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SPINNING PARTICLES MOTION AROUND THE REISSNER-NORDSTRÖM-LIKE BLACK HOLE

Annotation

In this study, we examine the motion of a spinning particle around a RN-like massive astronomical object. The dynamics of spinning test particles are analyzed using the Mathisson-Papapetrou-Dixon (MPD) equations. The effective potential is derived, and its dependence on the parameters of the Reissner-Nordström-like black hole is explored. The impact of spin of particle on the innermost stable circular orbit (ISCO), as well as the angular momentum and energy, is analyzed for various configurations of black holes.

Key words: charged black holes, spinning particle, MPD equations, innermost stable circular orbits, effective potential.

ДВИЖЕНИЕ ВРАЩАЮЩИХСЯ ЧАСТИЦ ВОКРУГ ЧЁРНОЙ ДЫРЫ, ПОДОБНОЙ РЕЙССНЕРА-НОРДСТРЁМА

Аннотация

В данном исследовании рассматривается движение вращающейся частицы вокруг черной дыры, подобной черной дыре Рейснера–Нордстрема. Динамика вращающихся тестовых частиц анализируется с использованием уравнений Матиссона–Папапетру–Диксона (MPD). Выводится эффективный потенциал и исследуется его зависимость от параметров черной дыры, подобной Рейснера–Нордстрему. Изучается влияние спина частицы на внутреннюю границу устойчивой круговой орбиты (ISCO), а также удельный угловой момент и энергия на ISCO для различных параметров «волосатых» черных дыр.

Ключевые слова: заряженные черные дыры, вращающаяся частица, уравнения MPD, внутренняя граница устойчивой круговой орбиты, эффективный потенциал.

REISSNER-NORDSTRÖM-GA O‘XSHASH QORA TUYNUK ATROFIDA AYLANUVCHI ZARRALARNING HARAKATI

Annotatsiya

Ushbu tadqiqotda Reissner-Nordström-ga o‘xshash qora tuynuk atrofida aylanayotgan aylanuvchi zarra harakati ko‘rib chiqiladi. Aylanuvchi sinov zarralarining dinamikasi Mathisson-Papapetrou-Dixon (MPD) tenglamalari yordamida tahlil qilinadi. Effektiv potensial chiqarilib, uning Reissner-Nordström-ga o‘xshash qora tuynuk parametrlarga bog‘liqligi o‘rganiladi. Zarrachaning aylanish momentining eng ichki barqaror aylana orbitasi (ISCO) ga ta’siri, shuningdek, "sochli" qora tuynuklarning turli parametrlari uchun ISCO dagi maxsus burchak momenti va energiyasi tadqiq etiladi.

Kalit so‘zlar: zaryadlangan qora tuynuklar, aylanuvchi zarra, MPD tenglamalari, eng ichki barqaror aylana orbita, effektiv potensial.

Introduction. Testing gravity theory is one of the important issues in exploring the theory as a whole. Particularly, it helps to understand the behavior of the field in different astrophysical scenarios [1-9]. The motivation for exploring modified and alternative theories of gravity stems from the fact that standard general relativity (GR) encounters several key unresolved issues. The existence of the singularity, inconsistency with quantum field theory, issues related to cosmology requiring the presence of dark energy and dark matter are few of them. In order to resolve these issues, one needs to consider the further modifications of the theory or suggest an alternative way of considering the gravitational interaction. One of the possible ways of modifying the standard gravity theory is to consider of breaking some symmetries. In particular, Lorentz symmetry is one of the features of the standard models and it is supported by the current observational and experimental data. In this study, we aim to investigate the dynamics of spinning particles around a charged Reissner-Nordström-like black hole.

Charged Hairy Black Hole. The spacetime metric of the Reissner-Nordström-like (RN-like) black hole in Boyer-Lindquist coordinates is given by

$$ds^2 = -f dt^2 + \frac{1}{f} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

where

$$f = \frac{1}{1-l} - \frac{2M}{r} + \frac{Q^2}{(1-l)^2 r^2} \tag{2}$$

Here, M represents the total mass of the black hole, Q denotes the electric charge, and l is a dimensionless parameter, whose value is constrained to be extremely small based on classical gravitational experiments within the Solar System. Notably, when $l = 0$, the solution reduces to the standard Reissner-Nordström (RN) metric, while setting $Q = 0$ recovers the Schwarzschild spacetime.

Now, let us analyze the characteristics of this RN-like black hole, which can be visualized through a graph illustrating the structure of its horizons. According to the metric (1), the radii of the horizons are determined by the relation:

$$r_{\pm} = (1-l)\left(M \pm \sqrt{M^2 - \frac{Q^2}{(1-l)^3}}\right). \tag{3}$$

As the parameter l increases, first panel in Fig. 1 demonstrates that the outer event horizon r_+ shrinks, whereas the inner Cauchy horizon $r_h$expands.

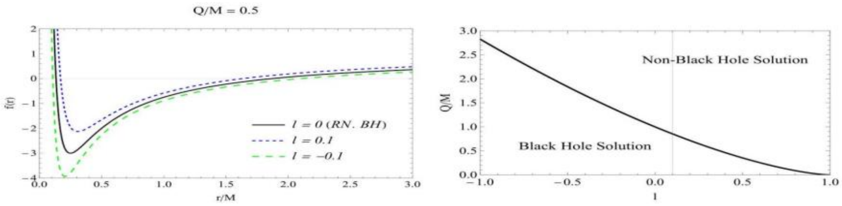

FIG. 1: First panel: Metric function $f(r)$ for the RN-like solution with varying l parameter and charge Q. Second panel: The black hole region for the RN-like solution, where the boundary separates the regions where black holes exist from the empty region, which contains naked singularities.

The dynamics of a particle. Particularly the equation of motion for a spinning particle, are governed by the Mathisson-Papapetrou-Dixon (MPD) equations in curved spacetime. These equations describe the motion of a particle with intrinsic spin and can be expressed as follows:

$$\frac{Dp^{\mu}}{d\tau} = -\frac{1}{2}R^{\mu}_{\nu\alpha\beta}u^{\nu}S^{\alpha\beta}, \frac{DS^{\alpha\beta}}{d\tau} = p^{\alpha}u^{\beta} - u^{\alpha}p^{\beta}, \tag{4}$$

where p^{μ} and u^{ν} are the canonical 4-momentum and the kinematical 4-velocity of the test particle, $R^{\alpha}_{\beta\delta\sigma}$ is the Riemann tensor, and τ is affine parameter. $S^{\alpha\beta}$ is the spin tensor that represents a quantity that defines the angular momentum of a test particle in terms of its spin. It should be noted that $S^{\alpha\beta}$ in Eq. (4) is an antisymmetric tensor, which means $S^{\alpha\beta} = -S^{\beta\alpha}$.

To solve Eq. (4), we introduce the the Tulczyjew spin supplementary condition (SSC). This condition states that the spin of a particle should be measured relative to its center of mass, which given by the relation:

$$S^{\mu\nu}p_{\mu} = 0. \tag{5}$$

It is evident from Eq. (5) that the spin and the canonical momentum of particles constitute two independent invariants, which are described by the following relations:

$$S^{\mu\nu}S_{\mu\nu} = 2S^2, p^{\mu}p_{\mu} = -m^2 \tag{6}$$

Given the existence of a Killing vector k, there exists a conserved quantity in the motion of a spinning particle, which is related by

$$C_{\xi} = P^{\mu}k_{\mu} + \frac{1}{2}S^{\mu\nu}\nabla_{\mu}k_{\nu} \tag{7}$$

Due to the symmetry of the metric, it is evident that the line element in Eq. (1) admits two Killing vector fields. One of these vector fields corresponds to time translation, while the other is related to rotational symmetry. These Killing vector fields are expressed as follows:

$$\xi^{\alpha} = \delta^{\alpha}_{t}, \xi^{\alpha} = \delta^{\alpha}_{\phi}. \tag{8}$$

In accordance with the conservation of energy E and angular momentum $J (J = L + S, L = \mathcal{L}m)$, respectively, for the first of these vectors, we find

$$C_t = E = -p_t + \frac{1}{2}g_{t\alpha,\beta}S^{\alpha\beta} = -p_t + \frac{1}{2}g_{tt,r}S^{tr}, \tag{9}$$

$$C_{\phi} = J = p_{\phi} - \frac{1}{2}g_{\phi\alpha,\beta}S^{\alpha\beta} = p_{\phi} - \frac{1}{2}g_{\phi\phi,r}S^{\phi r}. \tag{10}$$

In astrophysical studies, it's common to consider test particles moving in the equatorial plane, where $\theta = \pi/2$. In this plane, the metric functions only depend on the radial coordinate, and $p^{\theta} = 0$. As a result, the components of the spin tensor are:

$$S^{\theta\mu} = 0 \tag{11}$$

Now, employing the the Tulczyjew spin supplementary condition (SSC) (5), we determine the non-zero components of the spin tensor:

$$S^{t\phi} = -\frac{p_r}{p_{\phi}}S^{tr}, \tag{12}$$

$$S^{r\phi} = \frac{p_t}{p_{\phi}}S^{tr}. \tag{13}$$

From the Eq. (6) we find the radial momentum of the particle:

$$p_r^2 = g_{rr}\left(-g^{tt}p_t^2 - g^{\phi\phi}p_{\phi}^2 - m^2\right) \tag{14}$$

Expressing in terms of Eqs. (12), (13) and (14), in accordance with the law of conservation of spin Eq. (6), we can determine the S_{tr} component of the spin tensor:

$$S^{tr} = \pm \frac{p_\phi s}{\sqrt{-g_{tt} g_{rr} g_{\phi\phi}}} \tag{15}$$

Here, $s = S/m$ denotes the specific angular momentum, or spin, of the particle. This quantity can be positive or negative, depending on the direction of p_ϕ.

Now this expression can be rewritten as follows:

$$p_r^2 = \frac{\beta}{\alpha}\left(E - V_{eff}^{+}\right)\left(E - V_{eff}^{-}\right), \tag{22}$$

from which

$$V_{eff}^{\pm} = \frac{-\delta J \pm \sqrt{(\delta J)^2 - 4\gamma\beta}}{2\beta} \tag{23}$$

where $\gamma = \sigma J^2 - \rho$, and

$$\alpha = g_{rr}\left[1 - \frac{s^2 g_{tt,r} g_{\phi\phi,r}}{4 g_{tt} g_{rr} g_{\phi\phi}}\right]^2,$$

$$\beta = -g^{tt} + \frac{s^2 g^{\phi\phi}\left(g_{\phi\phi,r}\right)^2}{4 g_{tt} g_{rr} g_{\phi\phi}},$$

$$\sigma = -g^{\phi\phi} + \frac{s^2 g^{tt}\left(g_{tt,r}\right)^2}{4 g_{tt} g_{rr} g_{\phi\phi}},$$

$$\rho = m^2\left[1 - \frac{s^2 g_{tt,r} g_{\phi\phi,r}}{4 g_{tt} g_{rr} g_{\phi\phi}}\right]^2.$$

Before examining the spinning motion of test particles, it is essential to consider another critical factor influenced by the particle's spin: the superluminal bound on its dynamics. The canonical four-momentum p^α and four-velocity u^α of a spinning particle are not necessarily parallel. As a result, the normalization condition $u_\alpha u^\alpha = -1$ does not always hold, although $p_\alpha p^\alpha = -m^2$ remains satisfied. As the spinning particle approaches the center of spacetime, its four-velocity increases. For certain values of spin and radius, some components of the four-velocity may grow without limit, causing $u_\alpha u^\alpha$to approach +∞. Prior to this divergence, the particle's motion shifts from a time-like to a space-like trajectory. Since space-like (superluminal) motion is physically unattainable, real particles cannot cross into the region where $u_\alpha u^\alpha > 0$. Therefore, an additional constraint, known as the superluminal bound, must be imposed, ensuring that $u_\alpha u^\alpha = 0$. To ensure that the trajectory of spinning test particles retains a time-like nature, the following condition must be imposed (on the equatorial plane):

$$\frac{u^\alpha u_\alpha}{(u^t)^2} = g_{tt} + g_{rr}(u^r)^2 + g_{\phi\phi}\left(u^\phi\right)^2 \leqslant 0 \tag{32}$$

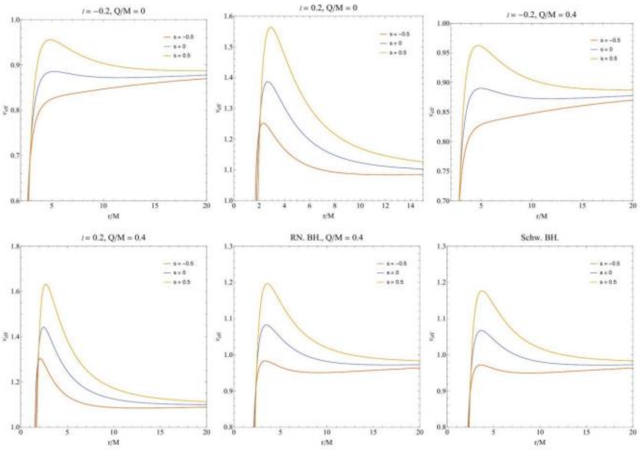

FIG. 2: The radial dependence of the effective potential $\mathcal{V}_{\rm eff}$ for spinning motion is shown for various values of the metric parameter l and charge Q, with the total angular momentum fixed at $\mathcal{L} = 4.5$.

Fig. 2 depicts the radial behavior of the effective potential $\mathcal{V}_{\rm eff}$ for a spinning particle, considering different values of the parameter l, the charge-to-mass ratio Q/M, and the particle's spin s, while maintaining the total angular momentum $\mathcal{L} = 4.5$.

Conclusion. This article investigates the motion of spinning particles around a charged black hole. By employing the Mathisson-Papapetrou-Dixon (MPD) equations and the Tulczyjew condition, we studied how the black hole's properties, charge, and particle spin affect motion. We analyzed the black hole's metric and found the limits of charge and parameters before it becomes a naked singularity. The MPD equations helped determine the effective potential, showing that charge and the metric parameter increase it. The investigate ISCO revealed that increasing charge, spin, or the metric parameter decreases the ISCO radius, angular momentum, and energy. This research highlights the role of spin, charge, and black hole properties in orbital stability and collisions. It has applications in modeling accretion disks, jets, and gravitational waves. Future work could explore dynamic black holes or higher dimensions for a deeper understanding of strong gravitational fields.

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