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GRAVITATIONAL LENSING OF SEN BLACK HOLES IN PLASMA MEDIUM

Annotation

We investigate the optical properties of the spacetime surrounding a Sen black hole, focusing on the photon sphere and weak gravitational lensing. Our analysis reveals that the effective charge of the Sen black hole significantly influences these phenomena. Specifically, an increase in the effective charge leads to a contraction in the radius of the photon sphere. Additionally, the bending angle of light rays diminishes as the effective charge increases. Our study provides observational bounds on the effective charge based on these optical characteristics. Our findings offer new insights into the impact of effective charge and plasma on the observational signatures of Sen black holes.
Key words: Sen black hole, plasma effects, photon motion, week gravitational lensing

ГРАВИТАЦИОННОЕ ЛИНЗИРОВАНИЕ ЧЁРНЫХ ДЫР СЕНА В ПЛАЗМЕННОЙ СРЕДЕ

Аннотация

Мы исследуем оптические свойства пространства-времени вокруг черной дыры Сена, сосредотачиваясь на фотонной сфере и слабом гравитационном линзировании. Наш анализ показывает, что эффективный заряд черной дыры Сена существенно влияет на эти явления. В частности, увеличение эффективного заряда приводит к уменьшению радиуса фотонной сферы. Кроме того, угол отклонения световых лучей уменьшается с увеличением эффективного заряда. Наше исследование предоставляет наблюдательные ограничения на эффективный заряд, основанные на этих оптических характеристиках. Наши результаты дают новые представления о влиянии эффективного заряда и плазмы на наблюдаемые характеристики черных дыр Сена.
Ключевые слова: черная дыра Сена, эффекты плазмы, движение фотонов, слабое линзирование

SEN QORA TUYNUKLARINING GRAVITATSION LINZALANISHI PLAZMA MUHITIDA

Annotatsiya

Biz Sen qora tuynugi atrofidagi fazo-vaqtning optik xususiyatlarini o‘rganamiz va foton orbitasi hamda zaif gravitatsion linzalanganlikka e’tibor qaratamiz. Tahlilimiz shuni ko‘rsatadiki, Sen qora tuynugidagi samarali zaryad ushbu hodisalarga sezilarli ta’sir ko‘rsatadi. Xususan, samarali zaryadning oshishi foton sferasining radiusining kamayishiga. Bundan tashqari, yorug‘lik nurlarining egilish burchagi samarali zaryad ortishi bilan kamayadi. Tadqiqotimiz ushbu optik xususiyatlarga asoslangan holda samarali zaryad bo‘yicha kuzatuv cheklovlarini taqdim etadi. Natijalarimiz Sen qora tuynugidagi samarali zaryad va plazmaning kuzatiladigan belgilarga ta’siri bo‘yicha yangi tushunchalarni taqdim etadi.
Kalit so‘zlar: Sen qora tuynugi, plazma ta’siri, foton harakati, zaif gravitatsion linzalanganlik

Introduction. Gravitational lensing, a key prediction of Einstein's general relativity, occurs when light bends around a massive object like a black hole [1]. This effect was first confirmed by Eddington’s observation of light bending around the Sun, highlighting its importance in astrophysics [2]. In black hole studies, gravitational lensing is particularly interesting due to strong gravitational fields [3]. Weak gravitational lensing, where light is slightly deflected at large distances from the black hole, provides valuable insights into their properties and helps test gravity theories [4]. It is also crucial for detecting dark matter and dark energy [5,6]. Studies on weak lensing by different black hole types, including rotating ones, have been extensively conducted [7,8]. Near black holes, surrounding plasma—such as from an accretion disk—can further affect light bending. Plasma modifies lensing properties and alters the black hole’s shadow [9]. Perlick, Bisnovatyi-Kogan, and Tsupko have made significant contributions to understanding these effects [10-13]. These studies are particularly relevant for interpreting high-resolution observations, such as those from the Event Horizon Telescope (EHT), which provide new insights into gravity in extreme conditions [14,15].

Dynamics around black hole. A. Sen Black Hole. The action describing the spacetime of the Sen black hole in the string frame is presented as follows:

$$S = \int d^4x\sqrt{-g}e^{-\phi}\mathcal{L} \tag{1}$$

The metric function derived in this configuration was found by Sen in Ref. [21] and reads:

$$f(r) = 1 - \frac{2M}{r + q_m^2/M}, \quad (2)$$

where q_m is an effective charge measured by a static observer at infinity, which characterizes specific hair. It is not difficult to notice that at $q_m = 0$, the Sen metric of the black hole coincides with the Schwarzschild metric. Here, we introduced a new variable like $q_m^2 = q$ just for convenience.

B. Null geodesic in dyonic Sen spacetime. In this subsection, we examine the dynamics of photon motion near a black hole when it is surrounded by plasma. To analyze this scenario, we can conveniently utilize the Hamilton-Jacobi equation, which is given by:

$$H(x^\alpha, p_\alpha) = \frac{1}{2}\tilde{g}^{\alpha\beta}p_\alpha p_\beta. \quad (3)$$

Here, x^α describes the spacetime coordinates. We can use definition of the effective metric tensor $\tilde{g}^{\alpha\beta}$ as

$$\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - (n^2 - 1)u^\alpha u^\beta, \quad (4)$$

where u^β represents the four-velocity of the photon, and n signifies the refractive index of the medium. It can be determined using the following formula

$$n^2 = 1 - \frac{4\pi e^2 N(r)}{m_e \omega(r)^2}, \quad (5)$$

where e and m_e represent the electron's charge and mass, respectively, and N denotes the electron number density. The photon frequency ω(r), as observed by a stationary observer, is determined using the gravitational redshift equation

$$\omega(r) = \frac{\omega_0}{\sqrt{f(r)}}. \quad (6)$$

Here $\omega_0 = \text{const}$ is the frequency measured by observer at infinity. Light can propagate in plasma only when its frequency is greater than the plasma frequency, therefore $n > 0$ condition must be done. The orbital radius of light encircling a black hole, specifically the one forming a photon sphere with a radius r_{ph}, is obtained by solving the following equation

$$\left.\frac{d(h^2(r))}{dr}\right|_{r=r_{ph}} = 0 \text{ and } h^2(r) \equiv r^2\left[\frac{1}{f(r)} - \frac{\omega_p^2(r)}{\omega_0^2}\right]. \quad (7)$$

C. Homegeneous plasma. First, we consider a homogeneous (uniform) plasma case with $\omega_p^2(r) = \text{const}$. In this case we can solve Eq. (8) analytically and it is written as

$$\frac{r_p}{M} = \frac{\left(1 - \frac{\omega_p^2}{\omega_0^2}\right)\left(2 - \frac{q}{M^2}\right) + \sqrt{\left(1 - \frac{\omega_p^2}{\omega_0^2}\right)\left(2 - \frac{q}{M^2}\right) + \frac{1}{4}} - \frac{1}{2}}{1 - \frac{\omega_p^2}{\omega_0^2}}. \quad (8)$$

From Fig. 1 (upper panel) it is easy to see that in the case of homogeneous plasma radius of photon sphere decreases with increasing parameter q/M and increases with increasing plasma frequency.

D. Inhomogeneous plasma. In this subsection we study photonic spheres in the presence of inhomogeneous plasma, where the plasma frequency must satisfy a simple power law of the form

$$\omega_p^2(r) = \frac{z_0}{r^k}, \quad (9)$$

Where z_0 and $k > 0$ are free parameters. From Fig. 1 (middle and lower panels) it is easy to see that in the case of inhomogeneous plasma photon sphere radius decreases with increasing parameter q/M and the parameter k, and increases with increasing ratio $z_0/M\omega_0^2$.

Effects of plasma medium on Gravitational weak lensing. The gravitational effects in the weak-field approximation can be analyzed by applying the following decomposition of the spacetime around a dense object [14]

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \quad (10)$$

Now we want to examine the plasma effects on the bending angle of the light rays. In the case of plasma medium, the deflection angle can be written as

$$\hat{\alpha}_i = \frac{1}{2}\int_{-\infty}^{\infty}\left(h_{33} + \frac{h_{00}\omega^2 - K_e N(x^i)}{\omega^2 - \omega_e^2}\right)_{,i} dz \quad (11)$$

where b is the impact parameter, which is characterized by the radius-vector $r^2 = b^2 + z^2$. Next, we analyze the influence of the plasma medium in conjunction with the gravitational field of the Sen black hole as an application of the previously outlined formalism. In the Cartesian coordinates, the components $h_{\alpha\beta}$ can be expressed as

$$h_{00} = \frac{R_s}{r} - \frac{2q}{r^2}, h_{ik} = \left(\frac{R_s}{r} - \frac{2q}{r^2}\right)n_i n_k, h_{33} = \left(\frac{R_s}{r} - \frac{2q}{r^2}\right)\cos^2 x \quad (12)$$

where $R_s = 2M, \cos x = z/\sqrt{b^2 + z^2}$ and $r = \sqrt{b^2 + z^2}$, b is the impact parameter signifying the closest approach of the photons to the black hole. By utilizing the previously stated expressions in equation (10), one can determine the light deflection angle in relation to b for a black hole enveloped by plasma:

$$\hat{\alpha}_b = \int_{-\infty}^{\infty}\frac{b}{2r}\left(\partial_r\left(\left(\frac{R_s}{r} - \frac{2q}{r^2}\right)\cos^2 x\right) + \partial_r\left(\frac{R_s}{r} - \frac{2q}{r^2}\right)\frac{\omega^2}{\omega^2 - \omega_e^2} - \frac{K_e}{\omega^2 - \omega_e^2}\partial_r N\right)dz \quad (13)$$

A. Deflection angle in the presence of plasma. It is evident from Eq. (13) that the expression accounts for both gravitational and plasma effects. Here, we reformulate the integral representation of the deflection angle by distinguishing these effects as follows:

$$\hat{\alpha}_1 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{dh_{33}}{dr}\right)dz, \hat{\alpha}_2 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr}\right)dz, \hat{\alpha}_3 = -\frac{1}{2}\int_{\infty}^{\infty}\frac{b}{r}\left(\frac{K_e}{\omega^2-\omega_e^2}\frac{dN(r)}{dr}\right)dz \tag{14}$$

Now, we calculate the integrals for the deflection angle for the different configurations of the plasma distribution.

B. Uniform plasma. First, we examine the influence of uniform plasma. We begin by computing the terms associated solely with the effects of spacetime curvature.

$$\hat{\alpha}_1 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{dh_{33}}{dr}\right)dz = -\frac{R_s}{b} + \frac{\pi q}{2b^2}. \tag{15}$$

One can see from Eq. (15) that when $q = 0$ we will get $\hat{\alpha}_1 = R_s/b$. The second term is a combined expression that represents the interaction between spacetime and plasma effects and is given by the following form:

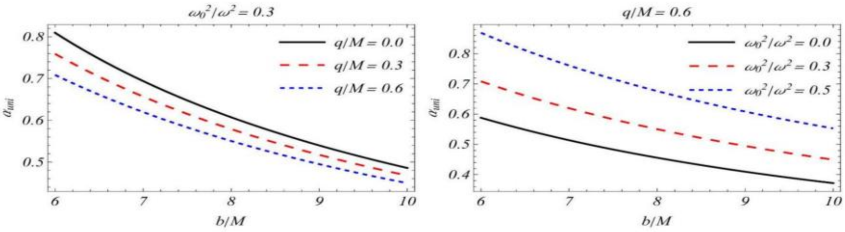

FIG. 1: Dependence of the deflection angle of the light rays in the presence of uniform plasma on impact parameter for different values of charge q and plasma frequency.

$$\hat{\alpha}_2 = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr}\right)dz = -\frac{R_s}{b(1-\omega_0^2/\omega^2)} + \frac{\pi q}{b^2(1-\omega_0^2/\omega^2)}. \tag{16}$$

Where $\omega_0^2 = \omega_e^2 = \text{const}$. The Eq. (16) also turns to $\hat{\alpha}_2 = R_s/b$ at $q = 0$. Thus, one may get the expression for the deflection

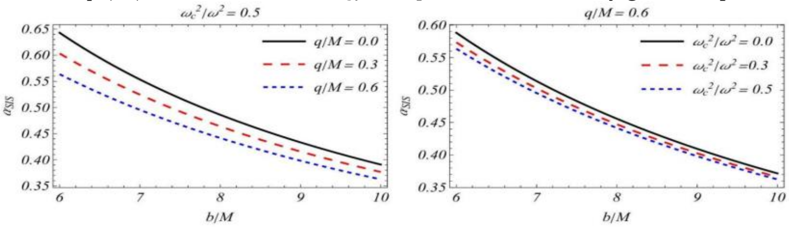

FIG. 2: Dependence of deflection angle of the light rays on impact parameter for different values of charge q and plasma frequency.

angle for Schwarschild case $\hat{\alpha}_b = 2Rs/b$ when $q = 0$. The deflection angle of light from Eq. (14) in the case of uniform plasma around the Sen black hole can be expressed as

$$\hat{\alpha}_{uni} = \left(1 + \frac{1}{1-\frac{\omega_0^2}{\omega^2}}\right)\frac{R_s}{b} - \left(1 + \frac{2}{1-\frac{\omega_0^2}{\omega^2}}\right)\frac{\pi q}{2b^2}. \tag{17}$$

Fig. 1 illustrates the photon deflection angle $\hat{\alpha}_{uni}$ as a function of the impact parameter b for various values of the coupling constant q (left panel) and the plasma parameter $\frac{\omega_0^2}{\omega^2}$ (right panel). An increase in the impact parameter and effective charge leads to a decrease in the deflection angle. Moreover, the deflection angle can be larger in the presence of uniform plasma compared to the vacuum scenario.

C. Singular isothermal sphere. The Singular Isothermal Sphere (SIS) serves as the most appropriate model for analyzing the properties of a photon's gravitational lens. It was initially proposed to study lensing behavior and galaxy clusters. The SIS represents a spherical gas cloud characterized by a density that extends infinitely at its core. The plasma frequency is expressed in the following form [14]

$$\omega_e^2 = K_e N(r) = \frac{K_e \sigma_v^2}{2\pi k m_p r^2}. \tag{20}$$

Here, m_p represents the proton mass, and k is a dimensionless constant typically linked to the dark matter universe. Now, the deflection angle can be determined in the presence of plasma with the SIS distribution surrounding the Sen black hole by applying Eq. (14). In this case, we have decomposed the deflection angle as in Eq. (14) and represented it as follows.

$$\hat{\alpha}_{SIS} = \hat{\alpha}_{SIS}^{(1)} + \hat{\alpha}_{SIS}^{(2)} + \hat{\alpha}_{SIS}^{(3)}. \tag{21}$$

As the first term excludes the influence of plasma, it retains the same structure as Eq. (14)

$$\hat{\alpha}_{SIS}^{1} = \hat{\alpha}_1$$

$$\hat{\alpha}_{SIS}^{2} = \frac{1}{2}\int_{-\infty}^{\infty}\frac{b}{r}\left(\frac{1}{1-\omega_e^2/\omega^2}\frac{dh_{00}}{dr}\right)dz = -\frac{R_s}{b} + \frac{3qR_s^2}{4b^4}\frac{\omega_c^2}{\omega^2} - \frac{2}{3}\frac{R_s^3}{b^3\pi}\frac{\omega_c^2}{\omega^2} + \frac{\pi q}{b^2}$$

To comprehend the impact of the SIS on the photon's path, we have graphed the deflection angle $\hat{\alpha}_{SIS}$ as a function of the impact parameter b, see Fig. 2. Interestingly, we observe that both the uniform plasma and the SIS medium exhibit similar characteristics concerning the parameter b. From Fig. 2, it can be seen that the deflection angle $\hat{\alpha}_{SIS}$ decreases due to plasma inhomogeneity exerting an opposing influence, which is significantly stronger than in a vacuum. We found that $\hat{\alpha}_{SIS}$ diminishes as $\frac{\omega_c^2}{\omega^2}$ increases (left panel), and conversely, $\hat{\alpha}_{SIS}$ decreases as q grows (right panel). Therefore, the presence of the SIS around the black hole notably affects the trajectory of passing massless particles.

Conclusion. We have studied how weak gravitational lensing and the photon sphere of Sen black holes change in a plasma medium. Our results show that the black hole's effective charge has a strong influence on these optical effects. We discovered that as the effective charge of a Sen black hole increases, the radius of the photon sphere decreases. The effective charge also influences the bending angle of light rays. Our results indicate that as the effective charge increases, the bending angle diminishes. In particular, we have taken into account uniformly distributed plasma and a singular isothermal sphere medium. We also demonstrated that the deflection angle grows as the plasma frequency rises in the case of a homogeneous plasma due to the medium's refractive properties. However, when the plasma is inhomogeneous (SIS), the deflection angle decreases with increasing plasma frequency.

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