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FURYE QATORLARINING ELLIPTIK QISMIY YIG‘INDILAR KETMA-KETLIGI UCHUN UMUMLASHGAN LOKALIZATSIYA MASALASI

Buvayev Q. T.
O‘zbekiston Milliy universiteti, Toshkent
buvayev@mail.ru
Davirova G. N.
O‘zbekiston Milliy universiteti, Toshkent
gulhayodaverova1@gmail.com

REZYUME

Ushbu ishda, yoyiluvchi funksiyaning tashuvchisi biror sohadan tashqarida bo‘lsin degan shart asosida, aynan shu sohada ikki karrali Furye qatorlarining elliptik qismiy yig‘indilar ketma-ketligini deyarli yaqinlashish masalasi o‘rganilgan. Bunday masalaga odatda umumlashgan lokalizatsiya masalasi deb ataladi.

Kalit so‘zlar: Furye qatori, elliptik qismiy yig‘indi, umumlashgan lokalizatsiya, maksimal operator.

Quyidagi

$$\sum_{n\in Z^2} f_n e^{inx} \tag{1}$$

ikki karrali Furye qatorini qaraymiz. Bu yerda Z^2 - tekislikdagi butun koordinatali vektorlar to‘plami, $f_n = (2\pi)^{-2}\int\limits_{T^2} f(y)e^{-iny}dy$- Furye koeffitsientlari, $T^2 = (-\pi,\pi]^2$, bunda $(-\pi,\pi]^2 = (-\pi,\pi]\times(-\pi,\pi]$- dekart ko‘paytma, $nx = n_1x_1 + n_2x_2$-skalyar ko‘paytma.

Ihtiyoriy $\lambda > 0$ son uchun $S_\lambda f(x)$ orqali (1) qatorning elliptik soha bo‘yicha qismiy yig‘indisini belgilaymiz, ya‘ni:

$$S_\lambda f(x) = \sum_{A(n)<\lambda} f_n e^{inx}, \tag{2}$$

bu yerda, $A(\xi) = a_{11}\xi_1^2 + 2a_{12}\xi_1\xi_2 + a_{22}\xi_2^2$- ikkinchi tartibli bir jinsli ko‘phad. Agar ihtiyoriy $\xi \neq 0$ vektor uchun ushbu $A(\xi) > 0$ tengsizlik o‘rinli bo‘lsa, u holda $A(\xi)$ ko‘phad elliptik ko‘phad deyiladi (yoki ushbu $a_{12}^2 - a_{11}a_{22} > 0$ tengsizlik bajarilsa).

$L_2(T^2)$- T^2 da kvadrati bilan integrallanuvchi funksiyalar sinfi bo‘lsin. Bu sinfda normani quyidagicha kiritish mumkin: $\|f\|_{L_2(T^2)} = \left(\int_{T^2}|f(x)|^2dx\right)^{\frac{1}{2}}$.

V.A. Il’in [1] ishida xos funksiyalar bo‘yicha ihtiyoriy yoyilma uchun umumlashgan lokalizatsiya prinsipi tushunchasini kiritdi. Il‘in tushinchasidan kelib chiqib, biz $L_2(T^2)$ da $S_\lambda f(x)$ uchun umumlashgan lokalizatsiya prinsipi o‘rinli deb aytamiz, agar ihtiyoriy $f \in L_2(T^2)$ funksiya uchun ushbu

$$\lim_{\lambda\to\infty} S_\lambda f(x) = 0 \tag{3}$$

tenglik $T^2 \setminus suppf$ da deyarli bajarilsa.

Ko‘rinib turibdiki, klassik lokalizatsiya prinsipidan farqli o‘laroq bu yerda (3) tenglikni $T^2 \setminus suppf$ da deyarli bajarilishi yetarli (hamma joyda emas). Ushbu ishning asosiy natijasi quyidagi teorema hisoblanadi.

1-teorema. *$\Omega \subset T^2$ biror ochiq soha, $A(\xi)$ ihtiyoriy ikkinchi tartibli bir jinsli elliptik ko‘pxad, $f(x) \in L_2(T^2)$ funksiya har bir argumentlari bo‘yicha davriy va davri 2π ga teng bo‘lsin. Agar ihtiyoriy $x \in \Omega$ da $f(x) = 0$ bo‘lsa, u holda (3) tenglik Ω ning deyarli hamma yerida bajariladi.*

Eslatib o‘tamiz, n karrali Furye qatorlarining shar bo‘yicha qismiy yig‘indisi uchun umumlashgan lokalizatsiya masalasini birinchi bo‘lib R.R. Ashurov tomonidan [2] ishda o‘rganilgan.

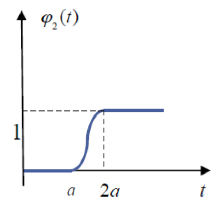
Deyarli yaqinlashish masalalarini o‘rganishda quyidagi maksimal operatorini kiritish maqsadga muvofiq: $S_*f(x) = \sup\limits_{\lambda>0} |S_\lambda f(x)|$. 1- teorema isboti maksimal operatorining quyidagi bahosiga asoslangan:

2-teorema. *$K_R = \{x \in R^2 : |x| < R\}$ - doira, $A(\xi)$ ihtiyoriy elliptik ko‘phad, $f(x) \in L_2(T^2)$ funksiya har bir argumentlari bo‘yicha davriy va davri 2π ga teng bo‘lsin. Agar istalgan $x \in K_R$ da $f(x) = 0$ bo‘lsa, u holda ihtiyoriy $r(r < R)$ uchun shunday $C = C(R, r)$ o‘zgarmas son mavjudki, quyidagi*

$$\int_{K_r} [S_*f(x)]^2 dx \le C \int_{T^2} [f(x)]^2 dx \tag{4}$$

tengsizlik o‘rinli bo‘ladi.

Biror $K_R \subset T^2$ doirada $f(x) = 0$ bo‘lsin va ixtiyoriy $r < R$ sonini tayinlaymiz. $\chi_b(t),\ t \ge 0$- $[0, b]$ segmentning xarakteristik funksiyasi bo‘lsin. $\varphi_1(t)$ orqali quyidagi $\chi_{\frac{R-r}{3}}(t) \le \varphi_1(t) \le \chi_{\frac{2(R-r)}{3}}(t)$ tengsizlikni qanoatlantiruvchi silliq funksiyani belgilaylik. $\varphi_1(t)$ funksiya orqali ifodalangan $\varphi_2(t)$ funksiyani ushbu $\varphi_2(t) = 1 - \varphi_1(t)$ ko‘rinishida aniqlaylik (1-chizma, $a = \frac{R-r}{3}$).



1-chizma

Endi quyidagi yangi funksiyani aniqlaymiz: $\psi(x) = \varphi_2(|x|), \quad x \in T^2$ va bu funksiyani butun tekislikka davriy davom ettiramiz.

Quyidagi belgilashni kiritamiz: $\theta(x, \lambda) = (2\pi)^{-2} \sum_{A(n)<\lambda} e^{inx}$. U holda tushunarliki: $S_\lambda f(x) = \int\limits_{T^2} \theta(x - y, \lambda) f(y) dy$.

Endi quyidagicha $\theta_\lambda(x) = \theta(x, \lambda)\psi(x)$ almashtirish kiritamiz.

1-xulosa. *Ixtiyoriy $x \in K_r$ uchun quyidagi*

$$S_\lambda f(x) = \int\limits_{T^2} \theta_\lambda(x - y) f(y) dy \tag{5}$$

tenglik o‘rinli.

Isbot.

$$S_\lambda f(x) = \int\limits_{T^2} \theta_\lambda(x - y) f(y) dy = \int\limits_{T^2} \theta(x - y)\psi(x - y) f(y) dy = \int\limits_{T^2} \theta(x - y) f(y) \left[1 - \varphi_1(|x - y|)\right] dy =$$

$$= \int\limits_{T^2} \theta(x - y) f(y) dy - \int\limits_{T^2} \theta(x - y) f(y) \varphi_1(|x - y|) dy = S_\lambda f(x) - I,$$

bu yerda $I = \int\limits_{T^2} \theta(x - y) f(y) \varphi_1(|x - y|) dy$.

Shunday qilib, yuqoridagi tenglikka ko‘ra $I = 0$ ekanligini ko‘rsatsak xulosa isbot bo‘ladi. Buning uchun ihtiyoriy $x \in K_r$ larda quyidagi

$$supp\varphi_1(|x - y|) \cap supp f(y) = \emptyset \tag{6}$$

munosabatni o‘rinli ekanligini ko‘rsatish yetarli.

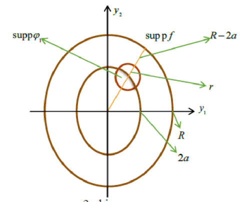
Shartga ko‘ra: $supp\varphi_1(|x - y|) = \left\{x - y \in T^2 : |x - y| < 2a, |x| < r\right\}$, bunda $a = \frac{R-r}{3}$ va $supp f(y) = \{y \in T^2 : |y| \ge R\}$.

2-chizmaga ko‘ra, 1-xulosani to‘g‘ri ekanligiga ishonch hosil qilish uchun $R - 2a \ge r$ ekanligini ko‘rsatish yetarli. Haqiqatan ham:

$$R - 2a = R - \frac{2(R - r)}{3} = \frac{R + 2a}{3} \ge \frac{3r}{3} = r.$$

Shunday qilib 1-xulosa isbot bo‘ldi.⧫

$0 < r < R$, $0 < 2a = \frac{2(R-r)}{3} < \frac{2R}{3} < R$ bo‘lgani uchun markazi koordinata boshida va radiusi $2a$ ga teng bo‘lgan doira, markazi koordinata boshida va radiusi R ga teng bo‘lgan doiraning ichida yotadi.
Boshqa tomondan, (6) ni ko‘rsatish uchun, markazi x bo‘lgan nuqtani ushbu $\{y : |y| \leq 2a\}$ doiraning ihtiyoriy nuqtasidan olib, r radiusli doira chizilganda bu doira quyidagi $\{y : |y| \geq R\}$ to‘plam bilan kesishmasligini ko‘rsatish yetarli (2- chizma).



2-chizma

Quyidagi $a(n) = [A(n)]^{\frac{1}{2}}$ belgilashni kiritamiz. Bu belgilashga ko‘ra $\theta(x, \lambda)$ ni quyidagi ko‘rinishida yozib olamiz:

$$\theta(x, \lambda) = (2\pi)^{-2} \sum_{a(n) < \sqrt{\lambda}} e^{inx}. \tag{7}$$

$A(n)$ elliptik ko‘phad bo‘lgani uchun, shunday a_1 va a_2 musbat o‘zgarmas sonlari mavjudki, ushbu

$$a_1|n| \leq a(n) \leq a_2|n|, \forall n \in Z^2 \tag{8}$$

tengsizlik o‘rinli bo‘lishini osongina ko‘rish mumkin.

O‘rama ta’rifidan foydalanib (5) ni quyidagicha yozish mumkin:

$$S_\lambda f(x) = \int\limits_{T^2} \theta_\lambda(x-y) f(y) dy = (\theta_\lambda * f)(x) = \theta_\lambda * f.$$

Oxirgi tenglikka ko‘ra:

$$S_* f(x) = \sup_{\lambda > 0} |S_\lambda f(x)| = \sup_{\lambda > 0} |\theta_\lambda * f|. \tag{9}$$

U holda (9) ga asosan quyidagi $\int\limits_{T^2} [S_* f(x)]^2 dx = \int\limits_{T^2} \left[\sup\limits_{\lambda>0} |\theta_\lambda * f|\right]^2 dx = \int\limits_{T^2} \sup\limits_{\lambda>0} [\theta_\lambda * f]^2 dx$ tenglikni yozish mumkin.

Shunday qilib, (4) ni isbot qilish uchun oxirgi tenglikka ko‘ra, ushbu

$$\int\limits_{T^2} \sup_{k>0} [\theta_k * f]^2 dx \leq C \int\limits_{T^2} [f(x)]^2 dx \tag{10}$$

tengsizlikni isbotlash yetarli, bu yerda supremum barcha natural sonlar bo‘yicha olinadi. Shuni e’tiboringizga havola qilish lozimki, bu yerda jamlash (2) tenglikka ko‘ra ushbu $\{n \in Z^2 : A(n) < \lambda\}$ to‘plam bo‘yicha olib borilayotganligi sababli, (10) tenglikda supremumni faqat barcha natural sonlar bo‘yicha olishni o‘zi yetarli bo‘ladi.

$(\theta_k)_n$ orqali $\theta_k(x)$ funksiyaning Furye koeffitsientlarini belgilaymiz, ya’ni: $(\theta_k)_n = (2\pi)^{-2} \int\limits_{T^2} \theta_k(x) e^{-inx} dx$. Bu Furye koeffitsientlari uchun quyidagi xulosa o‘rinli.

1-lemma. *Shunday $\varepsilon > 0$ son mavjudki, ixtiyoriy $k > 0$ butun son va ixtiyoriy $n \in Z^2$ uchun quyidagi*

$$|(\theta_k)_n| \leq \sum_{\varepsilon \left| a(n) - \sqrt{k} \right| < |j|} |\psi_j| \tag{11}$$

tengsizlik o‘rinli, bu yerda $\psi_j = (2\pi)^{-2} \int\limits_{T^2} \psi(y) e^{-ijy} dy$- $\psi(x)$ funksiyaning Furye koeffitsientlari.

Isbot.

$$(\theta_k)_n = (2\pi)^{-2} \int\limits_{T^2} \theta_k(y) e^{-iny} dy = (2\pi)^{-2} \int\limits_{T^2} \theta(y, k) \psi(y) e^{-iny} dy =$$

$$= (2\pi)^{-2} \int\limits_{T^2} \left((2\pi)^{-2} \sum_{a(j) < \sqrt{k}} e^{ijy} \right) \psi(y) e^{-iny} dy = (2\pi)^{-4} \sum_{a(j) < \sqrt{k}} \int\limits_{T^2} \psi(y) e^{-iy(n-j)} dy =$$

$$= (2\pi)^{-2} \sum_{a(j)<\sqrt{k}} \psi_{n-j} = (2\pi)^{-2} \sum_{a(n-j)<\sqrt{k}} \psi_j.$$

Shunday qilib, oxirgi tenglikdan, ushbu

$$|(\theta_k)_n| \le (2\pi)^{-2} \sum_{a(n-j)<\sqrt{k}} |\psi_j| \tag{12}$$

tengsizlik kelib chiqadi.

Dastlab $a(n) > \sqrt{k}$ holni qaraylik. Quyidagicha belgilashlar kiritamiz:

$$M = \left\{ j \in Z^2 : a(n-j) < \sqrt{k} \right\}, \quad N = \left\{ j \in Z^2 : |j| > \varepsilon_1 \left(a(n) - \sqrt{k} \right) \right\}.$$

(12) tengsizlikka ko‘ra (11) tengsizlikni isbotlash uchun quyidagi xulosani isbotlash yetarli:

2-xulosa. *Biror $\varepsilon_1 > 0$ son uchun, quyidagi*

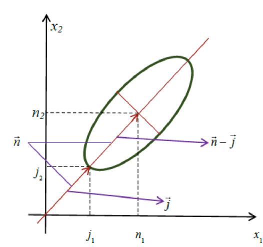
$$M \subset N \tag{13}$$

munosabat o‘rinli.

Isbot. O‘z navbatida (13) munosobatning o‘rinli ekanligini ko‘rsatish uchun, quyidagi

$$d_n = \min_{a(n-j)=\sqrt{k}} |j| = \varepsilon_1 \left(a(n) - \sqrt{k} \right) \tag{14}$$

tenglikni o‘rinli ekanligini ko‘rsatish yetarli, o‘z navbatida $A(\xi)$ elliptik ko‘phad bo‘lganligi sababli (14) tenglik koordinata boshidan markazi $n = (n_1, n_2)$ nuqtada bo‘lgan $a(n-j) = \sqrt{k}$ ellipsgacha bo‘lgan eng qisqa masofani topish masalasiga ekvivalentdir.
Faraz qilaylik $a(n-j) = \sqrt{k}$ tenglikni qanoatlantiruvchi (j_1, j_2) vektor $\omega = \frac{n}{|n|}$ vektoriga paralel bo‘lsin. U holda tushinarliki, $\frac{n}{|n|} = \frac{n-j}{|n-j|}$ boladi (3-chizma).



3-chizma

Bundan tashqari, yuqoridagi mulohazalarga asosan quyidagiga ega bo‘lamiz:

$$a\left(\frac{n-j}{|n-j|} \right) = a(\omega), \tag{15}$$

bu yerda $a(n) = [A(n)]^{\frac{1}{2}}$, $A(n) = a_{11}n_1^2 + 2a_{12}n_1n_2 + a_{22}n_2^2$.

Endi $a(n-j) = \sqrt{k}$ ekanligidan, quyidagilar

$$|n-j| = \frac{\sqrt{k}}{a(\omega)}, \tag{16}$$

$$d_k(n) = |j| = |n| - \frac{\sqrt{k}}{a(\omega)} = \frac{1}{a(\omega)} \left(a(n) - \sqrt{k} \right) \tag{17}$$

kelib chiqishini ko‘rsatamiz. Xaqiqatan ham:

$$\sqrt{k} = a(n-j) = a\left(\frac{n-j}{|n-j|} |n-j| \right) = |n-j| a\left(\frac{n-j}{|n-j|} \right) = |n-j| a(\omega) \Rightarrow |n-j| = \frac{\sqrt{k}}{a(\omega)}.$$

$$d_k(n) = |j| = |n| - |n-j| = |n| - \frac{\sqrt{k}}{a(\omega)} = \frac{1}{a(\omega)} \left(a\left(\frac{n}{|n|} \right) |n| - \sqrt{k} \right) = \frac{1}{a(\omega)} \left(a(n) - \sqrt{k} \right).$$

Endi $j=(j_1,j_2)$ uchun, ushbu
$$d_k(n) \geq a_2^{-1}\left(a(n)-\sqrt{k}\right) \tag{18}$$
tengsizlikni o‘rinli ekanligini ko‘rsatamiz, bu yerda $a_2>0$ son (8) tengsizlik orqali aniqlangan.

(8) ga ko‘ra ihtiyoriy $n\in Z^2$ uchun:
$$a_1|n| \leq a(n) \leq a_2|n| \Rightarrow a_1|n| \leq a\left(\frac{n}{|n|}|n|\right) \leq a_2|n| \Rightarrow a_1|n| \leq |n|a\left(\frac{n}{|n|}\right) \leq a_2|n| \Rightarrow$$
$$\Rightarrow^{(16)} a_1 \leq a(\omega) \leq a_2 \Rightarrow a(\omega) \geq a_2^{-1},$$
bu yerda (16) tengsizlik $|n|\neq 0$ ekanligini ta’minlab beradi.

Oxirgi va (17) tengsizliklardan (18) tengsizlik kelib chiqadi:
$$d_k(n)=\frac{1}{a(\omega)}\left(a(n)-\sqrt{k}\right) \geq a_2^{-1}\left(a(n)-\sqrt{k}\right).$$
Endi $(j_1,j_2)\parallel \omega\left(\frac{n}{|n|}\right)$ bo‘lsa, u holda ushbu
$$d_k(n)=\min_{a(n-j)=k}|j| = a_2^{-1}\left(|n|-\sqrt{k}\right) \tag{19}$$
munosabat o‘rinli bo‘lishini ko‘rsatamiz.

(16) tenglik va ushbu $|n-j|\geq|n|-|j|$ tengsizlikka ko‘ra:
$$|n|-|j| \leq |n-j| = \frac{\sqrt{k}}{a(\omega)} \Rightarrow |j| \geq |n| - \frac{\sqrt{k}}{a(\omega)} = \frac{1}{a(\omega)}\left(a(n)-\sqrt{k}\right) \geq a_2^{-1}\left(a(n)-\sqrt{k}\right) \Rightarrow$$
$$\Rightarrow d_k(n)=\min_{a(n-j)=k}|j| = a_2^{-1}\left(a(n)-\sqrt{k}\right).$$

Shunday qilib, biz (19) tenglikni bajarilishini ko‘rsatdik, o‘z navbatida $(j_1,j_2)\parallel\omega\left(\frac{n}{|n|}\right)$ hol uchun (18) va (19) ko‘ra (14) tengsizlikni o‘rinli ekanligi kelib chiqadi.

Endi j ihtiyoriy va k tayinlangan son bo‘lsin. $d_k(n)=(d_k^1,d_k^2)\in\left\{a(n-j)=\sqrt{k}\right\}$ orqali j ga minimum beradigan ihtiyoriy nuqtani belgilaylik. Oshkormas funksiyani mavjudligi haqidagi teoremaga ko‘ra $a(n)=\sqrt{k}$ ni qanoatlantiruvchi n lar uchun $a(n-j)=\sqrt{k}$ dan $j_2=f_k(j_1)\Rightarrow d_k^2=f_k(d_k^1)$ ni hosil qilamiz (bizning holimizda, $A(\xi)$ elliptik ko‘phad bo‘lgani uchun, ushbu $a(\xi)=\sqrt{k}$ tenglik bilan aniqlangan $\xi_2=f_k(\xi_1)$ oshkormas funksiya yuqoridagi tengsizlikni qanoatlantiruvchi barcha $(\xi_1\xi_2)$ nuqtalarning yetarlicha kichik atrofida mavjud va yagonadir).Oxirgi tenglikni hisobga olib, koordinata boshidan ellipsgacha bo‘lgan masofa uchun, quyidagini hosil qilamiz: $|d_k(n)|=\sqrt{(d_k^1)^2+(d_k^2)^2}\leq\sqrt{|d_k^1|^2+\left(f_k(d_k^1)+\delta\right)^2}$,
bu yerda $\delta > a_2^{-1}\left(a(n)-\sqrt{k}\right)$. Agar $|d_k^1|>l\delta$ bo‘lsa, u holda $|d_k(n)|\geq|d_k^1|>l\delta$. Boshqa tomondan, agar $|d_k^1|\leq l\delta$ va l yetarlicha kichik bo‘lsa, u holda bir o‘zgaruvchili funsiyalar uchun Teylor formulasiga ko‘ra:
$$f(x)=f(x_0)+\frac{f^{‘}(x_0}{1!}\Delta x+\frac{f^{“}(x_0}{2!}(\Delta x)^2+\cdots+\frac{f^{(m)}(x_0)}{m!}(\Delta x)^m+R_{m+1}(x), \tag{20}$$
bu yerda $R_{m+1}(x)=\frac{f^{m+1}(x_0+\theta\Delta x)}{(m+1)!}(\Delta x)^{m+1}$ - Lagranj ko‘rinishidagi qoldiq had, $0<\theta<1$, $m\geq 0$ butun son, $\Delta x = x-x_0$, $f(x)$ funksiya $x_0\in R^2$ nuqtaning biror atrofida aniqlangan va shu atrofda $m+1$ marta differensiallanuvchi funksiya. (20) formula $m=0$ da quyidagi ko‘rinishda bo‘ladi:
$$f(x)=f(x_0)+f^{‘}(x_0+\theta\Delta x)\Delta x. \tag{21}$$
Yoki $\Delta x = x-x_0$ ekanligini xisobga olib, (21) formulani $x_0=0$ uchun, ya’ni Makloren qatorini yozsak: $f(x)=f(0)+xf^{‘}(\theta x)$. Oxirgi tenglikni biz qarayotgan $j_2=f_k(j_1)$ funksiya uchun qo‘llasak, u holda quyidagini hosil qilamiz:
$$f_k(j_1)=f_k(0)+j_1f_k^{‘}(\theta j_1). \tag{22}$$

$a(n)=\sqrt{k}$ ni qanoatlantiruvchi barcha n lar uchun $f_k(0)=0$ bo‘lgani uchun (22) formula ushbu ko‘rinishida bo‘ladi:

$$f_k(j_1)=j_1 f_k^{`}(\theta j_1).$$

yoki

$$f_k(d_k^1)=d_k^1 f_k^{`}(\theta d_k^1). \tag{23}$$

l yetarlicha kichik bo‘lgani uchun (23) tenglikka ko‘ra quyidagini xosil qilamiz:

$$|d_k(n)|=\sqrt{|d_k^1|^2+\left(f_k(d_k^1)+\delta\right)^2}\geq |f_k(d_k^1)|+\delta=|d_k^1||f_k^{`}(\theta d_k^1)|+\delta.$$

d_k yetarlicha kichik va $f_k^{`}(\theta d_k^1)$ chegaranlangan bo‘lganligi uchun, oxirgi tenglikdan quyidagini olamiz: $|d_k(n)|\geq c\delta,\ c>0$. Shunday qilib, 2-xulosa o‘z navbatida $a(n)>\sqrt{k}$ hol uchun 1-lemma isbotlandi.

Endi $a(n)<\sqrt{k}$ bo‘lsin. Quyidagicha belgilashlar kiritamiz:

$$M=\left\{j\in Z^2: a(n-j)<\sqrt{k}\right\},\ N_1=\left\{j\in Z^2:|j|\leq\varepsilon_2\left(\sqrt{k}-a(n)\right)\right\}.$$

Shunday $\varepsilon_2>0$ soni mavjudki, ushbu $M\supset N_1$ munosabatni o‘rinli ekanligini yuqoridagiga o‘xshash ko‘rsatish mumkin.

$\psi(x)$ cheksiz marta differensiallanuvchi va davriy funksiya bo‘lgani uchun, uni Furye qatoriga yoyishimiz mumkin, ya‘ni $\psi(x)=(2\pi)^{-2}\sum\limits_{j\in Z^2}\psi_j e^{ijx}\Rightarrow\psi(0)=(2\pi)^{-2}\sum\limits_{j\in Z^2}\psi_j$.

$\psi(0)=0$ bo‘lgani uchun:

$$(2\pi)^{-2}\sum_{j\in Z^2}\psi_j=0\Rightarrow(2\pi)^{-2}\sum_{a(n-j)<\sqrt{k}}\psi_j+(2\pi)^{-2}\sum_{a(n-j)\geq\sqrt{k}}\psi_j=0\Rightarrow$$

$$\Rightarrow(2\pi)^{-2}\sum_{a(n-j)<\sqrt{k}}\psi_j=-(2\pi)^{-2}\sum_{a(n-j)\geq\sqrt{k}}\psi_j.$$

Shunday qilib, bu holda oxirgi tenglikdan foydalanib, quyidagini hosil qilamiz:

$$(2\pi)^{-2}\left|\sum_{a(n-j)<\sqrt{k}}\psi_j\right|=(2\pi)^{-2}\left|-\sum_{a(n-j)\geq\sqrt{k}}\psi_j\right|\leq(2\pi)^{-2}\sum_{a(n-j)\geq\sqrt{k}}|\psi_j|\leq$$

$$\leq(2\pi)^{-2}\sum_{|j|\geq\varepsilon_2\left(\sqrt{k}-a(n)\right)}|\psi_j|. \tag{24}$$

Agar ε sifatida ε_1 va ε_2 larni minimumini olsak, u holda $|(\theta_k)_n|\leq\sum\limits_{\varepsilon\left|a(n)-\sqrt{k}\right|<|j|}|\psi_j|$ tengsizlikni hosil qilamiz.

Shunday qilib 1-lemma to‘liq isbotlandi.♦

2-lemma. *Ixtiyoriy $l\in N$ son uchun l, r va R larga bog‘liq bo‘lgan shunday C o‘zgarmas son mavjudki, barcha $k\geq 0$ butun va $n\in Z^2$ lar uchun ushbu*

$$|(\theta_k)_n|\leq\frac{C}{\left(1+\varepsilon\left|a(n)-\sqrt{k}\right|\right)^l}$$

tengsizlik o‘rinli bo‘ladi.

Isbot. Ixtiyoriy $h\geq 0$ son uchun $R-r$ ga bog‘liq bo‘lgan shunday c_h o‘zgarmas son mavjudki, quyidagi

$$|\psi_j|\leq\frac{c_h}{(1+|j|)^h} \tag{25}$$

tengsizlik o‘rinli.

1-lemma va (25) tengizlikka ko‘ra:

$$\left|(\theta_k)_n\right| \le \sum_{\varepsilon\left|a(n)-\sqrt{k}\right|<|j|} |\psi_j| \le \sum_{\varepsilon\left|a(n)-\sqrt{k}\right|<|j|} \frac{c_h}{(1+|j|)^h} \le$$

$$\le \sum_{\varepsilon\left|a(n)-\sqrt{k}\right|<|j|} \frac{c_h}{\left(1+\varepsilon\left|a(n)-\sqrt{k}\right|\right)^h} \le \frac{c_l}{\left(1+\varepsilon\left|a(n)-\sqrt{k}\right|\right)^l}.$$

2- lemma isbotlandi.⧫

2-lemmadan quyidagi natija kelib chiqadi.

1-natija. *Quyidagi tengsizlik o‘rinli:*

$$\sum_{k=0}^{\infty}\left|(\theta_k)_n\right|^2 = \sum_{j=0}^{\infty}\sum_{j\le\sqrt{k}<j+1}\left|(\theta_k)_n\right|^2 \le ca(n).$$

Endi quyidagicha belgilashni kiritamiz: $(\Delta_k)_n = (\theta_{k+1})_n - (\theta_k)_n$. Bu belgilashga ko‘ra, $(\Delta_k)_n$ ni quyidagicha yozib olamiz:

$$(\Delta_k)_n = \int\limits_{T^2}\theta_{k+1}(x)e^{-inx}dx - \int\limits_{T^2}\theta_k(x)e^{-inx}dx = \int\limits_{T^2}\left[\theta_{k+1}(x)-\theta_k(x)\right]e^{-inx}dx. \tag{26}$$

$$\theta_{k+1}(x)-\theta_k(x) = \left[\theta(x,k+1)-\theta(x,k)\right]\psi(x). \tag{27}$$

$$\theta(x,k+1)-\theta(x,k) = (2\pi)^{-2}\sum_{A(j)<k+1}e^{ijx} - (2\pi)^{-2}\sum_{A(j)<k}e^{ijx} =$$

$$= (2\pi)^{-2}\left\{\sum_{A(j)<k}e^{ijx} + \sum_{A(j)=k}e^{ijx} - \sum_{A(j)<k}e^{ijx}\right\} = (2\pi)^{-2}\sum_{A(j)=k}e^{ijx} =$$

$$= (2\pi)^{-2}\sum_{a(j)=\sqrt{k}}e^{ijx}. \tag{28}$$

(28) ni (27) ga qo‘yib:

$$\theta_{k+1}(x)-\theta_k(x) = (2\pi)^{-2}\sum_{a(j)=\sqrt{k}}e^{ijx}\psi(x). \tag{29}$$

(29) ni (26) ga qo‘yib:

$$(\Delta_k)_n = (2\pi)^{-2}\int\limits_{T^2}\sum_{a(j)=\sqrt{k}}e^{-i(n-j)x}\psi(x)dx = (2\pi)^{-2}\sum_{a(n-j)=\sqrt{k}}\int\limits_{T^2}\psi(x)e^{-ijx}dx = \sum_{a(n-j)=\sqrt{k}}\psi_j.$$

Shunday qilib, quyidagi tenglikka ega bo‘lamiz:

$$(\Delta_k)_n = \sum_{a(n-j)=\sqrt{k}}\psi_j. \tag{30}$$

Agar $a(n-j)=\sqrt{k}$ tenglama yechimga ega bo‘lmasa, u holda $(\Delta_k)_n = 0$ deb olamiz.

1-natijadan ko‘rinib turibdiki, $(\theta_k)_n$ ni bahosi $a(n)$ bo‘yicha tekis bajarilmaydi. Bu holatni kompensatsiya qilish uchun $(\Delta_k)_n$ son uchun yaxshi baho olish zarur. Buning uchun $A^j = \{k : j \le \sqrt{k} < j+1\}$ sonli to‘plamni quyidagicha $Q^j = \left\{k : k = j^2+p, p = 0,1,2,\ldots,2j\right\}$ yozib olishimiz mumkin. Haqiqatan ham:

$$j \le \sqrt{k} < j+1 \Rightarrow j^2 \le k < j^2+2j+1 \Rightarrow k = j^2+p,\ \ p = 0,1,2,\ldots,2j.$$

Endi yuqorida kiritilgan Q^k to'plamga asoslanib $|(\Delta_k)_n|^2$ ni k bo'yicha quyidagicha yig'amiz:

$$\sum_{k=0}^{\infty}|(\Delta_k)_n|^2=\sum_{k=0}^{\infty}\sum_{p=0}^{2k}\left|(\Delta_{k^2+p})_n\right|^2. \tag{31}$$

3-lemma. *$n\in Z^2$ bo'yicha quyidagi*

$$\sum_{k=0}^{\infty}\sum_{p=0}^{2k}\left|(\Delta_{k^2+p})_n\right|^2\le C$$

tengsizlik tekis bajariladi.

Agar $(\Delta_{k^2+p})_n$ sonlarini p parametr bo'yicha mos tarzda guruhlansa, u holda yana ham kuchliroq natija olish mumkin. Bizning keyingi vazifamiz shu guruhlashni o'tkazishdan iborat.

x_0 orqali ushbu $\left\{x\in R^2: a(x-n)\le k+1\right\}$ to'plamning koordinata boshiga yaqin nuqtasini belgilaylik. T_{x_0}-$\left\{x\in R^2: a(x-n)\le k+1\right\}$ to'plamning x_0 nuqtasiga o'tkazilgan urinma bo'lsin. l orqali x_0 nuqtadan o'tuvchi va T_{x_0} urinmaga perpendikulyar bo'lgan to'g'ri chiziqni belgilaylik.

Quyidagi to'plamlarni tuzamiz:
$B_0=\{x\in T_{x_0}:|x-x_0|<1\}$ va $B_j=\left\{x\in T_{x_0}:\sqrt{j}\le|x-x_0|<\sqrt{j+1}\right\}$ bu yerda $j=1,2,\ldots,2k-1$. $C_j^k,\ j=1,2,\ldots,2k-1$ asosi B_j hamda balandligi l to'g'ri chiziqqa parallel va uzunligi $|n|$ ga teng bo'lgan to'g'ri to'rtburchak bo'lsin. Ushbu $K=\left\{x\in R^2: k\le a(x-n)<k+1\right\}$ xalqani qaraylik va uni quyidagi to'plamlarga ajratamiz: $P_j^k=K\cap C_j^k,\ j=0,1,\ldots,2k-1$.

$Q_q^k,\ q=0,1,\ldots,2k-1$ to'plamini quyidagicha aniqlaymiz. Q_q^k shunday butun $p,\ 0\le p\le 2k-1$, sonlari to'plamiki, qaysiki $A(n-j)=k^2+p$ tenglama P_q^k to'plamida yechimga ega. Agar biror p uchun P_n^k to'plam $A(n-j)=k^2+p$ tenglamaning yechimlarini o'z ichiga olmasa, u holda Q_q^k to'plamiga bitta ixtiyoriy p sonini qo'shamiz, qaysiki oldingi $Q_j^k,\ j=0,1,\ldots,q-1$ to'plamlariga kirmagan. Agar bunday p mavjud bo'lmasa, u holda $Q_j^k,\ j=q,q+1,\ldots,2k-1$ to'plamlarini bo'sh deb xisoblaymiz.

Bunday tanlashda Q_q^k to'plam ko'pi bilan $4\sqrt{q+1}$ elementlarga ega bo'lishini ko'rsatish mumkin. Bunga ishonch xosil qilish uchun P_q^k to'plamning Ox_1 o'qiga proeksiyasini uzunligi $2\sqrt{q+1}$ dan oshib ketmasligini ko'rsatish yetarli. O'z navbatida, agar $j\in P_q^k$ da har bir tayinlangan p uchun $A(n-j)=k^2+p$ tenglamaning yechimi mavjud bo'lsa, qaysiki j ning birinchi koordinatasi j_1, $[2\sqrt{q+1}]$ dan kichik bo'lgan turli qiymatlar qabul qiladi ($[a]$ - a butun qismi). Agarda p, 0 dan $2k-1$ gacha o'zgarsa, har bir j_1 soni uchun bu hol ko'pi bilan 2 marta takrorlanishi mumkin. O'z navbatida, har bir p parametr uchun Q_q^k to'plam ko'pi bilan $4[\sqrt{q+1}]$ ta elementlarga ega bo'lishi mumkin.

Q_q^k to'plamni bunday tanlashda biz quyidagi xulosaga ega bo'lamiz.

4-lemma. *$q=0,1,\ldots,2k-1$ va $S_p=\left\{j\in Z^2: A(n-j)=k^2+p\right\}$ bo'lsin, bunda $p=0,1,\ldots,2k$. Agar:*

1)$a(n)\ge k+1$ bo'lsa, u holda

$$\min_{j\in S_p,\ p\in Q_q^k}|j|\ge\sqrt{\left(a(n)-k-1\right)^2+q} \tag{32}$$

tengsizlik o'rinli bo'ladi;

2)$k<a(n)<k+1$ bo'lsa, u holda

$$\min_{j\in S_p,\ p\in Q_q^k}|j|=\sqrt{q} \tag{33}$$

tengsizlik o'rinli bo'ladi;

3)$a(n)\le k$ bo'lsa, u holda

$$\min_{j\in S_p,\ p\in Q_q^k}|j|\ge\frac{1}{2}\sqrt{\left(a(n)-k\right)^2+q} \tag{34}$$

tengsizlik o'rinli bo'ladi.

Isbot. Lemmani isbot qilish, ushbu $f(j) = |j|$ butun koordinatali funksiyaning P_q^k to‘plamida eng kichik qiymatini topish bilan bir xil. Buning uchun P_q^k to‘plamining koordinata boshiga eng yaqin nuqtasini topib, $f(j)$ funksiyaning shu nuqtadagi qiymatini topish yetarli.

1)$a(n) \geq k+1$ bo‘lsin. Bu holda kordinata boshidan B_q to‘plamgacha bo‘lgan masofa ga teng bo‘lishini osongina ko‘rish mumkin.

2)$k < a(n) < k+1$ bo‘lsin. Bu holda, koordinata boshidan P_q^k to‘plamigacha bo‘lgan eng qisqa masofa bu koordinata boshidan C_q^k- to‘g‘ri to‘rtburchakkacha bo‘lgan masofadir, vaholanki, bu masofa $\sqrt{q}$ ga tengdir. Shunday qilib, bu hol uchun ham 4-lemma o‘rinlidir.

3) $a(n) \leq k$ bo‘lsin. Bu holda koordinata boshidan P_q^k to‘plamigacha bo‘lgan eng qisqa masofa, C_q^k bilan $a(n-j) = k$ sirtning kesishish nuqtasigacha bo‘lgan masofadir. Bu holda ham isbot 1) qismi kabi isbotlanadi. Shunday qilib, 4-lemma to‘liq isbotlandi.⧫

5-lemma. *Ihtiyoriy l natural son uchun shunday c_l o‘zgarmas son mavjudki, quyidagi*

$$\sum_{q=0}^{2k-1}(q+1)^2\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)_n\right|^2 \leq \frac{c_l}{\left(1+|a(n)-k|\right)^l} \tag{35}$$

tengsizlik o‘rinli bo‘ladi.

Isbot. $a(n) \geq k+1$ bo‘lsin. $(\Delta_j)_n$ ni (30) tenglik bilan aniqlangan ta‘rifiga ko‘ra quyidagini xosil qilamiz:

$$\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)\right|^2 \leq c\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)\right| = c\sum_{p\in Q_q^k}\left|\sum_{a(n-j)=k^2+p}\psi_j\right| \leq c\sum_{p\in Q_q^k}\sum_{a(n-j)=k^2+p}|\psi_j|\,. \tag{36}$$

4-lemmaning 1) qismiga ko‘ra, quyidagi baho

$$\sum_{p\in Q_q^k}\sum_{a(n-j)=k^2+p}|\psi_j| \leq \sum_{|j|\geq\sqrt{(a(n)-k-1)^2+q}}|\psi_j| \tag{37}$$

o‘rinli. (37) da ψ_j uchun o‘rinli bo‘lgan (25) tengsizlikdan foydalanib, ushbu tengsizlikni hosil qilamiz:

$$\sum_{q=0}^{2k-1}(q+1)^2\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)_n\right|^2 \leq c\sum_{q=0}^{2k-1}(q+1)^2\sum_{p\in Q_q^k}\sum_{a(n-j)=k^m+p}|\psi_j| \leq$$

$$\leq c\sum_{q=0}^{2k-1}(q+1)^2\sum_{|j|\geq\sqrt{(a(n)-k-1)^2+q}}|\psi_j| \leq c\sum_{q=0}^{2k-1}(q+1)^2\sum_{|j|\geq\sqrt{(a(n)-k-1)^2+q}}\frac{c_l}{\left(1+|j|\right)^l} \leq$$

$$\leq c\sum_{q=0}^{2k-1}(q+1)^2\sum_{|j|\geq\sqrt{(a(n)-k-1)^2+q}}\frac{c_l}{\left(1+\sqrt{\left(a(n)-k-1\right)^2+q}\right)^l} \leq$$

$$\leq c\sum_{q=0}^{2k-1}(q+1)^2\sum_{|j|\geq\sqrt{(a(n)-k-1)^2+q}}\frac{c_l}{\left(1+a(n)-(k+1)\right)^l} \leq \frac{c_l}{\left(1+a(n)-k\right)^l}.$$

Boshqa hollarda ham 5-lemma xuddi shunga o‘xshash isbotlanadi. 5-lemma isbotlandi.⧫

5-lemmadan quyidagi natija kelib chiqadi:

2-natija. *Quyidagi*

$$\sum_{k=0}^{\infty}\sum_{q=0}^{2k-1}(q+1)^2\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)_n\right|^2 \leq c \tag{38}$$

tengsizlik n bo‘yicha tekis bajariladi.

Endi $(\theta_j)_n$ Furye koeffitsientlariga qaytamiz. 1-lemmadan quyidagi bahoni olamiz.

6-lemma. *Ushbu*

$$\sum_{k=0}^{\infty}\sum_{q=0}^{2k-1}(q+1)^{-2}\sum_{p\in Q_q^k}\left|\left(\theta_{k^2+p}\right)_n\right|^2\leq c \tag{39}$$

tengsizlik n bo‘yicha tekis bajariladi.

Isbot. Yuqorida Q_q^k to‘plamning elementlari soni $4\left[\sqrt{q+1}\right]$ dan oshib ketmasligi aytilgan edi. Ushbu mulohaza va 1-lemmaga ko‘ra quyidagini hosil qilamiz:

$$\sum_{k=0}^{\infty}\sum_{q=0}^{2k-1}(q+1)^{-2}\sum_{p\in Q_q^k}\left|\left(\theta_{k^2+p}\right)_n\right|^2\leq\sum_{k=0}^{\infty}\frac{c_l}{(1+|a(n)-k|)^l}\sum_{q=0}^{2k-1}\frac{4\left[\sqrt{q+1}\right]}{(q+1)^2}\leq c.$$

6-lemma isbotlandi. ⧫

2-teoremani isboti

Endi $\Delta_j(x)=\theta_{j+1}(x)-\theta_j(x)$ bo‘lsin. Bu tenglikni har ikkala tomoniga $2\theta_j(x)$ ni qo‘shamiz:

$$\theta_{j+1}(x)+\theta_j(x)=\Delta_j(x)+2\theta_j(x)$$

yoki

$$\theta_{j+1}*f+\theta_j*f=\Delta_j*f+2\theta_j*f. \tag{40}$$

Tushinarliki $\{a_q\}$ ketma-ketlik uchun quyidagi tenglik o‘rinli:

$$\sum_{j=0}^{q-1}\left(a_{j+1}^2-a_j^2\right)=a_q^2-a_0^2=a_q^2. \tag{41}$$

Bu yerda $a_0=0$ deb qabul qilamiz (bizning holda bu bajariladi $a_0=\theta_0*f=(\theta(0,x)\cdot\psi(x))*f=[\theta(0,x)=0]=0$).

(40) ni har ikkala tomonini $\Delta_j*f=\theta_{j+1}*f-\theta_j*f$ ga ko‘paytiramiz:

$$\left[\theta_{j+1}*f\right]^2-\left[\theta_j*f\right]^2=\left[\Delta_j*f\right]^2+2\left[\Delta_j*f\right]\left[\theta_j*f\right].$$

Oxirgi tenglikni j bo‘yicha 0 dan $q-1$ gacha yig‘ib va chap tomonida (41) tenglikdan foydalanib, quyidagi tenglikni hosil qilamiz:

$$\left[\theta_q*f\right]^2=\sum_{j=0}^{q-1}\left[\Delta_j*f\right]^2+2\sum_{j=0}^{q-1}\left[\Delta_j*f\right]\left[\theta_j*f\right]. \tag{42}$$

(42) tenglikka ko‘ra, ushbu tengsizlikka ega bo‘lamiz:

$$\left|\theta_q*f\right|^2\leq\sum_{j=0}^{q-1}\left|\Delta_j*f\right|^2+2\sum_{j=0}^{q-1}\left|\Delta_j*f\right|\left|\theta_j*f\right|\leq\sum_{j=0}^{\infty}\left|\Delta_j*f\right|^2+2\sum_{j=0}^{\infty}\left|\Delta_j*f\right|\left|\theta_j*f\right|=$$

$$=\sum_{j=0}^{\infty}\left|\Delta_j*f\right|^2+2\sum_{j=0}^{\infty}\sum_{q=0}^{2k-1}\sum_{p\in Q_q^k}\left|\Delta_{k^2+p}*f\right|(q+1)\times\left|\theta_{k^2+p}*f\right|(q+1)^{-1}. \tag{43}$$

(43) tengsizlikni har ikkala tomonidan supremum olib, quyidagi tengsizlikni hosil qilamiz:

$$\sup_{q>0}\left|\theta_q*f\right|^2\leq\sum_{j=0}^{\infty}\left|\Delta_j*f\right|^2+2\sum_{j=0}^{\infty}\sum_{q=0}^{2k-1}\sum_{p\in Q_q^k}\left|\Delta_{k^2+p}*f\right|(q+1)\times\left|\theta_{k^2+p}*f\right|(q+1)^{-1}. \tag{44}$$

(44) tenglikni T^2 bo‘yicha integrallab va Parseval tenglidan foydalanib, 1- qo‘shiluvchining yig‘indi ostidagi ifoda uchun quyidagi tengsizlikni hosil qilamiz:

$$\int_{T^2}\left|\left(\Delta_j*f\right)(x)\right|^2dx=\sum_{n\in Z^2}\left(\Delta_j*f\right)_n^2\leq\left[\left(\Delta_j*f\right)_n\leq\left(\Delta_j\right)_n\cdot f_n\right]\leq\sum_{n\in Z^2}\left|\left(\Delta_j\right)_n\right|^2\cdot\left|f_n\right|^2. \tag{45}$$

Ikkinchi qo‘shiluvchida esa dastlab, ushbu $2ab \leq a^2 + b^2$ tengsizlikni qo‘llab:

$$2\left|\Delta_{k^2+p} * f\right|(q+1) \times \left|\theta_{k^2+p} * f\right|(q+1)^{-1} \leq (q+1)^2\left|\Delta_{k^2+p} * f\right|^2 + (q+1)^{-2}\left|\theta_{k^2+p} * f\right|^2,$$

so‘ngra bu tengsizlikni ham har ikkala tomonini T^2 bo‘yicha integrallab va yuqoridagiga o‘xshash baholab, quyidagi tengsizlikni hosil qilamiz:

$$2\int\limits_{T^2}\left|\left(\Delta_{k^2+p} * f\right)(x)\right|(q+1) \times \left|\left(\theta_{k^2+p} * f\right)(x)\right|(q+1)^{-1}dx \leq$$

$$\leq \sum_{n\in Z^2} f_n^2(q+1)^2\left|\left(\Delta_{k^2+p}\right)_n\right|^2 + \sum_{n\in Z^2} f_n^2(q+1)^{-2}\left|\left(\theta_{k^2+p}\right)_n\right|^2. \tag{46}$$

(45) va (46) larga ko‘ra (44) ni quyidagicha yozamiz:

$$\int\limits_{T^2}\sup_{q>0}\left|\left(\theta_q * f\right)(x)\right|^2 dx \leq \sum_{n\in Z^2} f_n^2\sum_{j=1}^{\infty}\left|\left(\Delta_j\right)_n\right|^2 + \sum_{n\in Z^2} f_n^2\sum_{k=1}^{\infty}\sum_{q=0}^{2k-1}(q+1)^2\sum_{p\in Q_q^k}\left|\left(\Delta_{k^2+p}\right)_n\right| +$$

$$+\sum_{n\in Z^2} f_n^2\sum_{k=1}^{\infty}\sum_{q=0}^{2k-1}(q+1)^{-2}\sum_{p\in Q_q^k}\left|\left(\theta_{k^2+p}\right)_n\right|. \tag{47}$$

(47) ni o‘ng tomonidagi birinchi qo‘shiluvchi uchun 3-lemmani ikkinchi qo‘shiluvchi uchun 5-lemmani va uchinchi qo‘shiluvchi uchun 6-lemmani qo‘llab va yana Parseval tengligidan foydalanib, quyidagini olamiz:

$$\int\limits_{T^2}\sup_{q>0}\left|\left(\theta_q * f\right)(x)\right|^2 dx \leq c\|f\|_{L_2(T^2)}.$$

2- teorema isbotlandi.♦

1-teoremani isboti

$x \in \Omega$ da $f(x) = 0$ bo‘lsin, bu yerda $\Omega \subset T^2$ biror ochiq to‘plam. $f(x) \in C_0^\infty(T^2)$ bo‘lsin, u holda ushbu $\lim\limits_{\lambda\to\infty} S_\lambda f(x) = f(x)$ tenglik $x \in T^2$ bo‘yicha tekis bajariladi. Endi $f(x) \in L_2(T^2)$ bo‘lsin. Ihtiyoriy $\varepsilon > 0$ sonini tayinlaymiz. $C_0^\infty(T^2)$ fazosi $L_2(T^2)$ da zich, u holda shunday $g \in C_0^\infty(T^2)$ mavjudki, ushbu $\|f - g\|_{L_2(T^2)} < \varepsilon$ tengsizlik o‘rinli bo‘ladi. $S_\lambda g$ ni $g(x)$ ga T^2 da tekis yaqinlashishiga ko‘ra shunday $M = M(\varepsilon)$ soni mavjudki, barcha $\lambda > M$ lar uchun quyidagi

$$|S_\lambda g(x) - g(x)| < \varepsilon,\ x \in T^2$$

tengsizlik o‘rinli bo‘ladi.
U holda, $\lambda > M$ lar uchun quyidagi tengsizlikni olamiz:

$$|S_\lambda f - f| \leq |S_\lambda(f-g)| + |S_\lambda g - g| + |f-g| \leq S_*(f-g) + \varepsilon + |f-g|.$$

(4) tengsizlikdan foydalanib, quyidagi

$$\|\sup_{\lambda>M}|S_\lambda f - f|\,\|_{L_2(\Omega)} = \|\sup_{\lambda>M}|S_\lambda f|\,\|_{L_2(\Omega)} \leq c\|f-g)\|_{L_2(T^2)} + \varepsilon_1 + \|f-g\|_{L_2(T^2)} < \varepsilon$$

tengsizlikni xosil qilamiz.
O‘z navbatida, oxirgi tengsizlikdan T^2 dagi ihtiyoriy Ω uchun ushbu

$$\lim_{\lambda\to\infty}\|\sup_{\lambda>M}|S_\lambda f|\,\|_{L_2(\Omega)} = 0$$

munosabat o‘rinli ekanligi kelib chiqadi. Oxirgi munosabat $\lim\limits_{\lambda\to\infty} S_\lambda f(x) = 0$ tenglikni Ω da deyarli bajarilishini anglatadi. Shunday qilib, 1-teorema isbotlandi.♦

ADABIYOTLAR

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РЕЗЮМЕ

В данной работе изучается задача почти всюду сходимости последовательности эллиптических частичных сумм рядов Фурье в заданной области, исходя из условия, что носитель разлагаемой функции лежит вне этой области. Эту проблему обычно называют задачей обобщенной локализации.

Ключевые слова: Ряды Фурье, эллиптических частичных суммы, обобщенная локализация, максимальный оператор.

RESUME

In this paper, we study the problem of almost everywhere convergence of a sequence of elliptic partial sums of Fourier series in a given domain, based on the condition that the support of the function being expanded lies outside this domain. This problem is usually called the problem of generalized localization.

Key words: Fourier series, elliptic partial sums, generalized localization, maximal operator.