UDC 519.496

# NONLOCAL BOUNDARY VALUE PROBLEM FOR A SYSTEM OF NONHOMOGENEOUS PARABOLIC TYPE EQUATIONS WITH TWO DEGENERATE LINES

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## RESUME

This work is devoted to the study of the conditional correctness of the nonlocal boundary value problem for a system of nonhomogeneous parabolic type equations with two degeneration lines. In this article, based on the idea of A.N. Tikhonov, the conditional correctness of the problem are proved, namely, the theorems of uniqueness and conditional stability on the set of correctness are proved. For getting a priori estimate of the solution we used the logarithmic convexity method and the results of the spectral problem considered by S.G. Pyatkov.

***Key words:*** Nonlocal boundary value problem, ill-posed problem, a priori estimate, estimate of conditional stability, uniqueness of solution, set of correctness.

## Introduction

In this paper, we consider a nonlocal boundary value problem for a system of partial differential equations with changing the direction of time in space. Similar equations were considered by N. Kislov, S.G. Pyatkov, K.S. Fayazov, I.E. Egorov, S. Z. Djamalov, I.O. Khajiev and others. The correctness of nonlocal boundary value problems for some general differential and differential operator equations is studied in various aspects in the works of A.A. Dezin, V.K. Romanko, Y.I. Yurchuk and others.

The existence and uniqueness of solutions nonlocal boundary value problems for nonclassical equations were investigated in works by A. Ashyralyev and O. Yildirim [1], F. Zouyed, F. Rebbani, N. Boussetila [2], A.I. Kozhanov [3], K.B. Sabitov [4], S.Z. Djamalov[5], A.I. Shadrina [6] and others.

The parabolic equation with changing direction of time represents various physical processes. This is caused, in particular, by their applications in hydrodynamics by studying the motion of a fluid with an alternating coefficient of viscosity. Problems arising in gas dynamics lead to equations of this type. This class also includes equations that describe diffusion processes, electron scattering and many other processes in physics [7].

Ill-posed problems in the sense of J. Hadamard were studied in the works of E.M. Landis, S.G. Krein [8], M.M. Lavrent'ev [9], V. A. Morozov, V. Ya Arsenin, V. G Romanov, S. I.

Kabanikhin, B. A. Bubnov, H. A. Levine [10], I.E. Egorov and V.E. Fedorov, A.I. Kozhanov [11], S.G. Pyatkov, A.L. Buchheim [12], K.S. Fayazov [13],[14],[15], V.Isakov, M.Klibanov, A.Loiuse, P.Mass, E.Shock, A.Hasanugly, A.Amirov M.Kh. Alaminov, I.O. Khajiev [16],[17], Y.K. Khudayberganov [18] and others.

**Formulation of the problem**

Let $\Omega = \Omega_0 \times Q$, $\Omega_0 = \Omega_1 \times \Omega_2 \times \Omega_3$, $\Omega_1 = \{-1 < x < 1\}$, $\Omega_2 = \{-1 < y < 1\}$, $\Omega_3 = \{0 < z < \pi\}$, $Q = (0; T)$.

Consider the system of equations

$$\begin{cases} u_{1t} + \operatorname{sign}(x)u_{1xx} + \operatorname{sign}(y)u_{1yy} + u_{1zz} + a_1u_1 + b_1u_2 = f_1, \\ u_{2t} + \operatorname{sign}(x)u_{2xx} + \operatorname{sign}(y)u_{2yy} + u_{2zz} + a_2u_2 + b_2u_1 = f_2, \end{cases} \tag{1}$$

in the domain $\Omega \cap \{x, y \neq 0\}$, where $a_j, b_j$ are some constants, $b_2 \neq 0$, $(a_1 - a_2)^2 + 4b_1b_2 > 0$ and $j = 1, 2$, $f_j(x, y, z, t)$ are given sufficiently smooth functions.

**Problem.** Find a solution of the system of equations (1) satisfying the following conditions:
nonlocal

$$u_j(x, y, z, t)\,|_{t=0} + \alpha u_j(x, y, z, t)\,|_{t=T} = \varphi_j(x, y, z),\ (x, y, z) \in \bar{\Omega}_0,\ j = 1, 2, \tag{2}$$

boundary

$$u_j(x, y, z, t)|_{\partial\Omega_0} = 0,\ \ t \in [0; T], \tag{3}$$

and gluing

$$\begin{aligned} &\left.\frac{\partial^i u_j(x, y, z, t)}{\partial x^i}\right|_{x=-0} = \left.\frac{\partial^i u_j(x, y, z, t)}{\partial x^i}\right|_{x=+0},\ (y, z, t) \in \bar{\Omega}_2 \times \bar{\Omega}_3 \times \bar{Q}, \\ &\left.\frac{\partial^i u_j(x, y, z, t)}{\partial y^i}\right|_{y=-0} = \left.\frac{\partial^i u_j(x, y, z, t)}{\partial y^i}\right|_{y=+0},\ (x, z, t) \in \bar{\Omega}_1 \times \bar{\Omega}_3 \times \bar{Q}\,,\ \ i = 0, 1, \end{aligned} \tag{4}$$

conditions, where $\alpha$ is some constant, $\varphi_j(x, y, z)$ given sufficiently smooth functions.

In this paper, we study the correctness of the desired problem depending on the value of the parameter $\alpha$ and obtain a presentation of the solution, as well as an a priori estimate of the solution. In the case of well-posedness, conditional correctness is proved.

Let us introduce the notation

$$\begin{aligned} u_1(x, y, z, t) &= \frac{a_1 + \kappa_2}{b_2(\kappa_1 - \kappa_2)} e^{\kappa_1 t} v_1(x, y, z, t) + \frac{a_1 + \kappa_1}{b_2(\kappa_1 - \kappa_2)} e^{\kappa_2 t} v_2(x, y, z, t), \\ u_2(x, y, z, t) &= \frac{1}{(\kappa_1 - \kappa_2)} e^{\kappa_1 t} v_1(x, y, z, t) + \frac{1}{(\kappa_1 - \kappa_2)} e^{\kappa_2 t} v_2(x, y, z, t), \end{aligned} \tag{5}$$

where $\kappa_1, \kappa_2-$ roots of a quadratic equation

$$\kappa^2 + (a_1 + a_2)\kappa + a_1a_2 - b_1b_2 = 0,$$

then (1)-(4) is transformed to the following problems.

**Problem 1.** Find a solution to the equation

$$\frac{\partial v_j}{\partial t}+\operatorname{sign}(x)\frac{\partial^2 v_j}{\partial x^2}+\operatorname{sign}(y)\frac{\partial^2 v_j}{\partial y^2}+\frac{\partial^2 v_j}{\partial z^2}=\bar{f}_j,\ j=1,2, \tag{6}$$

in the domain $\Omega$, satisfying:
nonlocal

$$v_j(x,y,z,t)\,|_{t=0}+\alpha e^{\kappa_j T}v_j(x,y,z,t)\,|_{t=T}=\bar{\varphi}_j(x,y,z),\ (x,y,z)\in\bar{\Omega}_0,\ j=1,2, \tag{7}$$

boundary

$$v_j(x,y,z,t)|_{\partial\Omega_0}=0,\ \ t\in[0;T], \tag{8}$$

and gluing conditions

$$\begin{gathered}\left.\frac{\partial^i v_j(x,y,z,t)}{\partial x^i}\right|_{x=-0}=\left.\frac{\partial^i v_j(x,y,z,t)}{\partial x^i}\right|_{x=+0},\ (y,z,t)\in\bar{\Omega}_2\times\bar{\Omega}_3\times\bar{Q},\\ \left.\frac{\partial^i v_j(x,y,z,t)}{\partial y^i}\right|_{y=-0}=\left.\frac{\partial^i v_j(x,y,z,t)}{\partial y^i}\right|_{y=+0},\ (x,z,t)\in\bar{\Omega}_1\times\bar{\Omega}_3\times\bar{Q}\,,\ \ i=0,1.\end{gathered} \tag{9}$$

where

$$\begin{gathered}\bar{\varphi}_1(x,y,z)=((a_1+\kappa_1)\varphi_2(x,y,z)-b_2\varphi_1(x,y,z))\,,\\ \bar{f}_1(x,t)=(b_2f_1(x,y,z,t)-(a_1+\kappa_1)f_2(x,y,z,t))\,e^{-\kappa_1 t},\end{gathered}$$

$$\begin{gathered}\bar{\varphi}_2(x,y,z)=(b_2\varphi_1(x,y,z)-(a_1+\kappa_2)\varphi_2(x,y,z))\,,\\ \bar{f}_2(x,y,z,t)=(b_2f_1(x,y,z,t)-(a_1+\kappa_2)f_2(x,y,z,t))\,e^{-\kappa_2 t}.\end{gathered}$$

**Spectral problem**

Let $\left\{\lambda^{(1)}_{k,l,n}\right\}_{k,l,n=1}^{\infty}$, $\left\{\lambda^{(2)}_{k,l,n}\right\}_{k,l,n=1}^{\infty}$, $\left\{\lambda^{(3)}_{k,l,n}\right\}_{k,l,n=1}^{\infty}$, $\left\{\lambda^{(4)}_{k,l,n}\right\}_{k,l,n=1}^{\infty}$ eigenvalues and $\left\{\vartheta^{(j)}_{k,l,n}(x,y,z)\right\}_{k,l,n=1}^{\infty}$, $\left(j=\overline{1,4}\right)$ eigenfunctions of the following spectral problem

$$sign(x)\frac{\partial^2\vartheta}{\partial x^2}+sign(y)\frac{\partial^2\vartheta}{\partial y^2}+\frac{\partial^2\vartheta}{\partial z^2}+\lambda\vartheta=0,\ (x,y,z)\in\Omega_0\cap\{x,y\neq 0\}, \tag{10}$$

$$\begin{gathered}\vartheta(x,y,z)\,|_{x=\pm1}=0,\ (y,z)\in\bar{\Omega}_2\times\bar{\Omega}_3,\\ \vartheta(x,y,z)\,|_{y=\pm1}=0,\ (y,z)\in\bar{\Omega}_1\times\bar{\Omega}_3,\\ \vartheta(x,y,z)\left|_{\substack{z=0\\ z=\pi}}\right.=0,\ (x,y)\in\bar{\Omega}_1\times\bar{\Omega}_2,\\ \left.\frac{\partial^j\vartheta(x,y,z)}{\partial x^j}\right|_{x=-0}=\left.\frac{\partial^j\vartheta(x,y,z)}{\partial x^j}\right|_{x=+0},\ (y,z)\in\bar{\Omega}_2\times\bar{\Omega}_3,\\ \left.\frac{\partial^j\vartheta(x,y,z)}{\partial y^j}\right|_{y=-0}=\left.\frac{\partial^j\vartheta(x,y,z)}{\partial y^j}\right|_{y=+0},\ (x,z)\in\bar{\Omega}_1\times\bar{\Omega}_3\,,\ \ j=0,1.\end{gathered} \tag{11}$$

Let the solution of problem (10),(11) exist, we find presentations of the solution. We apply the method of separation of variables, we have

$$\vartheta(x,y,z)=X(x)\cdot Y(y)\cdot Z(z). \tag{12}$$

Conditions (12) imply

$$
\begin{aligned}
&X(-1) = X(+1) = 0,\\
&X(-0) = X(+0),\\
&X'(-0) = X'(+0),\\
&Y(-1) = Y(+1) = 0,\\
&Y(-0) = Y(+0),\\
&Y'(-0) = Y'(+0),\\
&\quad Z(0) = Z(\pi) = 0.
\end{aligned}
$$

Calculating partial derivatives of function (12), we obtain

$$
\begin{aligned}
\frac{\partial^2 \vartheta(x,y,z)}{\partial x^2} &= X''{}_1(x) \cdot Y(y) \cdot Z(z),\\
\frac{\partial^2 \vartheta(x,y,z)}{\partial y^2} &= X(x) \cdot Y''(y) \cdot Z(z),\\
\frac{\partial^2 \vartheta(x,y,z)}{\partial z^2} &= X(x) \cdot Y(y) \cdot Z''(z).
\end{aligned}
$$

Substituting expressions (10) into these data and separating the variables, we obtain

$$
\frac{sign(x)X''(x)}{X(x)} + \frac{sign(y)Y''(y)}{Y(y)} + \frac{Z''(z)}{Z(z)} = -\lambda.
$$

Further, since $\frac{sign(x)X''(x)}{X(x)}$ does not depend on $y, z$, $\frac{sign(y))Y''(y)}{Y(y)}$ on $x, z$ , etc. $\frac{Z''(z)}{Z(z)}$ on $x, y$, we have

$$
\frac{sign(x)X''(x)}{X(x)} = -\lambda_1, \quad \frac{sign(y)Y''(y)}{Y(y)} = -\lambda_2, \ \frac{Z''(z)}{Z(z)} = -\lambda_3.
$$

For the functions $X(x), Y(y), Z(z)$ we obtain the problems:

$$
\left.
\begin{aligned}
&sign(x)X''(x) = -\lambda_1 X(x),\\
&X(-1) = X(+1) = 0,\\
&X(-0) = X(+0),\\
&X'(-0) = X'(+0),
\end{aligned}
\right\} \tag{13}
$$

$$
\left.
\begin{aligned}
&sign(y)Y''(y) = -\lambda_2 Y(y),\\
&Y(-1) = Y(+1) = 0,\\
&Y(-0) = Y(+0),\\
&Y'(-0) = Y'(+0),
\end{aligned}
\right\} \tag{14}
$$

$$
\left.
\begin{aligned}
&Z''(z) = -\lambda_3 Z(z),\\
&Z(0) = Z(\pi) = 0.
\end{aligned}
\right\} \tag{15}
$$

Thus, solutions of a problems (13),(14),(15) will have the form:

at $\lambda_1 > 0,\ \lambda_2 > 0,\ \lambda_3 > 0,$

$$X_k^{(1)}(x) = \begin{cases} \sin\mu_k(x-1)/\cos\mu_k \quad ,0 \leqslant x \leqslant 1, \\ sh\mu_k(x+1)/ch\mu_k,\ -1 \leqslant x \leqslant 0, \end{cases} \quad k \in N,$$

$$Y_l^{(1)}(y) = \begin{cases} \sin\mu_l(y-1)/\cos\mu_l,\ 0 \leqslant y \leqslant 1, \\ sh\mu_l(y+1)/ch\mu_l\ ,\ -1 \leqslant y \leqslant 0, \end{cases} \quad l \in N,$$

$$Z_n(z) = \sin nz,\ \ n \in N,$$

and at $\lambda_1 < 0, \lambda_2 < 0,$

$$X_k^{(2)}(x) = \begin{cases} sh\mu_k(x-1)/ch\mu_k,\ 0 \leqslant x \leqslant 1, \\ \sin\mu_k(x+1)/\cos\mu_k,\ \ -1 \leqslant x \leqslant 0, \end{cases} \quad k \in N,$$

$$Y_l^{(2)}(y) = \begin{cases} sh\mu_l(y-1)/ch\mu_l\ ,0 \leqslant y \leqslant 1, \\ \sin\mu_l(y+1)/\cos\mu_l\ ,\ -1 \leqslant y \leqslant 0, \end{cases} \quad l \in N,$$

where $\lambda_{1k} = \mu_k^2 > 0, \lambda_{1k} = -\mu_k^2 < 0,\ \lambda_{2l} = \mu_l^2 > 0, \lambda_{2l} = -\mu_l^2 < 0.$

$\mu_k^2$ are eigenvalues corresponding to eigenfunctions $X_k^{(j)}(x), Y_l^{(j)}(y), j = 1, 2,$ respectively. In both cases $\mu_k$ are positive roots of the transcendental equation $tg\alpha = -th\alpha$.

We note that

$$\mu_k^2 + \mu_l^2 + n^2 = \lambda_{k,l,n}^{(1)},\ \ \mu_k^2 - \mu_l^2 + n^2 = \lambda_{k,l,n}^{(2)},$$
$$-\mu_k^2 + \mu_l^2 + n^2 = \lambda_{k,l,n}^{(3)},\ \ -\mu_k^2 - \mu_l^2 + n^2 = \lambda_{k,l,n}^{(4)}$$

and the systems of eigenfunctions corresponding to them can be represented as

$$\vartheta_{k,l,n}^{(1)}(x,y,z) = X_k^{(1)}(x) \cdot Y_l^{(1)}(y) \cdot Z_n(z),$$

$$\vartheta_{k,l,n}^{(2)}(x,y,z) = X_k^{(1)}(x) \cdot Y_l^{(2)}(l) \cdot Z_n(z),$$

$$\vartheta_{k,l,n}^{(3)}(x,y,z) = X_k^{(2)}(x) \cdot Y_l^{(1)}(y) \cdot Z_n(z),$$

$$\vartheta_{k,l,n}^{(4)}(x,y,z) = X_k^{(2)}(x) \cdot Y_l^{(2)}(y) \cdot Z_n(z),$$

where $k, l, n \in N.$

It follows from the results of [20] that problem (10),(11) has a nondecreasing sequence of eigenvalues $\left\{\lambda_{k,l,n}^{(1)}\right\}_{k,l,n=1}^{\infty}, \left\{\lambda_{k,l,n}^{(2)}\right\}_{k,l,n=1}^{\infty}, \left\{\lambda_{k,l,n}^{(3)}\right\}_{k,l,n=1}^{\infty},\ \left\{\lambda_{k,l,n}^{(4)}\right\}_{k,l,n=1}^{\infty}$ and a system of eigenfunctions corresponding to them $\left\{\vartheta_{k,l,n}^{(j)}(x,y,z)\right\}_{k,l,n=1}^{\infty},\ \left(j = \overline{1,4}\right).$

Let $\|u\|_0^2 = (u,u)$, where the inner product is $(u,v) = \int\limits_{\Omega_0} uv d\Omega_0$.

Moreover,

$$\left(sign(x)sign(y)\vartheta_{k,l,n}^{(p)}(x,y,z), \vartheta_{k_1,l_1,n_1}^{(q)}(x,y,z\right) = 0, p \neq q,\ (p,q = \overline{1,4}), \forall\, k,l,n,k_1,l_1,n_1 \in N,$$

$$\left(sign(x)sign(y)\vartheta^{(m)}_{k,l,n}(x,y,z),\vartheta^{(m)}_{k_1,l_1,n_1}(x,y,z)\right)=\begin{cases}1, \ k=k_1\wedge l=l_1\wedge n=n_1\\ 0, \ k\neq k_1\wedge l\neq l_1\wedge n\neq n_1\end{cases},(m=1,4),$$

$$\left(sign(x)sign(y)\vartheta^{(m)}_{k,l,n}(x,y,z),\vartheta^{(m)}_{k_1,l_1,n_1}(x,y,z)\right)=\begin{cases}-1, \ k=k_1\wedge l=l_1\wedge n_1=n_1\\ 0, \ k\neq k_1\wedge l\neq l_1\wedge n\neq n_1\end{cases},(m=2,3),$$

where $k,l,n,k_1,l_1,n_1\in N$.

The norm

$$\begin{aligned}\|u(x,y,z,t)\|_0^2=\sum_{k,l,n=1}^{\infty}\Bigg\{\left|\left(sign(x)sign(y)u(x,y,z,t),\vartheta^{(1)}_{k,l,n}(x,y,z)\right)\right|^2+\\ +\left|\left(sign(x)sign(y)u(x,y,z,t),\vartheta^{(2)}_{k,l,n}(x,y,z)\right)\right|^2+\\ +\left|\left(sign(x)sign(y)u(x,y,z,t),\vartheta^{(3)}_{k,l,n}(x,y,z)\right)\right|^2+\\ +\left|\left(sign(x)sign(y)u(x,y,z,t),\vartheta^{(4)}_{k,l,n}(x,y,z)\right)\right|^2\Bigg\},\end{aligned}\tag{16}$$

defined by the following formula is equivalent to the original norm in the space $H_0$.

According to Theorem 2.1 in [19] and Theorem 4.1. in [20], the eigenfunctions $\left\{\vartheta^{(j)}_{k,l,n}(x,y,z)\right\}$, $\left(j=\overline{1,4}\right)$ of problem [10],[11] normalized in $L_2\left((-1;1)^2\times(0;\pi)\right)$ form the Riesz basiss in $L_2\left((-1;1)^2\times(0;\pi)\right)$.

**A prior estimate**

Generalized solution of problem (6)-(7) is a function $\upsilon_j(x,y,z,t)\in C\left(L_2((-1;1)^2\times(0;\pi));[0;T]\right)$ which for any arbitrary function $V(x,y,z,t)\in W_2^{2,1}(\Omega)$, $V(x,y,z,T)+\alpha e^{\kappa_j T}V(x,y,z,0)=0$, $V(x,y,z,t)|_{\partial\Omega_0}=0$, $j=1,2$, satisfies the following integral identity

$$\begin{aligned}&\int_\Omega \upsilon_j\left(\operatorname{sign}(x)\operatorname{sign}(y)V_t-\operatorname{sign}(y)V_{xx}-\operatorname{sign}(x)V_{yy}-\operatorname{sign}(x)\operatorname{sign}(y)V_{zz}\right)d\Omega=\\ &-\int_\Omega \operatorname{sign}(x)\operatorname{sign}(y)V\bar{f}_j d\Omega-\int_{\Omega_0}\operatorname{sign}(x)\operatorname{sign}(y)V|_{t=0}\bar{\varphi}_j d\Omega_0.\end{aligned}$$

Let $V(x,y,z,t)=\mu_{k,l,n}(t)\vartheta^{(j)}_{k,l,n}(x,y,z)$, $\left(j=\overline{1,4}\right)$, and $\mu_{k,l,n}(T)+\alpha e^{\kappa_j T}\mu_{k,l,n}(0)=0$, $\mu_{k,l,n}(t)\in W_2^1(0;T)$. Then

$$\begin{aligned}0=\int_\Omega \operatorname{sign}(x)\operatorname{sign}(y)\vartheta^{(j)}_{k,l,n}(x,y,z)v_i(x,y,z,t)\left(\mu'_{k,l,n}(t)+\lambda^{(j)}_{k,l,n}\mu_{k,l,n}(t)\right)d\Omega\\ +\mu_{k,l,n}(t)\int_\Omega \operatorname{sign}(x)\operatorname{sign}(y)\vartheta^{(j)}_{k,l,n}(x,y,z)\bar{f}_i(x,y,z,t)d\Omega\\ +\mu_{k,l,n}(0)\int_{\Omega_0}\operatorname{sign}(x)\operatorname{sign}(y)\vartheta^{(j)}_{k,l,n}(x,y,z)\bar{\varphi}_i(x,y,z)d\Omega_0\end{aligned}\tag{17}$$

From (17) it follows that

$$\int_0^T v_{i,k,l,n}^{(j)}(t)\left({\mu'}_{k,l,n}(t)+\lambda_{k,l,n}^{(j)}\mu_{k,l,n}(t)\right)dt=$$
$$-\int_0^T\left(\mu_{k,l,n}(t)\bar{f}_{i,k,l,n}^{(j)}(t)\right)dt-\mu_{k,l,n}(0)\bar{\varphi}_{k,l,n}^{(j)},\, k,l,n\in N,$$

where

$$v_{i,k,l,n}^{(j)}(t)=(sign(x)sign(y)v_i(x,y,z,t),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=1,4)\,,$$
$$v_{i,k,l,n}^{(j)}(t)=-(sign(x)sign(y)v_i(x,y,z,t),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=2,3)\,,i=1,2,$$

$$\bar{\varphi}_{i,k,l,n}^{(j)}=(sign(x)sign(y)\bar{\varphi}_i(x,y,z),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=1,4)\,,$$
$$\bar{\varphi}_{i,k,l,n}^{(j)}=-(sign(x)sign(y)\bar{\varphi}_i(x,y,z),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=2,3)\,,$$

$$f\bar{(t)}_{i,k,l,n}^{(j)}=(sign(x)sign(y)\bar{f}_i(x,y,z,t),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=1,4)\,,$$
$$f\bar{(t)}_{i,k,l,n}^{(j)}=-(sign(x)sign(y)\bar{f}_i(x,y,z,t),\vartheta_{k,l,n}^{(j)}(x,y,z)),\ (j=2,3)\,,\ i=1,2.$$

Finally we have

$$\begin{aligned}&\left(v_{i,k,l,n}^{(j)}(t)\right)_t-\lambda_{k,l,n}^{(j)}v_{i,k,l,n}^{(j)}(t)=\bar{f}_{i,k,l,n}^{(j)}(t),\\&v_{i,k,l,n}^{(j)}(0)+\alpha e^{\kappa_i T}v_{i,k,l,n}^{(j)}(T)=\bar{\varphi}_{i,k,l,n}^{(j)},\ i=1,2,\ j=1,2,3,4,\ k,l,n\in N.\end{aligned}\tag{18}$$

From (18) we obtain

$$v_{i,k,l,n}^{(j)}(t)=\frac{\bar{\varphi}_{i,k,l,n}^{(j)}e^{\lambda_{k,l,n}^{(j)}t}}{1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(j)}T}}-\frac{\alpha e^{\lambda_{k,l,n}^{(j)}T+\kappa_i T}}{1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(j)}T}}\int_t^T e^{\lambda_{k,l,n}^{(j)}(t-\tau)}\bar{f}_{i,k,l,n}^{(j)}(\tau)d\tau+$$
$$+\frac{1}{1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(j)}T}}\int_0^t e^{\lambda_{k,l,n}^{(j)}(t-\tau)}\bar{f}_{i,k,l,n}^{(j)}(\tau)d\tau,$$

where $i=1,2,\ j=1,2,3,4,\ k,l,n\in N$.

**Lemma 1.** *Let $v\left(x,y,z,t\right)$ satisfies the equation*

$$\frac{\partial v}{\partial t}+\mathrm{sign}(x)\frac{\partial^2 v}{\partial x^2}+\mathrm{sign}(y)\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=\bar{f},$$

*and conditions $v(x,y,z,0)=\bar{\varphi}(x,y,z)$, (8)-(9), then for $v\left(x,y,z,t\right)$ at $t\in(0;T)$*

$$\left\|v(x,y,z,t)\right\|_0\leqslant 2(\left\|v(x,y,z,0)\right\|_0+\beta)^{1-\frac{t}{T}}\cdot(\left\|v(x,y,z,T)\right\|_0+\beta)^{\frac{t}{T}}+\beta,\tag{19}$$

*estimate is valid, where $\beta=\left(\int\limits_0^T\left\|\bar{f}(x,y,z,t)\right\|_0^2dt\right)^{\frac{1}{2}}$.*

Proof of lemma 1 one can find in [18].

**Lemma 2.** *Let $\alpha = 0$. Then for any generalized solution to the problem (1)-(4) for $t \in (0,T)$ the following inequalities hold:*

$$\|u_1(x,y,z,t)\|_0 \leqslant \left|\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_1 t}\omega(t,\kappa_1,\gamma_1) + \left|\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_2 t}\omega(t,\kappa_2,\gamma_2),$$

$$\|u_2(x,y,z,t)\|_0 \leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\omega(t,\kappa_1,\gamma_1) + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\omega(t,\kappa_2,\gamma_2),$$

*where*

$$\omega(t,\kappa_i,\gamma_i) = 2(|b_2|\,\|\varphi_1\|_0 + |a_1+\kappa_i|\,\|\varphi_2\|_0 + \gamma_i)^{1-\frac{t}{T}}\times$$
$$\times\big((|b_2|\,\|u_1(x,y,z,T)\|_0 + |a_1+\kappa_i|\,\|u_2(x,y,z,T)\|_0)\,e^{-\kappa_i T} + \gamma_i\big)^{\frac{t}{T}} + \gamma_i,$$

$$\gamma_i = \left(2\int_0^T \big(|b_2|^2\,\|f_1(x,y,z,t)\|_0^2 + |a_1+\kappa_i|^2\,\|f_2(x,y,z,t)\|_0^2\big)e^{-\kappa_i t}dt\right)^{\frac{1}{2}}, i = 1,2.$$

**Proof.** From (5) we have

$$\begin{aligned} v_1(x,y,z,t) &= ((a_1+\kappa_1)u_2(x,y,z,t) - b_2u_1(x,y,z,t))\,e^{-\kappa_1 t}, \\ v_2(x,y,z,t) &= (b_2u_1(x,y,z,t) - (a_1+\kappa_2)u_2(x,y,z,t))\,e^{-\kappa_2 t}. \end{aligned} \tag{20}$$

From inequalities (19) and taking into account [20] we have

$$\begin{aligned} \|v_1(x,y,z,t)\|_0 &\leqslant 2(|b_2|\,\|u_1(x,y,z,0)\|_0 + |a_1+\kappa_1|\,\|u_2(x,y,z,0)\|_0 + \beta_1)^{1-\frac{t}{T}}\times \\ &\times\big((|b_2|\,\|u_1(x,y,z,T)\|_0 + |a_1+\kappa_1|\,\|u_2(x,y,z,T)\|_0)\,e^{-\kappa_1 T} + \beta_1\big)^{\frac{t}{T}} + \beta_1, \end{aligned} \tag{21}$$

$$\begin{aligned} \|v_2(x,y,z,t)\|_0 &\leqslant 2(|b_2|\,\|u_1(x,y,z,0)\|_0 + |a_1+\kappa_2|\,\|u_2(x,y,z,0)\|_0 + \beta_2)^{1-\frac{t}{T}}\times \\ &\times\big((|b_2|\,\|u_1(x,y,z,T)\|_0 + |a_1+\kappa_2|\,\|u_2(x,y,z,T)\|_0)\,e^{-\kappa_2 T} + \beta_2\big)^{\frac{t}{T}} + \beta_2. \end{aligned} \tag{22}$$

Taking the norm from both sides of equalities (5), we obtain the following estimates

$$\begin{aligned} \|u_1(x,y,z,t)\|_0 &\leqslant \left|\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_1 t}\|v_1(x,y,z,t)\|_0 + \left|\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_2 t}\|v_2(x,y,z,t)\|_0, \\ \|u_2(x,y,z,t)\|_0 &\leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\|v_1(x,y,z,t)\|_0 + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\|v_2(x,y,z,t)\|_0. \end{aligned} \tag{23}$$

Finally, from these estimates and taking into account (21), (22), (23) the required inequalities are derived

$$\|u_1(x,y,z,t)\|_0 \leqslant \left|\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_1 t}\omega(t,\kappa_1,\gamma_1) + \left|\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_2 t}\omega(t,\kappa_2,\gamma_2),$$

$$\|u_2(x,y,z,t)\|_0 \leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\omega(t,\kappa_1,\gamma_1) + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\omega(t,\kappa_2,\gamma_2),$$

where

$$\omega(t,\kappa_i,\gamma_i)=2(|b_2|\,\|\varphi_1\|_0+|a_1+\kappa_i|\,\|\varphi_2\|_0+\gamma_i)^{1-\frac{t}{T}}\times$$
$$\times\left((|b_2|\,\|u_1(x,y,z,T)\|_0+|a_1+\kappa_i|\,\|u_2(x,y,z,T)\|_0)\,e^{-\kappa_i T}+\gamma_i\right)^{\frac{t}{T}}+\gamma_i,$$

$$\beta_i=\left(\int\limits_0^T\|\overline{f}_i(x,y,z,t)\|_0^2dt\right)^{\frac{1}{2}}\leqslant\left(2\int\limits_0^T\left(|b_2|^2\,\|f_1\|_0^2+|a_1+\kappa_i|^2\,\|f_2\|_0^2\right)e^{-\kappa_i t}dt\right)^{\frac{1}{2}}=\gamma_i, i=1,2.$$

The lemma 2 is proved.

Let $M$ denote the correctness set defined as follows

$$M=\left\{u_j(x,y,z,t):\|u_1(x,y,z,T)\|_0+\|u_2(x,y,z,T)\|_0<m,\ m<\infty\right\},\ j=1,2.$$

**Lemma 3.** *Let $\alpha\neq 0$. Then for any solution to the problem (1)-(4) for $t\in(0,T)$ the following inequalities hold:*

$$\|u_1\left(x,y,z,t\right)\|_0^2\leqslant 2\left(\frac{a_1+\kappa_2}{b_2\left(\kappa_1-\kappa_2\right)}\right)^2e^{2\kappa_1 t}\,\|v_1\left(x,y,z,t\right)\|_0^2+2\left(\frac{a_1+\kappa_1}{b_2\left(\kappa_1-\kappa_2\right)}\right)^2e^{2\kappa_2 t}\,\|v_2\left(x,y,z,t\right)\|_0^2,$$

$$\|u_2\left(x,y,z,t\right)\|_0^2\leqslant\frac{2}{\left(\kappa_1-\kappa_2\right)^2}e^{2\kappa_1 t}\,\|v_1\left(x,y,z,t\right)\|_0^2+\frac{2}{\left(\kappa_1-\kappa_2\right)^2}e^{2\kappa_2 t}\,\|v_2\left(x,y,z,t\right)\|_0^2.$$

*where*

$$\begin{aligned}&\|v_j(x,y,z,t)\|_0^2\leqslant C\left(1+\alpha^{-2}\right)e^{-2\kappa_i T}\,\|\bar{\varphi}_i(x,y,z)\|_0^2\\&+C\left(1+\alpha^2\right)(T-t)\,e^{2\kappa_i T}\int_t^T\left\|\bar{f}_i(x,y,z,t)\right\|_0^2d\tau\\&+C\left(1+\alpha^{-2}\right)te^{-2\kappa_i T}\int_0^t\left\|\bar{f}_i(x,y,z,t)\right\|_0^2d\tau, i=1,2,\end{aligned}$$

*$C-$ some constant depending on $\alpha, T$.*

**Proof.** If a solution to the problem (1)-(4) exists and belongs to $M$, then it has the form

$$\begin{aligned}u_1\left(x,y,z,t\right)&=\frac{a_1+\kappa_2}{b_2\left(\kappa_1-\kappa_2\right)}e^{\kappa_1 t}v_1\left(x,y,z,t\right)+\frac{a_1+\kappa_1}{b_2\left(\kappa_1-\kappa_2\right)}e^{\kappa_2 t}v_2\left(x,y,z,t\right),\\u_2\left(x,y,z,t\right)&=\frac{1}{\left(\kappa_1-\kappa_2\right)}e^{\kappa_1 t}v_1\left(x,y,z,t\right)+\frac{1}{\left(\kappa_1-\kappa_2\right)}e^{\kappa_2 t}v_2\left(x,y,z,t\right),\end{aligned}$$

where

$$\begin{aligned}v_i(x,y,z,t)&=\sum_{k,l,n=1}^{\infty}v_{i,k,l,n}^{(1)}(t)\vartheta_{k,l,n}^{(1)}(x,y,z)+\sum_{k,l,n=1}^{\infty}v_{i,k,l,n}^{(2)}(t)\vartheta_{k,l,n}^{(2)}(x,y,z)\\&+\sum_{k,l,n=1}^{\infty}v_{i,k,l,n}^{(3)}(t)\vartheta_{k,l,n}^{(3)}(x,y,z)\\&+\sum_{k,l,n=1}^{\infty}v_{i,k,l,n}^{(4)}(t)\vartheta_{k,l,n}^{(4)}(x,y,z),\ i=1,2.\end{aligned}\tag{24}$$

According to (15) we have

$$\|v_i(x,y,z,t)\|_0^2 = \sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(1)}(t)\right\}^2 + \sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(2)}(t)\right\}^2 + \sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(3)}(t)\right\}^2 + \sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(4)}(t)\right\}^2,\ i=1,2. \tag{25}$$

We estimate the first sum on the right-hand side of equality (25). In this case, we take into account the inequality $(a+b+c)^2 \leq 3(a^2+b^2+c^2)$, the Cauchy-Bunkhouses inequality for the integral, and we have

$$\sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(1)}(t)\right\}^2 \leqslant \sum_{k,l,n=1}^{\infty}\left\{\frac{3\left(\bar{\varphi}_{i,k,l,n}^{(1)}\right)^2 e^{2\lambda_{k,l,n}^{(1)}t}}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2} + \right.$$
$$+\frac{3\alpha^2 e^{2\kappa_i T+2\lambda_{k,l,n}^{(1)}T}}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2}\int_t^T e^{2\lambda_{k,l,n}^{(1)}(t-\tau)}d\tau\int_t^T\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau +$$
$$\left. + \frac{3}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2}\int_0^t e^{2\lambda_{k,l,n}^{(1)}(t-\tau)}d\tau\int_0^t\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau\right\}$$

From here, evaluating each participant separately for the amounts, then we can easily get the following

$$\sum_{k,l,n=1}^{\infty}\frac{3\left(\bar{\varphi}_{i,k,l,n}^{(1)}\right)^2 e^{2\lambda_{k,l,n}^{(1)}t}}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2} = \sum_{k,l,n=1}^{\infty}\frac{3\left(\bar{\varphi}_{i,k,l,n}^{(1)}\right)^2 e^{2\lambda_{k,l,n}^{(1)}(t-T)}}{\left(e^{-\lambda_{k,l,n}^{(1)}T}+\alpha e^{\kappa_i T}\right)^2} \leqslant C_{i,1}\alpha^{-2}e^{-2\kappa_i T}\sum_{k,l,n=1}^{\infty}\left(\bar{\varphi}_{i,k,l,n}^{(1)}\right)^2$$

where

$$C_{i,1} = \begin{cases} 3, \quad \alpha > 0 \\ 3\left(\dfrac{1}{\alpha}e^{-\lambda_{1,1,1}^{(1)}T-\kappa_i T}+1\right)^2, \alpha < -1 \\ 3D_{i,1},\ -1 < \alpha < 0, \end{cases}$$

$$D_{i,1} = \begin{cases} \left(\dfrac{1}{\alpha}e^{-\lambda_{k_0,l_0,n_0}^{(1)}T-\kappa_i T}+1\right)^{-2}, k \geqslant k_0, l \geqslant l_0, n \geqslant n_0,\ \lambda_{k,l,n}^{(1)} > \dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|, \\ \max\left(\left(\dfrac{1}{\alpha}e^{-\lambda_{k,l,n}^{(1)}T-\kappa_i T}+1\right)^{-2}\right),\ \lambda_{k,l,n}^{(1)} < \dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\ i=1,2. \end{cases}$$

Further we have

$$\sum_{k,l,n=1}^{\infty}\frac{3\alpha^2 e^{2\kappa_i T+2\lambda_{k,l,n}^{(1)}T}}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2}\int\limits_t^T e^{2\lambda_{k,l,n}^{(1)}(t-\tau)}d\tau\int\limits_t^T\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau$$

$$\leqslant C_{i,1}(T-t)\sum_{k,l,n=1}^{\infty}\int\limits_t^T\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau,$$

$$\sum_{k,l,n=1}^{\infty}\frac{3}{\left(1+\alpha e^{\kappa_i T+\lambda_{k,l,n}^{(1)}T}\right)^2}\int\limits_0^t e^{2\lambda_{k,l,n}^{(1)}(t-\tau)}d\tau\int\limits_0^t\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau$$

$$\leqslant C_{i,1}t\alpha^{-2}e^{-\kappa_i T}\sum_{n=1}^{\infty}\int\limits_0^t\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau.$$

Summing up these estimates, we get

$$\sum_{k,l,n=1}^{\infty}\left\{v_{i,k,l,n}^{(1)}(t)\right\}^2\leqslant C_1\alpha^{-2}e^{-2\kappa_i T}\sum_{n=1}^{\infty}\left(\bar{\varphi}_{i,k,l,n}^{(1)}\right)^2+C_{i,1}(T-t)\sum_{k,l,n=1}^{\infty}\int\limits_t^T\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau$$
$$+C_{i,1}t\alpha^{-2}e^{-2\kappa_i T}\sum_{k,l,n=1}^{\infty}\int\limits_0^t\left(\bar{f}_{i,k,l,n}^{(1)}(\tau)\right)^2 d\tau \tag{26}$$

We estimate for the second sum on the right-hand side of inequality (25) for two cases:$\lambda_{k,l,n}^{(2)}>0$ and $\lambda_{k,l,n}^{(2)}<0$. Let $\lambda_{k,l,n}^{(2)}>0$, then we get

$$\sum_{\lambda_{k,l,n}^{(2)}>0}\left\{v_{i,k,l,n}^{(2)}(t)\right\}^2\leqslant C_{i,2}\alpha^{-2}e^{-2\kappa_i T}\sum_{\lambda_{k,l,n}^{(2)}>0}\left(\bar{\varphi}_{i,k,l,n}^{(2)}\right)^2$$
$$+C_{i,2}(T-t)\sum_{\lambda_{k,l,n}^{(2)}>0}\int\limits_t^T\left(\bar{f}_{i,k,l,n}^{(2)}(\tau)\right)^2 d\tau \tag{27}$$
$$+C_{i,2}t\alpha^{-2}e^{-2\kappa_i T}\sum_{\lambda_{k,l,n}^{(2)}>0}\int\limits_0^t\left(\bar{f}_{i,k,l,n}^{(2)}(\tau)\right)^2 d\tau,$$

where

$$C_{i,2}=\begin{cases}3,\quad \alpha>0\\ 3\left(\dfrac{1}{\alpha}e^{-\lambda_{1,1,1}^{(2)}T-\kappa_i T}+1\right)^2,\alpha<-1\\ 3D_{i,2},\ -1<\alpha<0,\end{cases}$$

$$D_{i,2}=\begin{cases}\left(\dfrac{1}{\alpha}e^{-\lambda^{(2)}_{k_0,l_0,n_0}T-\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(2)}_{k,l,n}>\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\\ \max\left(\left(\dfrac{1}{\alpha}e^{-\lambda^{(2)}_{k,l,n}T-\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(2)}_{k,l,n}<\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\ i=1,2.\end{cases}$$

Let $\lambda^{(2)}_{k,l,n}<0$. Then

$$\begin{gathered}\sum_{\lambda^{(2)}_{k,l,n}<0}\left\{v^{(2)}_{i,k,l,n}(t)\right\}^2\leqslant C_{i,3}\sum_{\lambda^{(2)}_{k,l,n}<0}\left(\bar{\varphi}^{(2)}_{i,k,l,n}\right)^2+\\ +C_{i,3}\alpha^2e^{2\kappa_iT}(T-t)\sum_{\lambda^{(2)}_{k,l,n}<0}\int_t^T\left(\bar{f}^{(2)}_{i,k,l,n}(\tau)\right)^2d\tau\\ +C_{i,3}t\sum_{\lambda^{(2)}_{k,l,n}<0}\int_0^t\left(\bar{f}^{(2)}_{i,k,l,n}(\tau)\right)^2d\tau,\end{gathered}\tag{28}$$

where

$$C_{i,3}=\begin{cases}3,\quad \alpha>0\\ 3\left(\alpha e^{\lambda^{(2)}_{1,1,1}T+\kappa_iT}+1\right)^2, \alpha<-1\\ 3D_{i,3},\ -1<\alpha<0,\end{cases}$$

$$D_{i,3}=\begin{cases}\left(\alpha e^{\lambda^{(2)}_{k_0,l_0,n_0}T+\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(2)}_{k,l,n}<\dfrac{1}{T}\ln|\alpha|,\\ \max\left(\left(\alpha e^{\lambda^{(2)}_{k,l,n}T+\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(2)}_{k,l,n}>\dfrac{1}{T}\ln|\alpha|,\ i=1,2.\end{cases}$$

We estimate for the third sum on the right-hand side of inequality (25) for two cases:$\lambda^{(3)}_{k,l,n}>0$ and $\lambda^{(3)}_{k,l,n}<0$. Let $\lambda^{(3)}_{k,l,n}>0$, then we get

$$\begin{gathered}\sum_{\lambda^{(3)}_{k,l,n}>0}\left\{v^{(3)}_{i,k,l,n}(t)\right\}^2\leqslant C_{i,4}\alpha^{-2}e^{-2\kappa_iT}\sum_{\lambda^{(3)}_{k,l,n}>0}\left(\bar{\varphi}^{(3)}_{i,k,l,n}\right)^2+\\ +C_{i,4}(T-t)\sum_{\lambda^{(3)}_{k,l,n}>0}\int_t^T\left(\bar{f}^{(3)}_{i,k,l,n}(\tau)\right)^2d\tau\\ +C_{i,4}t\alpha^{-2}e^{-2\kappa_iT}\sum_{\lambda^{(3)}_{k,l,n}>0}\int_0^t\left(\bar{f}^{(3)}_{i,k,l,n}(\tau)\right)^2d\tau,\end{gathered}\tag{29}$$

where

$$C_{i,4}=\begin{cases}3,\quad \alpha>0\\ 3\left(\dfrac{1}{\alpha}e^{-\lambda^{(3)}_{1,1,1}T-\kappa_iT}+1\right)^2, \alpha<-1\\ 3D_{i,4},\ -1<\alpha<0,\end{cases}$$

$$D_{i,4}=\begin{cases}\left(\dfrac{1}{\alpha}e^{-\lambda^{(3)}_{k_0,l_0,n_0}T-\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(3)}_{k,l,n}>\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\\ \max\left(\left(\dfrac{1}{\alpha}e^{-\lambda^{(3)}_{k,l,n}T-\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(3)}_{k,l,n}<\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\ i=1,2.\end{cases}$$

Let $\lambda^{(3)}_{k,l,n}<0$. We have

$$\begin{aligned}\sum_{\lambda^{(3)}_{k,l,n}<0}\left\{v^{(3)}_{i,k,l,n}(t)\right\}^2 &\leqslant C_{i,5}\sum_{\lambda^{(3)}_{k,l,n}<0}\left(\bar{\varphi}^{(3)}_{i,k,l,n}\right)^2+\\ &+C_{i,5}\alpha^2e^{2\kappa_i T}(T-t)\sum_{\lambda^{(3)}_{k,l,n}<0}\int\limits_t^T\left(\bar{f}^{(3)}_{i,k,l,n}(\tau)\right)^2d\tau\\ &+C_{i,5}t\sum_{\lambda^{(3)}_{k,l,n}<0}\int\limits_0^t\left(\bar{f}^{(3)}_{i,k,l,n}(\tau)\right)^2d\tau,\end{aligned}\tag{30}$$

where

$$C_{i,5}=\begin{cases}3,\quad \alpha>0\\ 3\left(\alpha e^{\lambda^{(3)}_{1,1,1}T+\kappa_i T}+1\right)^2, \alpha<-1\\ 3D_{i,5},\ -1<\alpha<0,\end{cases}$$

$$D_{i,5}=\begin{cases}\left(\alpha e^{\lambda^{(3)}_{k_0,l_0,n_0}T+\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(3)}_{k,l,n}<\dfrac{1}{T}\ln|\alpha|,\\ \max\left(\left(\alpha e^{\lambda^{(3)}_{k,l,n}T+\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(3)}_{k,l,n}>\dfrac{1}{T}\ln|\alpha|,\ i=1,2.\end{cases}$$

We estimate for the fourth sum on the right-hand side of inequality (25) for two cases:$\lambda^{(4)}_{k,l,n}>0$ and $\lambda^{(4)}_{k,l,n}<0$. Let $\lambda^{(4)}_{k,l,n}>0$, then we get

$$\begin{aligned}\sum_{\lambda^{(4)}_{k,l,n}>0}\left\{v^{(4)}_{i,k,l,n}(t)\right\}^2 &\leqslant C_{i,6}\alpha^{-2}e^{-2\kappa_i T}\sum_{\lambda^{(4)}_{k,l,n}>0}\left(\bar{\varphi}^{(4)}_{i,k,l,n}\right)^2+\\ &+C_{i,6}(T-t)\sum_{\lambda^{(4)}_{k,l,n}>0}\int\limits_t^T\left(\bar{f}^{(4)}_{i,k,l,n}(\tau)\right)^2d\tau\\ &+C_{i,6}t\alpha^{-2}e^{-2\kappa_i T}\sum_{\lambda^{(4)}_{k,l,n}>0}\int\limits_0^t\left(\bar{f}^{(4)}_{i,k,l,n}(\tau)\right)^2d\tau,\end{aligned}\tag{31}$$

where

$$C_{i,6}=\begin{cases}3,\quad \alpha>0\\ 3\left(\dfrac{1}{\alpha}e^{-\lambda^{(4)}_{1,1,3}T-\kappa_i T}+1\right)^2, \alpha<-1\\ 3D_{i,6},\ -1<\alpha<0,\end{cases}$$

$$D_{i,6}=\begin{cases}\left(\dfrac{1}{\alpha}e^{-\lambda^{(4)}_{k_0,l_0,n_0}T-\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(4)}_{k,l,n}>\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\\ \max\left(\left(\dfrac{1}{\alpha}e^{-\lambda^{(4)}_{k,l,n}T-\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(4)}_{k,l,n}<\dfrac{1}{T}\ln\left|\dfrac{1}{\alpha}\right|,\ i=1,2.\end{cases}$$

Let $\lambda^{(4)}_{k,l,n}<0$. We have

$$\begin{aligned}\sum_{\lambda^{(4)}_{k,l,n}<0}\left\{v^{(4)}_{i,k,l,n}(t)\right\}^2 &\leqslant C_{i,7}\sum_{\lambda^{(4)}_{k,l,n}<0}\left(\bar{\varphi}^{(4)}_{i,k,l,n}\right)^2+\\ &+C_{i,7}\alpha^2e^{2\kappa_i T}(T-t)\sum_{\lambda^{(4)}_{k,l,n}<0}\int\limits_t^T\left(\bar{f}^{(4)}_{i,k,l,n}(\tau)\right)^2d\tau \\ &+C_{i,7}t\sum_{\lambda^{(4)}_{k,l,n}<0}\int\limits_0^t\left(\bar{f}^{(4)}_{i,k,l,n}(\tau)\right)^2d\tau,\end{aligned}\tag{32}$$

where

$$C_{i,7}=\begin{cases}3,\quad \alpha>0\\ 3\left(\alpha e^{\lambda^{(4)}_{1,1,3}T+\kappa_i T}+1\right)^2, \alpha<-1\\ 3D_{i,7},\ -1<\alpha<0,\end{cases}$$

$$D_{i,7}=\begin{cases}\left(\alpha e^{\lambda^{(4)}_{k_0,l_0,n_0}T+\kappa_i T}+1\right)^{-2}, k\geqslant k_0, l\geqslant l_0, n\geqslant n_0,\ \lambda^{(4)}_{k,l,n}<\dfrac{1}{T}\ln|\alpha|,\\ \max\left(\left(\alpha e^{\lambda^{(4)}_{k,l,n}T+\kappa_i T}+1\right)^{-2}\right)\ ,\ \lambda^{(4)}_{k,l,n}>\dfrac{1}{T}\ln|\alpha|,\ i=1,2.\end{cases}$$

Combining the last estimate and (26)-(27), we find that for the expression $\|v_j(x,y,z,t)\|_0^2$ the following inequality holds:

$$\begin{aligned}\|v_j(x,y,z,t)\|_0^2 &\leqslant C\left(1+\alpha^{-2}\right)e^{-2\kappa_i T}\|\bar{\varphi}_i(x,y,z)\|_0^2+\\ &+C\left(1+\alpha^2\right)(T-t)\,e^{2\kappa_i T}\int_t^T\left\|\bar{f}_i(x,y,z,t)\right\|_0^2d\tau \\ &+C\left(1+\alpha^{-2}\right)te^{-2\kappa_i T}\int_0^t\left\|\bar{f}_i(x,y,z,t)\right\|_0^2d\tau,\end{aligned}\tag{33}$$

where $C=max(C_1,C_2,C_3,C_4,C_5,C_6,C_7)$. Taking the norm from both sides of estimates (23),(33), we obtain the following estimates

$$\|u_1(x,y,z,t)\|_0^2\leqslant 2\left(\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right)^2e^{2\kappa_1 t}\|v_1(x,y,z,t)\|_0^2+2\left(\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right)^2e^{2\kappa_2 t}\|v_2(x,y,z,t)\|_0^2,$$

$$\|u_2(x,y,z,t)\|_0^2\leqslant\frac{2}{(\kappa_1-\kappa_2)^2}e^{2\kappa_1 t}\|v_1(x,y,z,t)\|_0^2+\frac{2}{(\kappa_1-\kappa_2)^2}e^{2\kappa_2 t}\|v_2(x,y,z,t)\|_0^2.$$

The lemma 3 is proved.

**Uniqueness and conditional stability**

**Theorem 1.** *Let* $\alpha = 0$. *If a solution to the problem (1)-(4) exists and* $(u_1(x,y,z,t), u_2(x,y,z,t)) \in M$, *then the solution to the problem (1)-(4) unique.*

**Proof.** Let pairs of functions $(u_{1,1}(x,y,z,t),\ u_{1,2}(x,y,z,t))$, $(u_{2,1}(x,y,z,t), u_{2,2}(x,y,z,t))$, are solutions of the problem (1)-(4). Let us introduce the notation $U_1(x,y,z,t) = u_{1,1}(x,y,z,t) - u_{1,2}(x,y,z,t)$, $U_2(x,y,z,t) = u_{2,1}(x,y,z,t) - u_{2,2}(x,y,z,t)$. Then the pair of functions $(U_1(x,y,z,t),\ U_2(x,y,z,t))$ satisfies the system of equations

$$\begin{cases} U_{1t} + sign(x)U_{1xx} + sign(y)U_{1yy} + U_{1zz} + a_1U_1 + b_1U_2 = 0, \\ U_{2t} + sign(x)U_{2xx} + sign(y)U_{2yy} + U_{2zz} + a_2U_2 + b_2U_1 = 0, \end{cases}$$

with conditions

$$U_i|_{t=0} + \alpha U_i|_{t=T} = 0,\ i = 1,2$$

and (3),(4). According to **lemma 2**, we have

$$\| U_1(x,y,z,t) \|_0 = 0$$

and

$$\| U_2(x,y,z,t) \|_0 = 0.$$

Hence, for any $(x,y,z,t) \in \Omega$, we have $u_{1,1}(x,y,z,t) \equiv u_{1,2}(x,y,z,t)$, $u_{2,1}(x,y,z,t) \equiv u_{2,2}(x,y,z,t)$. The theorem 1 is proved.

Let $U_{i\varepsilon}(x,y,z,t) = u_i(x,y,z,t) - u_{i\varepsilon}(x,y,z,t)$, $i = 1,2$, where the pair of functions $(u_1(x,y,z,t),\ u_2(x,y,z,t))$ is a solution to the problem (1)-(4) with exact data, and the pair of functions $(u_{1\varepsilon}(x,y,z,t), u_{2\varepsilon}(x,y,z,t))$ as a solution to the problem (1)-(4) with approximate data.

**Theorem 2.** *Let* $\alpha = 0, (u_1,u_2) \in M$, $(u_{1\varepsilon}, u_{2\varepsilon}) \in M$. *Moreover,* $\|\varphi_j - \varphi_{j\varepsilon}\|_0 \leq \varepsilon$, $\|f_j - f_{j\varepsilon}\|_0 \leq \varepsilon$, $j = 1,2$. *Then for the functions* $U_{1\varepsilon}$, $U_{2\varepsilon}$ *for any* $t \in (0;T)$ *following estimates is valid*

$$\|U_{1\varepsilon}\|_0 \leqslant \left|\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_1 t}\omega_\varepsilon(t,\kappa_1,\gamma_{1\varepsilon},\varepsilon,m) + \left|\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_2 t}\omega_\varepsilon(t,\kappa_2,\gamma_{2\varepsilon},\varepsilon,m),$$

$$\|U_{2\varepsilon}\|_0 \leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\omega_\varepsilon(t,\kappa_1,\gamma_{1\varepsilon},\varepsilon,m) + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\omega_\varepsilon(t,\kappa_2,\gamma_{2\varepsilon},\varepsilon,m),$$

*where*

$$\omega_{1\varepsilon}(t,\kappa_i,m,\varepsilon,\gamma_{i\varepsilon}) = 2(\varepsilon(|b_2| + |a_1+\kappa_i|) + \gamma_{i\varepsilon})^{1-\frac{t}{T}} \left(2m(|b_2| + |a_1+\kappa_i|)e^{-\kappa_i T} + \gamma_{i\varepsilon}\right)^{\frac{t}{T}} + \gamma_{i\varepsilon},$$

$$\gamma_{i\varepsilon} = \left(\frac{\varepsilon^2\left(|b_2|^2 + |a_1+\kappa_i|^2\right)e^{-2\kappa_i T}}{|\kappa_i|}\right)^{\frac{1}{2}}$$

**Proof.** A pair of functions $(U_{1\varepsilon}, U_{2\varepsilon})$ satisfies system

$$\begin{cases} U_{1\varepsilon t} + sign(x)U_{1\varepsilon xx} + sign(y)U_{1\varepsilon yy} + U_{1\varepsilon zz} + a_1U_{1\varepsilon} + b_1U_{2\varepsilon} = f_1 - f_{1\varepsilon}, \\ U_{2\varepsilon t} + sign(x)U_{2\varepsilon xx} + sign(y)U_{2\varepsilon yy} + U_{2\varepsilon zz} + a_2U_{2\varepsilon} + b_2U_{1\varepsilon} = f_2 - f_{2\varepsilon}, \end{cases}$$

with initial conditions

$$U_{i\varepsilon}|_{t=0} + \alpha U_{i\varepsilon}|_{t=T} = \varphi_i - \varphi_{i\varepsilon},\ i = 1, 2,$$

and homogeneous boundary conditions (3), as gluing conditions (3). It's easy to see that

$$\begin{gathered} \|\bar{\varphi}_{1\varepsilon}(x,y,z)\|_0 \leqslant |a_1+\kappa_1| \, \|\varphi_2(x,y,z)-\varphi_{2\varepsilon}(x,y,z)\|_0 \\ +|b_2| \, \|\varphi_1(x,y,z)-\varphi_{1\varepsilon}(x,y,z)\|_0 \leqslant \\ \leqslant \varepsilon\left(|a_1+\kappa_1|+|b_2|\right), \end{gathered}$$

$$\begin{gathered} \|\bar{\varphi}_{2\varepsilon}(x,y,z)\|_0 \leqslant |b_2| \, \|\varphi_1(x,y,z)-\varphi_{1\varepsilon}(x,y,z)\|_0 \\ +|a_1+\kappa_2| \, \|\varphi_2(x,y,z)-\varphi_{2\varepsilon}(x,y,z)\|_0 \leqslant \\ \leqslant \varepsilon\left(|b_2|+|a_1+\kappa_2|\right). \end{gathered}$$

Now, using the condition $(u_1, u_2) \in M$, $(u_{1\varepsilon},\ u_{2\varepsilon}) \in M$, we have

$$\begin{gathered} |b_2| \, \|U_{1\varepsilon}(x,y,z,T)\|_0 + |a_1+\kappa_1| \, \|U_{2\varepsilon}(x,y,z,T)\|_0 = |b_2| \, \|u_1(x,y,z,T)-u_{1\varepsilon}(x,y,z,T)\|_0 \\ +|a_1+\kappa_1| \, \|u_2(x,y,z,T)-u_{2\varepsilon}(x,y,z,T)\|_0 \leqslant |b_2|\left(\|u_1(x,y,z,T)\|_0+\|u_{1\varepsilon}(x,y,z,T)\|_0\right) \\ +|a_1+\kappa_1|\left(\|u_2(x,y,z,T)\|_0+\|u_{2\varepsilon}(x,y,z,T)\|_0\right) \leqslant 2m\left(|b_2|+|a_1+\kappa_1|\right), \end{gathered}$$

$$\begin{gathered} |b_2| \, \|U_{1\varepsilon}(x,y,z,T)\|_0 + |a_1+\kappa_2| \, \|U_{2\varepsilon}(x,y,z,T)\|_0 = |b_2| \, \|u_1(x,y,z,T)-u_{1\varepsilon}(x,y,z,T)\|_0 \\ +|a_1+\kappa_2| \, \|u_2(x,y,z,T)-u_{2\varepsilon}(x,y,z,T)\|_0 \leqslant |b_2|\left(\|u_1(x,y,z,T)\|_0+\|u_{1\varepsilon}(x,y,z,T)\|_0\right) \\ +|a_1+\kappa_2|\left(\|u_2(x,y,z,T)\|_0+\|u_{2\varepsilon}(x,y,z,T)\|_0\right) \leqslant 2m\left(|b_2|+|a_1+\kappa_2|\right), \end{gathered}$$

and

$$\left(\int_0^T \left\|\overline{f}_i(x,y,z,t)-\overline{f}_{i\varepsilon}(x,y,z,t)\right\|_0^2 d\tau\right)^{\frac{1}{2}} \leqslant \left(\frac{\varepsilon^2\left(|b_2|^2+|a_1+\kappa_i|^2\right)e^{-2\kappa_i T}}{|\kappa_i|}\right)^{\frac{1}{2}}.$$

According to **lemma 2**, we obtain

$$\|U_{1\varepsilon}\|_0 \leqslant \left|\frac{a_1+\kappa_2}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_1 t}\omega_\varepsilon(t,\kappa_1,\gamma_{1\varepsilon},\varepsilon,m) + \left|\frac{a_1+\kappa_1}{b_2(\kappa_1-\kappa_2)}\right| e^{\kappa_2 t}\omega_\varepsilon(t,\kappa_2,\gamma_{2\varepsilon},\varepsilon,m),$$

$$\|U_{2\varepsilon}\|_0 \leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\omega_\varepsilon(t,\kappa_1,\gamma_{1\varepsilon},\varepsilon,m) + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\omega_\varepsilon(t,\kappa_2,\gamma_{2\varepsilon},\varepsilon,m),$$

where

$$\omega_{1\varepsilon}(t,\kappa_i,m,\varepsilon,\gamma_{i\varepsilon}) = 2(\varepsilon\left(|b_2|+|a_1+\kappa_i|\right)+\gamma_{i\varepsilon})^{1-\frac{t}{T}}\left(2m\left(|b_2|+|a_1+\kappa_i|\right)e^{-\kappa_i T}+\gamma_{i\varepsilon}\right)^{\frac{t}{T}}+\gamma_{i\varepsilon},$$

$$\gamma_{i\varepsilon} = \left(\frac{\varepsilon^2\left(|b_2|^2+|a_1+\kappa_i|^2\right)e^{-2\kappa_i T}}{|\kappa_i|}\right)^{\frac{1}{2}}$$

The theorem 2 is proved.

**Theorem 3.** *Let $\alpha \neq 0$. If a solution to the problem (1)-(4) exists and $(u_1(x,y,z,t), u_2(x,y,z,t)) \in M$, then the solution to the problem (1)-(4) unique.*

**Proof.** Let pairs of functions $(u_{1,1}(x,y,z,t),\ u_{1,2}(x,y,z,t))$, $(u_{2,1}(x,y,z,t), u_{2,2}(x,y,z,t))$, are solutions of the problem (1)-(4). Let us introduce the notation $V_1(x,y,z,t) = u_{1,1}(x,y,z,t) - u_{1,2}(x,y,z,t)$, $V_2(x,y,z,t) = u_{2,1}(x,y,z,t) - u_{2,2}(x,y,z,t)$. Then the pair of functions $(V_1(x,y,z,t), V_2(x,y,z,t))$ satisfies the system of equations

$$\begin{cases} V_{1t} + sign(x)V_{1xx} + sign(y)V_{1yy} + V_{1zz} + a_1V_1 + b_1V_2 = 0, \\ V_{2t} + sign(x)V_{2xx} + sign(y)V_{2yy} + V_{2zz} + a_2V_2 + b_2V_1 = 0, \end{cases}$$

with conditions

$$V_i|_{t=0} + \alpha V_i|_{t=T} = 0, \ i = 1, 2$$

and (3),(4). According to **lemma 3**, we have

$$\| V_i(x,y,z,t) \|_0 = 0, i = 1, 2.$$

Hence, for any $(x,y,z,t) \in \Omega$, we have $u_{1,1}(x,y,z,t) \equiv u_{1,2}(x,y,z,t)$, $u_{2,1}(x,y,z,t) \equiv u_{2,2}(x,y,z,t)$. The theorem 3 is proved.

Let $V_{i\varepsilon}(x,y,z,t) = u_i(x,y,z,t) - u_{i\varepsilon}(x,y,z,t)$, $i = 1, 2$, where the pair of functions $(u_1(x,y,z,t), u_2(x,y,z,t))$ is a solution to the problem (1)-(4) with exact data, and the pair of functions $(u_{1\varepsilon}(x,y,z,t), u_{2\varepsilon}(x,y,z,t))$ as a solution to the problem (1)-(4) with approximate data.

**Theorem 4.** *Let $\alpha \neq 0, (u_1, u_2) \in M$, $(u_{1\varepsilon}, u_{2\varepsilon}) \in M$. Moreover, $\|\varphi_j - \varphi_{j\varepsilon}\|_0 \leq \varepsilon$, $\|f_j - f_{j\varepsilon}\|_0 \leq \varepsilon$, $j = 1, 2$. Then for the functions $V_{1\varepsilon}$, $V_{2\varepsilon}$ for any $t \in (0;T)$ estimates is valid*

$$\|V_{1\varepsilon}\|_0 \leqslant \left|\frac{a_1 + \kappa_2}{b_2(\kappa_1 - \kappa_2)}\right| e^{\kappa_1 t}\omega_\varepsilon(t, \kappa_1, \gamma_{1\varepsilon}, \varepsilon, m) + \left|\frac{a_1 + \kappa_1}{b_2(\kappa_1 - \kappa_2)}\right| e^{\kappa_2 t}\omega_\varepsilon(t, \kappa_2, \gamma_{2\varepsilon}, \varepsilon, m),$$

$$\|V_{2\varepsilon}\|_0 \leqslant \frac{1}{|\kappa_1 - \kappa_2|} e^{\kappa_1 t}\omega_\varepsilon(t, \kappa_1, \gamma_{1\varepsilon}, \varepsilon, m) + \frac{1}{|\kappa_1 - \kappa_2|} e^{\kappa_2 t}\omega_\varepsilon(t, \kappa_2, \gamma_{2\varepsilon}, \varepsilon, m),$$

**Proof.** A pair of functions $(V_{1\varepsilon}, V_{2\varepsilon})$ satisfies system

$$\begin{cases} V_{1\varepsilon t} + sign(x)V_{1\varepsilon xx} + sign(y)V_{1\varepsilon yy} + V_{1\varepsilon zz} + a_1V_{1\varepsilon} + b_1V_{2\varepsilon} = f_1 - f_{1\varepsilon}, \\ V_{2\varepsilon t} + sign(x)V_{2\varepsilon xx} + sign(y)V_{2\varepsilon yy} + V_{2\varepsilon zz} + a_2V_{2\varepsilon} + b_2V_{1\varepsilon} = f_2 - f_{2\varepsilon}, \end{cases}$$

with initial conditions

$$V_{i\varepsilon}|_{t=0} + \alpha V_{i\varepsilon}|_{t=T} = \varphi_i - \varphi_{i\varepsilon}, \ i = 1, 2,$$

and homogeneous boundary conditions (3), as gluing conditions (4). It's easy to see that

$$\begin{aligned} &C\left(1+\alpha^{-2}\right)e^{-2\kappa_i T}\left\|\bar{\varphi}_i(x,y,z) - \bar{\varphi}_{i\varepsilon}(x,y,z)\right\|_0^2 \\ &+C\left(1+\alpha^2\right)(T-t)\,e^{2\kappa_i T}\int_t^T \left\|\bar{f}_i(x,y,z,t) - \bar{f}_{i\varepsilon}(x,y,z,t)\right\|_0^2 d\tau \\ &+C\left(1+\alpha^{-2}\right)te^{-2\kappa_i T}\int_0^t \left\|\bar{f}_i(x,y,z,t) - \bar{f}_{i\varepsilon}(x,y,z,t)\right\|_0^2 d\tau \\ &\leqslant C\varepsilon^2\left((a_1+\kappa_i)^2 + (b_2)^2\right) \\ &\times\left(\left(1+\alpha^{-2}\right)e^{-2\kappa_i T} + \frac{(1+\alpha^2)(T-t) + (1+\alpha^{-2})\,te^{-2\kappa_i(T+t)}}{|\kappa_i|}\right) = \varpi_\varepsilon(\kappa_i, t) \end{aligned}$$

According to **lemma 3**, we have

$$\|V_{1\varepsilon}\|_0 \leqslant \left|\frac{a_1+\kappa_2}{b_2\left(\kappa_1-\kappa_2\right)}\right| e^{\kappa_1 t}\varpi_\varepsilon(\kappa_i,t) + \left|\frac{a_1+\kappa_1}{b_2\left(\kappa_1-\kappa_2\right)}\right| e^{\kappa_2 t}\varpi_\varepsilon(\kappa_i,t),$$

$$\|V_{2\varepsilon}\|_0 \leqslant \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_1 t}\varpi_\varepsilon(\kappa_i,t) + \frac{1}{|\kappa_1-\kappa_2|} e^{\kappa_2 t}\varpi_\varepsilon(\kappa_i,t).$$

The theorem 4 is proved.

## REFERENCES

1. Ashyralyev A, Yildirim O., On nonlocal boundary value problems for hyperbolic-parabolic equations. Taiwanese Journal of Mathematics,2007, 11, Issue 4,P. 1075-1089.

2. Zouyed F., Rebbani F. , Boussetila N., Ona Class of Multitime Evolution Equations with Nonlocal Initial Conditions. Abstract and Applied Analysis, Article ID-16938, 2007,P. 1-26.

3. Kozhanov A.I., On the solvability of some spatial nonlocal boundary value problems for linear parabolic equations., Vestn. Samar. Un. Ser. natural science, 3,2008, Issue 62,P. 165-174.

4. Sabitov K.B., Nonlocal problem for an equation of parabolic-hyperbolic type in a rectangular domain., Mat. Notes, 2011, 89, Issue 4,P. 596-602.

5. Djamalov S.Z., A nonlocal boundary value problem for a mixed type equation of the second kind in a multidimensional space., Bulletin of the Institute of Mathematics, 2018,No 1 P. 1-8.

6. Shadrina I.A., Nonlocal boundary value problems of Samarskii for parabolic equations with changing direction of time., Mathematical Notes of NEFU, 2012, No19, Issue 2, P. 171-186.

7. Larkin A.N., Novikov V.A., Yanenko N.N, Nonlinear equations of variable type. Novosibirsk, Science 1983.

8. Krein S.G.,Prozorovskaya O.I., Approximate methods of solving ill-posed problems, U.S.S.R. Comput. Math. Math. Phys., 1963. Vol.3 No1, 1963, P. 153-167.

9. Lavrent‘ev M.M.,Saveliev L.Y., Theory of operators and ill-posed problems., Publishing House of the Institute of Mathematics, Novosibirsk, 2010.

10. Levine H.A., Logarithmic convexity, first order differential inequalities and some applications, Trans. Amer. Math. Soc., Issue. 152. 1970, P. 299-320.

11. Kozhanov A.I. Composite Type Equations and Inverse Problems., VSP, Utrecht, The Netherlands, 1999.

12. Bukhgeim A.L., Ill-posed problems, number theory and tomography, Sib. Math. Journal.,Vol 33. Issue. 3. 1992, P. 389-402.

13. Fayazov K.S., Lavrent'ev M.M., Couchy problem for partial differential equations with operator coefficients in space, Journal inverse and ill-posed problems., Vol. 2., 1994, P. 283-296.

14. Fayazov K.S., The Cauchy problem for an elliptic equation with operator coefficients, Siberian Math. Journal, Vol. 36., 1995 P. 404-411.

15. Fayazov K.S., Khudayberganov Y.K., Ill-posed boundary-value problem for a system of partial differential equations with two degenerate lines, Journal of Siberian Federal University. Mathematics and Physics., Vol. 12., 2019, P. 392-401

16. Fayazov K.S.,Khazhiev I.O., Conditional correctness of the initial-boundary value problem for a system of high-order mixed-type equations, Izv. Vyssh. Uchebn. Zaved. Mat., No22, 202), P. 62-75.

17. Khajiev I.O., Conditional correctness and approximate solution of boundary value problem for the system of second order mixed-type equations, J. Sib. Fed. Univ. Math. Phys.,No11, Issue. 2., 2018, P. 231-241.

18. Fayazov K.S., Khudayberganov Y.K., Ill-posed boundary value problem for mixed type system equations with two degenerate lines, Siberian Electronic Mathematical Reports, Vol.17. 2020, P. 647-660.

19. Fayazov K.S.,Khudayberganov Y.K., An ill-posed boundary value problem for a mixed type second-order differential equation with two degenerate lines, Mathematical notes of nefu, Vol. 30 2003, No1, P. 51-62.

20. Pyatkov S.G., Properties of Eigen functions of a certain spectral problem and their applications, Some Applications of Functional Analysis to Equations of Mathematical Physics. Inst. Mat. Novosibirsk., 1986, P. 65-84.

21. Krein M.G.,Gokhberg I.T., Introduction to the theory of linear non-self-adjoint operators in Hilbert space.,Moscow: Scinse 1965.

22. Pyatkov S.G., Some properties of proper and attached functions of unfamiliar Sturm-Liouville problems, Nonclassical equations of mathematical physics. Collection of scientific papers. Novosibirsk: Publishing House of the Institute of Mathematics, 2005, P. 240-251.

## REZYUME

Ushbu ish ikkita buzilish chizig'iga ega bo'lgan bir jinsli bo'lmagan parabolik tipdagi tenglamalar sistemasi uchun nolokal chegaraviy masalaning shartli turg'riligini o'rganishga bag'ishlangan. Ushbu ishda A.N. Tixonov bo'yicha masalaning shartli turg'inligi isbotlangan, ya'ni korrektlik to'plamida yagonalik va shartli turg'unlik teoremalari isbotlangan. Yechim uchun Aprior baho olishda logarifmik qavariqlik usulidan va S.G.Pyatkov tomonidan qaralgan spektral masala xossalaridan foydalanilgan.

***Kalit so'zlar:*** Nolokal chegaraviy masala, nokorrekt masala, Aprior baho, shartli turg'unlik bahosi, yechimning yagonaligi, korrektlik to'plami.

## РЕЗЮМЕ

Данная работа посвящена исследованию условной корректности нелокальной краевой задачи для системы неоднородных уравнений параболического типа с двумя линиями вырождения. В данной работе на основе идеи А.Н. Тихонова доказана условная корректность задачи, а именно, доказаны теоремы единственности и условной устойчивости на множестве корректности. Для получения априорной оценки решения использован метод логарифмической выпуклости и результаты спектральной задачи, рассмотренной С.Г. Пятковым.


***Ключевые слова:*** Нелокальная краевая задача, некорректная задача, априорная оценка, оценка условной устойчивости, единственность решения, множество корректности.