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BIRINCHI TIP MATRITSAVIY SHAR AVTOMORFIZMLARINING XOSSALARI VA ULARNING BA’ZI TATBIQLARI

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REZYUME

Bu maqolada Rudin [2] kitobidagi Teorema 2.2.5 ning $\mathbb{C}^n\,[m,m]$ sohadagi birinchi tip matritsaviy shar uchun analogi keltirilgan. Birinchi tip matritsaviy shar avtomorfizmlari xossalarining ba’zi tatbiqlari keltirilgan. Jumladan, birinchi tip matritsaviy sharga bigolomorf ekvivalent bo‘lgan sohalarning avtomorfizmlarining ba’zi xossalari isbotlangan.

Kalit so‘zlar: Blok matritsa, bigolomorf akslantirish, ermit matritsa, klassik soha, avtomorfizm.

Ma’lumki bir jinsli, simmetrik, qavariq va chegaralangan kompleks sohalar turli nuqtai nazardan katta qiziqish uyg‘otadi. Buning sababi shundaki, ular yordamida $\mathbb{C}^n$ sohalari uchun bir qator muhim, asosan ko‘p o‘lchovli natijalar olingan ([1], [2], [3] va boshqalar).

Bir jinsli sohalarda integral formulalar qurishda avtomorfizmlar gruppasidan foydalaniladi. Bunda avtomorfizmlar gruppalari([4], [5]) keng bo‘lgan elementi matritsalardan iborat sohalar ([1], [6]) qaraladi. Matritsaviy sohalar birinchi bo‘lib E.Kartan va K.Zigel tomonidan chuqur o‘rganilgan. Jumladan, ular to‘rtta klassik sohalar avtomorfizmlarining umumiy ko‘rinishlarini tasvirlashgan. Xua Lo-Ken esa klassik sohalar uchun ko‘p kompleks o‘zgaruvchili funksiyalar nazariyasida garmonik analizni qurgan (1944-1957yillarda) va ular bo‘yicha natijalar Xua Lo-Kenning 1958-yilda xitoy tilida chop etilgan (1959-yilda rus tilida chop etilgan [1]) monografiyasida keltirilgan.

$\mathbb{C}^{mn}$ fazoni qaraylik. Bu fazodagi $z \in \mathbb{C}^{mn}$ nuqtalarni quyidagicha yozamiz: $z = (z_{11}, z_{12}, ..., z_{1n}; ...; z_{m1}, z_{m2}, ..., z_{mn})$, bu yerda, $z_{\mu\nu} = x_{\mu\nu} + iy_{\mu\nu}$, $\mu = 1, 2, ..., m$; $\nu = 1, 2, ..., n$. Ba’zi amaliy masalalarda $\mathbb{C}^{mn}$ fazoning nuqtalarini $[m\times n]$ matritsa sifatida yozish qulay bo‘ladi:

$$Z = \begin{pmatrix} z_{11} & z_{12} ... z_{1n} \\ . & . \; ... \; . \\ . & . \; ... \; . \\ z_{m1} & z_{m2} ... z_{mn} \end{pmatrix}.$$

U holda ushbu $\mathbb{C}^{mn} \simeq \mathbb{C}[m \times n]$ izomorfizm bo‘ladi.

Ba’zi hollarda $\mathbb{C}[m \times m]$ fazo bilan birgalikda n ta $\mathbb{C}[m \times m]$ fazoning dekart ko‘paytmalaridan tashkil topgan $\mathbb{C}^n[m \times m]$ fazo ham keltiriladi:

$$\mathbb{C}^n[m \times m] = \underbrace{\mathbb{C}[m \times m] \times ... \times \mathbb{C}[m \times m]}_{n-ta}.$$

Ushbu

$$B_{m,n} = \{Z = (Z_1, ..., Z_n) \in \mathbb{C}^n[m \times m] : \ I^{(m)} - \langle Z, Z \rangle > 0\}$$

to‘plam matritsaviy shar deyiladi, bunda, $\langle Z, Z \rangle = Z_1 {Z_1}^* + Z_2 {Z_2}^* + ... + Z_n {Z_n}^*$– "matritsaviy skalyar"ko‘paytma, $I^{(m)}$ – birlik $[m \times m]$-matritsa, ${Z_\nu}^* = \overline{Z}_\nu'$ – matritsa esa, Z_ν, $(\nu = 1, 2, ..., n)$– ga nisbatan qo‘shma va transponirlangan matritsa. $I - \langle Z, Z \rangle > 0$ tengsizlik esa $I - \langle Z, Z \rangle$ – ermit matritsaning musbat aniqlanganligini, ya’ni barcha xos sonlari musbatligini bildiradi.

Aytaylik,

$$H = \begin{pmatrix} I^{(m)} & 0 & ... & 0 \\ 0 & -I^{(m)} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & -I^{(m)} \end{pmatrix}, \ A = \begin{pmatrix} A_{00} & A_{01} & ... & A_{0n} \\ A_{10} & A_{11} & ... & A_{1n} \\ ... & ... & ... & ... \\ A_{n0} & A_{n1} & ... & A_{nn} \end{pmatrix},$$

matritsalar ushbu

$$AHA^* = H \tag{1}$$

shartni qanoatlantiruvchi $n+1$-tartibli blok kvadrat matritsalar bo‘lsin, bu yerda, $A_{i\,j}$-$[m, m]$ tartibli kvadrat matritsa. (1) munosabatdan quyidagi tengliklar kelib chiqadi:

$$A_{00}A_{00}^* - \sum_{s=1}^{n} A_{0s}A_{0s}^* = I^{(m)},$$

$$A_{j0}A_{k0}^* = \sum_{s=1}^{n} A_{js}A_{ks}^*, \ \ j \neq k, \tag{2}$$

$$A_{j0}A_{j0}^* - \sum_{s=1}^{n} A_{js}A_{js}^* = -I^{(m)}, \ \ j \geq 1.$$

Endi ushbu $\zeta = (\zeta_0, \zeta_1, ..., \zeta_n)$ matritsaviy vektorni qaraylik, bu yerda ζ_j – $[m \times m]$ –tartibli kvadrat matritsalar bo‘lib, barcha $j = 0, ..., n$ sonlar uchun quyidagi

$$\zeta H \zeta^* > 0, \tag{3}$$

munosabatni qanoatlantiradi ([7], [8]). Oxirgi (3) munosabatdan

$$\zeta_0\zeta_0^* > \zeta_1\zeta_1^* + ... + \zeta_n\zeta_n^*$$

tengsizlikka ega bo‘lamiz. A matritsa qatnashgan

$$\omega = \zeta A, \tag{4}$$

chiziqli akslantirishni qaraylik. Bu akslantirish (1) munosabat yordamida berilgan matritsalar to‘plamini o‘ziga akslantiradi, ya’ni

$$\omega H \omega^* = \zeta AHA^* \zeta^* = \zeta H \zeta^*.$$

Endi (4) tenglikni

$$\omega_0 = \sum_{j=0}^{n} \zeta_j A_{j0},$$

$$\omega_k = \sum_{j=0}^{n} \zeta_j A_{jk} \ , \ k = 1, 2, ..., n \tag{5}$$

ko‘rinishda yozib olamiz va

$$\{Z = (Z_1, ..., Z_n), Z_k = \zeta_0^{-1} \zeta_k \ , \ (k = 1, 2, ...n)\}$$

vektorlar to‘plamini qaraymiz.

Ushbu bir-biriga ekvivalent

$$\zeta H \zeta^* > 0$$

yoki

$$\zeta_0 \zeta_0^* > \zeta_1 \zeta_1^* + \zeta_n \zeta_n^*$$

tengsizliklardan quyidagi munosabatlarga ega bo‘lamiz:

$$\zeta_1 \zeta_1^* + \zeta_n \zeta_n^* < \zeta_0 \zeta_0^* \Leftrightarrow {\zeta_0}^{-1} \zeta_1 \zeta_1^* {\zeta_0^*}^{-1} + ... + {\zeta_0}^{-1} \zeta_n \zeta_n^* {\zeta_0^*}^{-1} < I^{(m)} \Leftrightarrow$$
$$\Leftrightarrow Z_1 Z_1^* + ... + Z_n Z_n^* < I^{(m)} \Leftrightarrow \langle Z, Z \rangle < I^{(m)}.$$

Demak, ushbu

$$\left\{ Z = (Z_1, ..., Z_n) : \langle Z, Z \rangle < I^{(m)} \right\}$$

matritsalar to‘plami $\mathbb{C}^n[m \times m]$ fazodagi matritsaviy sharni hosil qiladi. U holda (5) tenglikka asosan quyidagi munosabatga ega bo‘lamiz:

$$W_k = \omega_0^{-1} \omega_k = \left(A_{00} + \sum_{j=1}^{n} Z_j A_{j0} \right)^{-1} \left(A_{0k} + \sum_{j=1}^{n} Z_j A_{jk} \right), \ k = 1, 2, ..., n. \tag{6}$$

Bundan esa [9], (6) akslantirishning $B_{m,n}$ matritsaviy sharning avtomorfizmi bo‘lishi uchun, $A_{i\,j}, \ \ i, j = 0, 1, ..., n,$ koeffitsientlar (2) shartlarni qanoatlantirishi kerakligini ko‘rish mumkin. Matritsaviy sharning (6) ko‘rinishdagi avtomorfizmini

$$P = (P_1, ..., P_n)$$

nuqtani O nuqtaga akslantiruvchi quyidagi ko‘rinishda ham yozib olishimiz mumkin:

$$W_k = R^{-1} (I^{(m)} - \langle Z, P \rangle)^{-1} \sum_{s=1}^{n} (Z_s - P_s) Q_{sk} \quad (k = 1, ..., n). \tag{7}$$

U holda (2) munosabatlardan, m-tartibli

$$R \, va \, Q_{sk} \, (s, k = 1, ..., n),$$

matritsalarning ushbu

$$R^* (I^{(m)} - \langle P, P \rangle) R = I^{(m)}$$
$$Q^* (I^{(mn)} - P^* P) Q = I^{(mn)}. \tag{8}$$

shartlarni qanoatlantirishi zarurligi kelib chiqadi. Bu yerda Q va P matritsalar blok matritsa bo‘lib,

$$Q = \begin{pmatrix} Q_{11}\, Q_{12} \,...Q_{1n} \\ ... \;...\;\; ...\; ... \\ Q_{n1}\, Q_{n2}\, ...Q_{nn} \end{pmatrix}$$

,

$$P^*P = \begin{pmatrix} P_1^*P_1 & P_1^*P_2 & ... & P_1^*P_n \\ P_2^*P_1 & P_2^*P_2 & ... & P_2^*P_n \\ . & . & ... & . \\ P_n^*P_1 & P_n^*P_2 & ... & P_n^*P_n \end{pmatrix}.$$

ko‘rinishda aniqlangan. Aytaylik, $W = \varphi_P(Z) - B_{m,n}$ matritsaviy sharning $P \in B_{m,n}$ nuqtasini O nuqtaga o‘tkazuvchi avtomorfizmi bo‘lsin. U holda quyidagi tasdiq o‘rinli ([10]):

Tasdiq. Barcha $Z, W \in B_{m,n}$ nuqtalar uchun quyidagi tengliklar o‘rinli:

$$I^{(m)} - \langle \varphi_P(Z), \varphi_P(W)\rangle = R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}\left(I^{(m)} - \langle Z, W\rangle\right)\left(I^{(m)} - \langle P, W\rangle\right)^{-1}R^{*-1}.$$

Teorema 1. $\varphi_P(Z)$ avtomorfizm uchun ushbu

$$RP_i + P_iQ_{ii} = \mathrm{O}, \quad i = 1, 2, ..., n \tag{9}$$

munosabatlar o‘rinli bo‘lsin. U holda quyidagi xossalar o‘rinli.

1^0. $\varphi_P(P) = \mathrm{O}$, $\varphi_P(\mathrm{O}) = P$;

2^0. $\frac{\partial W_k(P)}{\partial Z_i} = -R^*Q_{ik}$, $\frac{\partial W_k(\mathrm{O})}{\partial Z_i} = R^{-1}(P_i^*RP_k - Q_{ik})$;

3^0. Ixtiyoriy $Z, W \in B_{m,n}^{(1)}$ uchun quyidagi munosabat o‘rinli

$$\det\left(I^{(m)} - \langle\varphi_P(Z), \varphi_P(W)\rangle\right) = \frac{\det\left(I^{(m)} - \langle P, P\rangle\right)\cdot\det\left(I^{(m)} - \langle Z, W\rangle\right)}{\det\left(I^{(m)} - \langle Z, P\rangle\right)\cdot\det\left(I^{(m)} - \langle P, W\rangle\right)}; \tag{10}$$

4^0. Ixtiyoriy $Z \in B_{m,n}^{(1)}$ uchun quyidagi munosabat o‘rinli

$$\det\left(I^{(m)} - \langle\varphi_P(Z), \varphi_P(Z)\rangle\right) = \frac{\det\left(I^{(m)} - \langle P, P\rangle\right)\cdot\det\left(I^{(m)} - \langle Z, Z\rangle\right)}{\det\left(I^{(m)} - \langle Z, P\rangle\right)\cdot\det\left(I^{(m)} - \langle P, Z\rangle\right)};$$

5^0. $\varphi_P(\varphi_P(Z)) = Z$ (involyutsiya bo‘lish xossasi);

6^0. $\varphi_P(Z)$ gomeomorfizm bo‘ladi.

Isbot. 1^0. $W_k = R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}\sum\limits_{s=1}^{n}(Z_s - P_s)Q_{sk}$ $k = 1, ..., n$ avtomorfizmning P nuqtadagi qiymatini hisoblaymiz

$$W_K(P) = R^{-1}\big(I^{(m)} - \langle P, P\rangle\big)^{-1}\sum_{s=1}^{n}(P_s - P_s)Q_{sk} = R^{-1}\big(I^{(m)} - \langle P, P\rangle\big)^{-1}\cdot \mathrm{O} = \mathrm{O}.$$

Endi (7) avtomorfizmning nol nuqtadagi qiymatini hisoblaymiz.

$$W_K(\mathrm{O}) = R^{-1}\big(I^{(m)} - \langle \mathrm{O}, P\rangle\big)^{-1}\sum_{s=1}^{n}(\mathrm{O} - P_s)Q_{sk} = R^{-1}(-P_sQ_{sk})$$

(9) tengliklarni inobatga olsak

$$W_k\left(\mathrm{O}\right) = P_k\left(k = 1, 2, ..., n\right)$$

munosabatlarning o'rinli ekani kelib chiqadi.

2^0. (7) avtomorfizm differensialini hisoblaymiz.

$$d\left(W_k\right) = d\left(R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk}\right) =$$

$$= R^{-1}d\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk} + R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}d\left(\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk}\right) =$$

$$= R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}d\left\langle Z, P\right\rangle\left(I^{(m)} - \langle Z, P\rangle\right)^{-1}\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk} +$$

$$+ R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}d\left(\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk}\right) =$$

$$= R^{-1}\big(I^{(m)} - \langle Z, P\rangle\big)^{-1}\left(d\left\langle Z, P\right\rangle\left(I^{(m)} - \langle Z, P\rangle\right)^{-1}\cdot\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk} + d\left(\sum_{s=1}^{n}\left(Z_s - P_s\right)Q_{sk}\right)\right).$$

(7) avtomorfizmning ixtiyoriy Z_k elementi bo'yicha xususiy hosilasining Pva O nuqtalardagi qiymatini hisoblaymiz.

$$\frac{\partial W_k\left(P\right)}{\partial Z_i} = R^{-1}\big(I^{(m)} - \langle P, P\rangle\big)^{-1}\left(P_i^*\big(I^{(m)} - \langle P, P\rangle\big)^{-1}\cdot\sum_{s=1}^{n}\left(P_s - P_s\right)Q_{sk} + \left(-Q_{ik}\right)\right)$$

(8) munosabatlarga ko'ra

$$R^*\left(I^{(m)} - \langle P, P\rangle\right)R = I^{(m)},\ R^* = R^{-1}\big(I^{(m)} - \langle P, P\rangle\big)^{-1}$$

tenglik o'rinli. Bundan

$$\frac{\partial W_k\left(P\right)}{\partial Z_i} = -R^* Q_{ik}$$

munosabatning o'rinli ekani kelib chiqadi.

$$\frac{\partial W_k\left(\mathrm{O}\right)}{\partial Z_i} = R^{-1}\big(I^{(m)} - \langle \mathrm{O}, P\rangle\big)^{-1}\left(P_i^*\big(I^{(m)} - \langle \mathrm{O}, P\rangle\big)^{-1}\cdot\sum_{s=1}^{n}\left(\mathrm{O} - P_s\right)Q_{sk} + \left(-Q_{ik}\right)\right) =$$

$$= R^{-1}\left(P_i^*\cdot\sum_{s=1}^{n}\left(-P_s Q_{sk}\right) + \left(-Q_{ik}\right)\right) = R^{-1}\left(P_i^* R P_k - Q_{ik}\right)$$

3^0. Tasdiqga ko'ra

$$\det\left(I^{(m)} - \left\langle\varphi_P\left(Z\right), \varphi_P\left(W\right)\right\rangle\right) = \frac{\det\left(R^{-1}\right)\cdot\det\left(I^{(m)} - \langle Z, W\rangle\right)\det\left(R^*\right)^{-1}}{\det\left(I^{(m)} - \langle Z, P\rangle\right)\cdot\det\left(I^{(m)} - \langle P, W\rangle\right)}$$

tenglik o‘rinli. (8) munosabatga ko‘ra

$$\det\left(I^{(m)}-\langle P,P\rangle\right)=\det\left(R^{*}\right)^{-1}\cdot\det\left(R^{-1}\right)$$

tenglik o‘rinli. Bundan

$$\det\left(I^{(m)}-\langle\varphi_P(Z),\varphi_P(W)\rangle\right)=\frac{\det\left(I^{(m)}-\langle P,P\rangle\right)\cdot\det\left(I^{(m)}-\langle Z,W\rangle\right)}{\det\left(I^{(m)}-\langle Z,P\rangle\right)\cdot\det\left(I^{(m)}-\langle P,W\rangle\right)}$$

munosabatning o‘rinli ekani kelib chiqadi.

4^0. (10) tenglikda $W=Z$ almashtirish bajarsak ushbu

$$\det\left(I^{(m)}-\langle\varphi_P(Z),\varphi_P(Z)\rangle\right)=\frac{\det\left(I^{(m)}-\langle P,P\rangle\right)\cdot\det\left(I^{(m)}-\langle Z,Z\rangle\right)}{\det\left(I^{(m)}-\langle Z,P\rangle\right)\cdot\det\left(I^{(m)}-\langle P,Z\rangle\right)}$$

tenglikga ega bo‘lamiz.

5^0. $F=\varphi_P\circ\varphi_P$ akslantirishni qaraylik. $F(Z)$ akslantirish uchun quyidagilar o‘rinli

a) $F(Z)\in Aut\left(B^{(1)}_{m,n}\right)$

b) $F(\mathrm{O})=\mathrm{O}$

c) $F(P)=P$

yuqoridagi a) va b) xossalarga ko‘ra

$$F(Z):W_k=\sum_{j=1}^{n}Z_jA_{jk}\left(k=\overline{1,n}\right)$$

munosabatlar o‘rinli.

$F(P)=P$ ekanini inobatga olsak ushbu

$$\left\{\begin{array}{c}P_1A_{11}+P_2A_{21}+P_3A_{31}+...+P_nA_{n1}=P_1\\ P_1A_{12}+P_2A_{22}+P_3A_{32}+...+P_nA_{n2}=P_2\\ ...\\ ...\\ P_1A_{1n}+P_2A_{2n}+P_3A_{3n}+...+P_nA_{nn}=P_n\end{array}\right. \tag{11}$$

matritsaviy chiziqli tenglamalar sistemasiga ega bo‘lamiz. (11) tenglamalar sistemasi yagona yechimga ega va quyidagicha aniqlanadi:

$$A_{ij}=\left\{\begin{array}{c}I^{(m)},\ \text{agar } i=j\\ \mathrm{O},\ \text{agar } i\neq j\end{array}\right.$$

Demak, $F(Z):W_k=Z_k\ \left(k=\overline{1,n}\right)$ ya’ni $\varphi_P\circ\varphi_p=Z$ munosabat o‘rinli.

6^0. $\varphi_P(Z)\in Aut\left(B^{(1)}_{m,n}\right)$ demak $\varphi_P(Z)$ gomeomorfizm bo‘ladi.

Izoh. Yuqorida keltirilgan teorema U. Rudin [2] kitobidagi 2.2.2 teoremaning anoligi.

1-teoremada keltirilgan xossalarni qanoatlantiruvchi avtomorfizmlar gruppasini $\Gamma_0(\Gamma_0\subset Aut\left(B^{(1)}_{m,n}\right))$ orqali belgilaymiz.

Lemma. Γ_0 avtomorfizmlar gruppasi $B_{m,n}^{(1)}$ sohada tranzitiv.

Isbot. Ixtiyoriy $Z^{(1)}$, $Z^{(2)} \in B_{m,n}^{(1)}$ elementlar berilgan bo'lsin. U holda

$$\psi = \varphi_{Z^{(2)}} \circ \varphi_{Z^{(1)}}$$

avtomorfizm $Z^{(1)} \in B_{m,n}^{(1)}$ nuqtani $Z^{(2)} \in B_{m,n}^{(1)}$ nuqtaga akslantiradi.

Yuqorida keltirilgan 1-teorema $\varphi_P(Z)$ akslantirish uchun o'rinli bo'ladi. Keyingi keltiriladigan teorema birinchi tip matritsaviy shar $B_{m,n}^{(1)}$ ning ixtiyoriy golomorf avtomorfizmi uchun o'rinli bo'ladi.

Teorema 2. Ushbu $F(P) = \mathrm{O}$ munosabatni qanoatlantiruvchi ixtiyoriy $F \in Aut\left(B_{m,n}^{(1)}\right)$ golomorf avtomorfizm berilgan bo'lsin. U holda shunday yagona U unitar matritsa topiladiki quyidagi tengliklar o'rinli bo'ladi.

1. $F = U \circ \varphi_P$;
2. Ixtiyoriy $Z^{(1)}$, $Z^{(2)} \in B_{m,n}^{(1)}$ lar uchun

$$\det\left(I - \left\langle F\left(Z^{(1)}\right), F\left(Z^{(2)}\right)\right\rangle\right) = \frac{\det\left(I - \langle P, P\rangle\right) \cdot \det\left(I - \left\langle Z^{(1)}, Z^{(2)}\right\rangle\right)}{\det\left(I - \langle Z^{(1)}, P\rangle\right) \cdot \det\left(I - \langle P, Z^{(2)}\rangle\right)}.$$

Isbot. Ushbu $F \circ \varphi_P$ akslantirish birinchi tip matritsaviy shar $B_{m,n}^{(1)}$ ning avtomorfizmi nol nuqtani saqlaydi ya'ni $(F \circ \varphi_P)(0) = 0$.

U holda A.Kartanning yagonalik teoremasiga ([2] 2.1.3-teorema) ko'ra $F \circ \varphi_P$ akslantirish chiziqli. Ma'lumki ixtiyoriy chiziqli akslantirishni shunday yagona U unitar matritsa orqali ifodalash mumkin [9].

Demak, ushbu

$$F \circ \varphi_P = U$$

munosabat o'rinli bo'ladi. 1-teoremaning 5°-xossasiga ko'ra

$$\varphi_P(\varphi_P(Z)) = Z$$

involyutsiya xossasi o'rinli. Bundan

$$F \circ \varphi_P \circ \varphi_P = U \circ \varphi_P$$

ya'ni

$$F = U \circ \varphi_P$$

tenglik o'rinli bo'ladi.

2. Ixtiyoriy $Z, W \in B_{m,n}^{(1)}$ nuqtalar uchun quyidagilar o'rinli

$$\langle F(Z), F(W)\rangle = \langle U\varphi_P(Z), U\varphi_P(W)\rangle = \langle \varphi_P(Z), \varphi_P(W)\rangle$$

ya'ni

$$\langle F(Z), F(W)\rangle = \langle \varphi_P(Z), \varphi_P(W)\rangle.$$

Bundan

$$\det\left(I - \langle F(Z), F(W)\rangle\right) = \det\left(I - \langle \varphi_P(Z), \varphi_P(W)\rangle\right)$$

tenglik o‘rinli ekani kelib chiqadi. 1-teoremaning 3°- xossasiga ko‘ra

$$\det\left(I - \langle F(Z), F(W)\rangle\right) = \frac{\det\left(I - \langle P, P\rangle\right) \cdot \det\left(I - \langle Z, W\rangle\right)}{\det\left(I - \langle Z, P\rangle\right) \cdot \det\left(I - \langle P, W\rangle\right)}$$

munasabat o‘rinli.

Bizga $f : B^{(1)}_{m,n} \to \Omega$ bigolomorf akslantirish berilgan bo‘lsin. U holda quyidagi teorema o‘rinli.

Teorema 3. Ushbu $\psi_P = f \circ \varphi_P \circ f^{-1}$ akslantirish uchun quyidagi xossalar o‘rinli.

1^0. $\psi_P \circ \psi_P = Z$(Involyutsiya bo‘lish xossasi);

2^0. $\psi_P \in Aut\left(\Omega\right);$

3^0. ψ_P akslantirish Ω sohada tranzitiv;

4^0. $\forall \psi \in Aut\left(\Omega\right)$ uchun φ_U unitar almashtirish mavjudki ushbu

$$\psi = \varphi_U \circ \psi_P$$

munosabat o‘rinli bo‘ladi.

Isbot. 1^0. Quyidagi

$$F\left(Z\right) = \psi_P \circ \psi_P$$

akslantirish uchun ushbu

$$F\left(Z\right) = \psi_P \circ \psi_P = f \circ \varphi_P \circ f^{-1} \circ f \circ \varphi_P \circ f^{-1} = f \circ \varphi_P \circ \varphi_P \circ f^{-1}$$

munosabat o‘rinli. 1-teoremaning 4^0- xossasiga ko‘ra

$$F\left(Z\right) = f \circ \varphi_P \circ \varphi_P \circ f^{-1} = Z$$

ya’ni

$$\psi_P \circ \psi_P = Z.$$

2^0. $\varphi_P \in Aut\left(B^{(1)}_{m,n}\right)$ va $f : B^{(1)}_{m,n} \to \Omega$ bigolomorf akslantirish ekanligidan $\psi_P \in Aut\left(\Omega\right)$.

3^0. Ushbu $\varphi_P\left(Z\right)$akslantirish $\Re_1\left(m, n\right)$da tranzitiv va $f : B^{(1)}_{m,n} \to \Omega$ akslantirish bigolomorf ekanligidan ψ_P akslantirish Ω sohada tranzitiv bo‘ladi.

4^0. $\forall \psi \in Aut\left(\Omega\right)$ avtomorfizm berilgan bo‘lsin, u holda ushbu

$$\varphi = f^{-1} \circ \psi \circ f$$

akslantirish uchun $\varphi \in Aut\left(B^{(1)}_{m,n}\right)$ munosabat o‘rinli. 2-teoremaning 1^0-xossasiga ko‘ra

$$\varphi = U \circ \varphi_P$$

o‘rinli. Bundan

$$U \circ \varphi_P = f^{-1} \circ \psi \circ f$$

$$\psi = f \circ U \circ \varphi_P \circ f^{-1} = f \circ U \circ f^{-1} \circ f \circ \varphi_P \circ f^{-1} = \varphi_U \circ \psi_P.$$

Ya’ni

$$\psi = \varphi_U \circ \psi_P,$$

bu yerda φ_U ushbu

$$\varphi_U = f \circ U \circ f^{-1}.$$

ko‘rinishdagi unitar akslantirish.

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РЕЗЮМЕ

В данной статье получен аналог Теоремы 2.2.5 из книги Рудина [2] для матричного шара первого типа в области $\mathbb{C}^n[m, m]$. Приведены некоторые применения свойств автоморфизмов матричного шара первого типа. В частности, доказаны некоторые свойства автоморфизмов областей, биголоморфно эквивалентных матричному шару первого типа.

Ключевые слова: Блочная матрица, биголоморфное отображение , эрмитова матрица, классическая область, автоморфизма.

RESUME

In this article, an analogue of Theorem 2.2.5 from Rudin's book [2] is obtained for the first-type matrix ball in the domain $\mathbb{C}^n[m, m]$. Some applications of the properties of the automorphisms of the first-type matrix ball are presented. In particular, certain properties of the automorphisms of domains that are biholomorphically equivalent to the first-type matrix ball are proven.

Key words: Block matrix, biholomorphic mapping, Hermitian matrix, classical domain, automorphism.