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A GENERIC STUDY OF THIN-SHELL VIA REGULAR CHARGED BLACK HOLE

Annotation

We present a thin-shell in the background of a regular charged static black hole solution in the presence of T-duality effects for the Einstein-Maxwell system. The thin-shells are created by aligning the inner and outer surfaces using the well-established Visser method of cutting and pasting. We manufacture two varieties of thin-shells an outer regular charged black hole from T-duality and different choices of inner manifolds named: 1. Thin-shell (inner flat spacetime), 2. Thin-shell Garvastars (inner de Sitter spacetime).
Key words: Thin-shell wormholes, Gravastar, T-Duality, Stability analysis.

ОБЩЕЕ ИССЛЕДОВАНИЕ ТОНКОЙ ОБОЛОЧКИ ЧЕРЕЗ РЕГУЛЯРНУЮ ЗАРЯЖЕННУЮ ЧЕРНУЮ ДЫРУ

Аннотация

Мы представляем тонкую оболочку в фоне регулярного заряженного статического решения черной дыры с учетом эффектов Т-дуальности в системе Эйнштейна-Максвелла. Тонкая оболочка формируется путем согласования внутреннего и внешнего многообразий с использованием известного метода Виссера «вырезать и вставить». Мы получаем два типа тонких оболочек с внешней регулярной заряженной черной дырой из Т-дуальности и различными выборками внутренних многообразий: 1. Тонкая оболочка (внутреннее плоское пространство-время), 2. Тонкая оболочка гравастаров (внутреннее пространство-время де Ситтера).
Ключевые слова: тонкие оболочки кротовых нор, гравастар, Т-дуальность, анализ устойчивости.

MUNTAZAM ZARYADLANGAN QORA TUYNUK ORQALI UMUMIY YUPQA QOBIQ TADQIQOTI

Annotatsiya

Biz Eynshteyn-Maksvell sistemasida T-duallik ta’sirlarini hisobga olgan holda, muntazam zaryadlangan statsionar qora tuynuk fonida yupqa qobiqni taqdim etamiz. Yupqa qobiq Visserning mashhur "kesib-yopishtirish" usuli orqali ichki va tashqi manifoldlarni moslashtirish orqali hosil qilinadi. Biz T-duallikdan olingan tashqi muntazam zaryadlangan qora tuynuk va turli ichki manifold tanlovlari bilan ikki turdagi yupqa qobiqlarni hosil qilamiz: 1. Yupqa qobiq (ichki tekis fazo-vaqt), 2. Yupqa qobiq gravastarlar (ichki de Sitter fazo-vaqti).
Kalit so‘zlar: yupqa qobiq chuqurchalar, gravastar, T-duallik, barqarorlik tahlili.

Introduction. Black holes (BHs) stand out as particularly remarkable manifestations of strong-gravitational fields in contemporary research. Black holes possess an event horizon from which nothing can escape because of the immense gravitational pull, while simultaneously absorbing everything within their vicinity. Quantum fluctuations have consistently yielded miraculous consequences regarding the physical properties of BH geometries. The presence of singularities represents a fundamental challenge in BH physics.

Incorporating an NLED Lagrangian ensures a singularity cut-off in the context of the magnetic charge of BHs [1, 2]. The literature also conquered this limitation by a process of BH engineering, i.e., considering regular BH models with a de Sitter central core [3]-[6].

A vital breakthrough in removing the singularity comes with a family of regular BHs developed in a string-inspired manner by noncommutative geometry [7, 8]. Apart from the singularity eradication, these BHs present an interesting scenario for the final phase of evaporation. Instead of a divergent Hawking emission, there exists a phase characterized by a gradual cooling towards a zero-temperature extremal configuration, even in the absence of charge and angular momentum [9]. Interestingly, noncommutative effects undergo an equivalence framework to a non-local gravity that softens the curvature singularity [10]-[14].

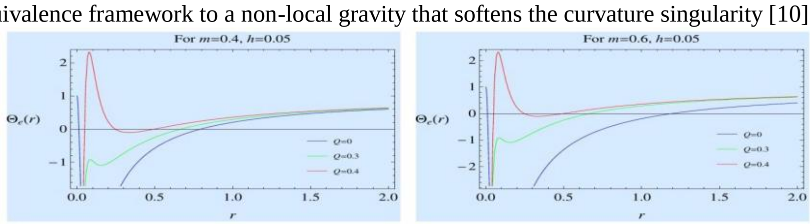

FIG. 1: Graphical analysis of metric function of charged BH from T-duality verses r with suitable physical parameters.

THIN-SHELL WITH EXTERIOR REGULAR CHARGED BLACK HOLE FROM T-DUALITY. This section is devoted to analysing the thin-shell formalism for charged black holes (BHs) arising from T-duality. The line element for the regular charged BH resulting from T-duality is given by

$$ds^2 = -\Theta_e(r)dt^2 + \Theta_e(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where

$$\Theta_e(r) = \frac{Q^2r^2F(r)}{(h^2+r^2)^2} - \frac{2mr^2}{(h^2+r^2)^{3/2}} + 1, \quad (2)$$

with

$$F(r) = \frac{3\pi}{16h}\sqrt{h^2+r^2}\left(1 - \frac{(h^2+r^2)^{3/2}\left(2\tan^{-1}\left(\frac{r}{h}\right)\right)}{\pi r^3}\right) + \frac{3h^2}{8r^2} + \frac{5}{8} \quad (3)$$

Here, m represents the mass of Schwarzschild BH, Q is the charge, h denotes the zero point length. $F(r)$ is monotonically increasing function and,

$$F(r) = \begin{Bmatrix} 3\pi/16, & r \ll h \\ 1, & r \gg h \end{Bmatrix}$$

i.e., $3\pi/16 < F(r) < 1$.

It should be pointed out that the metric can be equivalent to the following solutions: 1)Bardeen metrics if the length is redefined as $h \to Q$, 2)Ayon-Beato and Garcia spacetime by considering $h \to Q$ and $F(r) = 1$ everywhere.

A. Thin-Shell. Now, we examine $(2+1) - D$ a charged thin-wall, represented by Σ, with its radius $r = b(\tau)(\tau$ being the proper time). The metric for these spaces can be expressed :

$$ds_\pm^2 = -\Theta_n(r)dt^2 + (\Theta_n(r))^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (4)$$

The quantitative measure for these two regions can be summarized as follows

$$\Theta_i(r) = 1, \Theta_e(r) = \frac{F(r)(Q^2r^2)}{(h^2+r^2)^2} - \frac{2mr^2}{(h^2+r^2)^{3/2}} + 1 \quad (5)$$

In the case of a thin shell, the metric function corresponds to a time-like two-dimensional sphere, whose coordinates can be expressed as $y^i = (\tau, \theta, \phi)$:

$$ds^2 = h_{ij}dy^idy^j = -d\tau^2 + b^2d\theta^2 + b^2\sin^2\theta d\phi^2 \quad (6)$$

The constituent elements of the thin-shell can be expressed as:

$$S_\tau^\tau \equiv \rho(b) = -\frac{1}{4\pi b}(\chi_e(b) - \chi_i(b)) \quad (7)$$

$$S_\theta^\theta = S_\phi^\phi \equiv \mathfrak{P}(b) = \frac{-\chi_i(b)+\chi_e(b)}{8\pi b} + \frac{2\ddot{b}+\Theta_e'(b)}{16\pi\chi_e(b)} - \frac{2\ddot{b}+\Theta_i'(b)}{16\pi\chi_i(b)} \quad (8)$$

where

$$\chi_i(b) = \sqrt{\Theta_i(b) + \dot{b}^2}, \chi_e(b) = \sqrt{\Theta_e(b) + \dot{b}^2} \quad (9)$$

$\rho(b)$ and $\mathfrak{P}(b)$ -the energy density and the tangential pressure are represented by the dot and dash, respectively, which correspond to the derivatives with respect to the proper time and the radial coordinate. Here, subindex zero belongs to the physical entities at the static state of thin-shell, i.e., $b = b_0$. Employing equations (7) and (9), we establish connections between the gravitational mass and the mass of the shell.

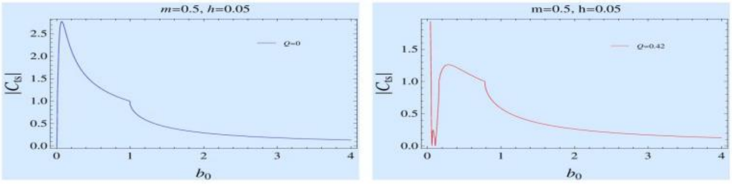

FIG. 2: The compactness of thin-shell with exterior Bardeen (left plot) and charged BHs from T-duality versus equilibrium shell radius.

We choose a compactness parameter for the stable arrangement, which is calculated as the ratio of the shell's mass to its radius., i.e., $C_{ts} = M/b_0$. The compactness parameter yields

$$C_{ts} = 1 - \frac{\sqrt{b_0^2(b_0^2+h^2)^2\left(b_0^4 + b_0^2Q^2F(b_0) + 2b_0^2\left(h^2 - m\sqrt{b_0^2+h^2}\right) + h^4\right)}}{b_0(b_0^2+h^2)^2} \quad (10)$$

Fig (2) shows the behaviour of compactness for allowed values of parameters. We can conlude that the thin-shell is less compact near the origin for exterior Bardeen BH while more compact near the origin for the case of charged BH from T-duality. The compactness tends to decrease as the shell radius increases.

B. Thin-Shell Gravastars. In this subsection we utilize Visser's cut-and-paste approach to form a charged thin-shell gravastar with the inner manifold as a de Sitter spacetime and the exterior manifold as charged BH from T-duality. We joined the interior and exterior regions at the timelike hypersurface known as a gravastar shell with radius $r = b$.

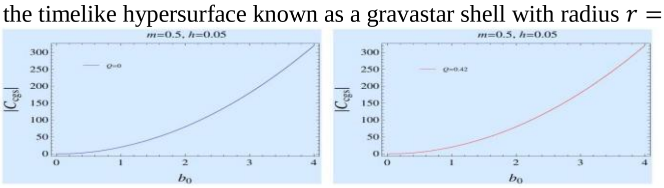

FIG. 3: The compactness of the thin-shell gravastars with exterior Bardeen (left plot) and charged BHs from T-duality versus equilibrium shell radius.

here $\Theta_i(r)$ is the metric function of de Sitter geometry with α is a nonzero positive constant.

The surface stresses for thin-shell gravastar at equilibrium state can be written as

$$\rho_0 = -\frac{1}{4\pi b_0}\left(\sqrt{\frac{F(b_0)(Q^2 b_0^2)}{(h^2+b_0^2)^2} - \frac{2mb_0^2}{(h^2+b_0^2)^{3/2}} + 1} - \sqrt{1-\frac{b_0^2}{\alpha^2}}\right) \quad (11)$$

$$\mathfrak{P}_0 = \frac{4b_0^2 - 2\alpha^2}{8\pi b_0 \alpha^2 \sqrt{1-\frac{b_0^2}{\alpha^2}}} + \frac{b_0\left(\frac{2b_0 Q^2 F(b_0)}{(b_0^2+h^2)^2} + \Theta_e'(b_0)\right) - \frac{4b_0^2 m}{(b_0^2+h^2)^{3/2}} + 2}{8\pi b_0 \sqrt{\frac{b_0^2 Q^2 F(b_0)}{(b_0^2+h^2)^2} - \frac{2b_0^2 m}{(b_0^2+h^2)^{3/2}} + 1}} \quad (12)$$

We define the compactness of thin-shell gravastar by $C_{cgs} = 4\pi b^2 \rho(b)$ with the subindex denoting charged thin-shell gravastar. Fig (3) illustrates that the thin-shell gravastar is less compact near the origin, and compactness is enhanced as the shell radius increases for both choices of exterior geometries.

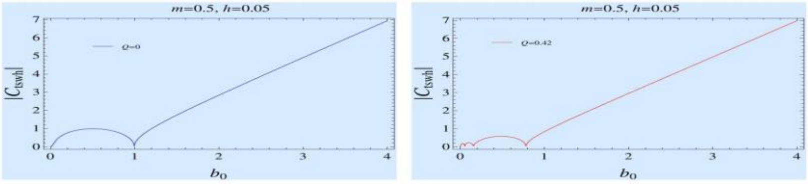

FIG. 4: The compactness of thin-shell WHs with exterior Bardeen (left plot) and charged BHs from T-duality versus equilibrium shell radius.

Conclusions. In this work, we have studied the thin-shell theory with three different options for its inner geometries, namely flat, de Sitter, and regular charged black hole obtained from T-duality while the latter has been chosen as an exterior geometry for all three cases. Here, the following approaches have been utilized through investigating: First, all configurations are developed using a cut-and-paste approach, which is a valuable method for eliminating singularities and event horizons in the constructed structure; Furthermore, the elements of the stress-energy tensor are calculated using a simplified version of Einstein's field equations on the surface.

We have employed the barotropic and generalized Chaplygin gas models to analyze the stable and unstable structures by determining the critical values of Π_{20} for each choice of inner manifolds.

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