 -momentum of the particle $u^\mu u_\mu=-1$, and obtain the resulting dimensionless equations:

$$\left(\frac{d\rho}{d\sigma}\right)^2=\mathcal{E}^2-U \tag{7}$$

$$\frac{dt}{d\sigma}=\frac{\mathcal{E}\rho}{\rho-e^a} \tag{8}$$

$$\rho\frac{d\phi}{d\sigma}=\beta,\beta=\frac{1}{\rho}\left(l+\frac{2ab}{(r_he^a)^2}\right)-b\rho \tag{9}$$

where $U$ is the effective potential given by the equation:

$$U=\left(1-\frac{e^{-a}}{\rho}\right)(1+\beta^2) \tag{10}$$

The equations $U_{,\rho}=0$ and $U_{,\rho\rho}=0$ define the innermost stable circular orbits (ISCO). By solving this system of equations, we obtain an explicit expression for $l$ and :

$$b^2_\pm=\frac{9\mp N-e^a\rho(4e^{2a}\rho^2-21e^a\rho+30)}{8\rho^2(e^a\rho-1)^2(4e^{2a}\rho^2-10e^a\rho+3)} \tag{11}$$

$$l_\pm=b_\pm e^{-2a}\frac{e^{2a}\rho^2r_h^2N\pm 2a(e^a\rho-3)^2}{(e^a\rho-3)^2r_h^2} \tag{12}$$

where

$$N=\sqrt{(1-3e^a\rho)(e^a\rho-3)^3} \tag{13}$$

In the case of Schwarzschild spacetime, the equation exists within the interval $\rho\in(1/3,3]$.

## COLLISION OF CHARGED PARTICLES

We study the energy of particles that collide around a black hole in the Schwarzschild-like metric using Frolov's method [15]. It is well known that particles moving along ISCO have a minimum magnitude of energy and angular momentum, and their velocities become close to light speed. Because of this, we get the nonzero components of the momentum of the particles found in the form:

$$p^t=m\gamma e^t_{(t)} \tag{14}$$

$$p^\phi=m\gamma v e^\phi_{(\phi)} \tag{15}$$

$$e^t_{(t)}=\sqrt{\eta_{tt}g^{tt}}=\frac{1}{\sqrt{f}} \tag{16}$$

$$e^\phi_{(\phi)}=\sqrt{\eta_{\phi\phi}g^{\phi\phi}}=\frac{1}{r}, \tag{17}$$

where Eq. (16) and Eq. (17) are orthonormal tetrad components in equatorial motion around fixed orbit. Here $v$ (which can be both positive and negative) is a velocity of the particle with respect to a rest frame, and $\gamma$ is the Lorentz gamma factor:

$$\frac{d\phi}{d\tau}=\frac{p^\phi}{m}=\frac{v\gamma}{r} \tag{18}$$

From Eq. (5) one may easily get the following

$$\frac{d\phi}{d\tau}=\frac{\beta}{r},\gamma=\sqrt{1+\beta^2}. \tag{19}$$

Here we can use expressions (11), (12) and rewrite $\beta$ as:

$$\gamma_\pm=\sqrt{\frac{-9\pm N+e^a\rho(4e^{2a}\rho^2-21e^a\rho+30)}{(e^a\rho-3)(4e^{2a}\rho^2-10e^a\rho+3)}} \tag{20}$$

The dependence of the gamma factor on $\rho$ for fixed values of $a$ is shown in Figs. 2 and 3.

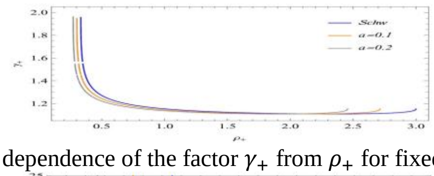


FIG. 2: The dependence of the factor $\gamma_+$ from $\rho_+$ for fixed values of $a$.

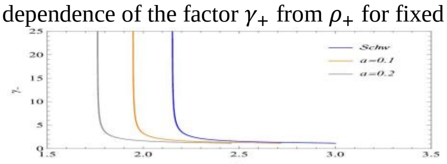


FIG. 3: The dependence of the $\gamma_-$ at the position of the ISCO $\rho_-$.

As we can observe in Fig. 2, the gamma factor takes values slightly greater than 1, but in Fig. 3$\gamma_-$assumes quite high values. For $\gamma_+$, the energy cannot be high, as $\gamma_+$is not significantly different from 1. However, for $\gamma_-$, the energy increases significantly. It can be seen from Figs. 3 and 2 that the black hole parameter $a$ helps to reduce the radius, while having little effect on the values of $\gamma$. In Table I shown the numerical results for the maximum and minimum values of the quantities of $\rho_\pm$ and $\gamma_\pm$, for various values of the parameter $a$.

We learn a state of a collision of two particles with the same mass $m$, with charges $+q, -q$, and moving along the same circular orbit in opposite directions. After the collision the four-momentum will be equal to:

$$P^0 = 2m\gamma\frac{1}{\sqrt{f}}. \tag{21}$$

| $a = 0$ | $a = 0.1$ | $a = 0.2$ |
|---|---|---|
| $\rho_{+\max} = 3$ | $\rho_{+\max} = 2.714$ | $\rho_{+\max} = 2.456$ |
| $\rho_{+\min} = 0.333$ | $\rho_{+\min} = 0.302$ | $\rho_{+\min} = 0.273$ |
| $\rho_{-\max} = 3$ | $\rho_{-\max} = 2.715$ | $\rho_{-\max} = 2.456$ |
| $\rho_{-\min} = 2.151$ | $\rho_{-\min} = 1.947$ | $\rho_{-\min} = 1.761$ |
| $\gamma_{+\max} = 1.959$ | $\gamma_{+\max} = 1.951$ | $\gamma_{+\max} = 1.963$ |
| $\gamma_{+\min} = 1.1541$ | $\gamma_{+\min} = 1.154$ | $\gamma_{+\min} = 1.154$ |
| $\gamma_{-\max} = 200$ | $\gamma_{-\max} = 586$ | $\gamma_{-\max} = 503$ |
| $\gamma_{-\min} = 1.155$ | $\gamma_{-\min} = 1.155$ | $\gamma_{-\min} = 1.155$ |

TABLE I: Maximum and minimum values of $\rho_\pm$ and $\gamma_\pm$ at fixed values of $a$.

The value of the center of mass-energy after the collision:

$$\mathcal{M} = 2m\gamma \tag{22}$$

From analysis given in Tab.I, it can be seen that in some cases it is possible to observe high energies in a collision corresponding to large gamma values. In this case, we gave a similar result to previous work [15]. We do not consider high gamma values, as a particle in an ISCO orbit would need to possess such energy even at a great distance from the black hole in order to maintain a stable orbit.

**Conclusion.** Now, the energy within a center of mass system can reach significant values even with relatively low initial energies for the colliding particles. Under certain conditions, scattering may result in a significant increase in the energy of one particle, significantly exceeding its initial value. This confirms the effectiveness of acceleration mechanisms, although their implementation depends on interaction parameters, such as the energy and direction of particle motion. We can conclude that the collision energy is low when two particles collide along the same trajectory near the innermost stable circular orbit (ISCO) in opposite directions. However, the collision energy can be high if a particle falling from infinity collides with a charged particle orbiting around the ISCO. A black hole can serve as an effective particle accelerator, contributing to high collision energies, but the success of this process depends on the interaction parameters, such as the energy and direction of motion of the colliding particles. Our analysis is somewhat simplified, as we used a basic model of particle motion and collision without considering plasma and other relevant effects. In the future, we intend to investigate the impact of this parameter on other physical phenomena. It is crucial to acknowledge that the results presented in this article are primarily based on the parameter involved in the black hole solution, which has a minimal effect.

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