UDC 514.18

# PIFAGOR UCHLIGINING FRAKTAL XOSSALARI

**Ibragimov H. H.**
Denov tadbirkorlik va pedagogika instituti, Surxondaryo, Denov
husniddinhikmatovich@gmail.com

**REZYUME**

To‘g‘ri burchakli uchburchak gipotenuzasida katetlari yordamida ajratilgan kesmalar bilan teng yonli uchburchak orasidagi bog‘lanishlar, teng yonli uchburchak va $\mathbb{R}^n$ fazoda $n$ o‘lchamli Pifagor g‘ishti orasidagi bog’lanishlar ko‘rsatilgan. Hamda bu ishda teng yonli va to‘g‘ri burchakli uchburchaklar orasidagi fraktal bog’lanishlar chizma hamda sxemalarda aks ettirilgan. Eyler g‘ishtining qirralari va diagonalini hosil qiluvchi qo‘shimcha $r \in Z$ parametrli tenglama keltirilgan.

***Kalit so‘zlar:*** Pifagor soni, Pifagor taxtasi, Pifagor g‘ishti, Eyler uchligii, Eyler g‘ishti, diagonal, to‘g‘ri burchakli uchburchak, teng yonli uchburchak, parametrli tenglama.

Agar butun musbat $a$, $b$ va $c$ sonlari uchun $a^2 + b^2 = c^2$ tenglik o‘rinli bo‘lsa, $(a, b; c)$ uchlikka Pifagor uchligi deyiladi.

Bizga, $ABC$ to‘g‘ri burchakli uchburchak berilgan bo‘lsin (1-chizma).
Agar $AC = b$, $BC = a$ va $AB = c$ ga teng bo‘lsin. $A$ nuqtani aylana markazi deb $AC = AE = b$ ni, $B$ nuqtani aylana markazi deb $BC = BD = a$ ni chizamiz. Demak:

$$AC = AE = b,\ BC = BD = a,\ AD = AB - DB = c - a,\ BE = AB - AE = c - b,$$

$$DE = AB - AD - BE = c - (c - a) - (c - b) = a + b - c,$$

$$AD + BE = c - a + c - b = 2c - a - b$$

larga ega bo‘lamiz.

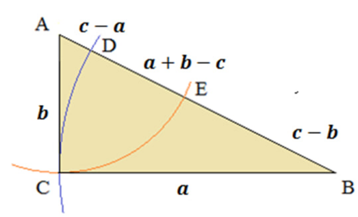


Рис. 7: 1-chizma

Demak, to‘g‘ri burchakli uchburchak gipotenuzasini (1-chizma) $c - a$ , $c - b$ va $a + b - c$ kabi kesmalarga ajratamiz, gipotenuzadagi ikki chetki $c - a$ va $c - b$ kesmalar yig‘indisi $2c - a - b$ uchun:

$$(c - a)^2 + (a + b - c)^2 + (c - b)^2 = (2c - a - b)^2$$

tenglik o’rinli. Bu tenglik quyida 1-teoremada $n = 3$ desak kelib chiqadi.

**1-teorema.** Agar $a$, $b$ *va* $c$ Pifagor sonlari bo‘lsa, u holda ixtiyoriy $n$ uchun quyidagi

$$(n-2)\,(c-a)^2 + (b-(n-2)\,(c-a))^2 +$$

$$+\left(\frac{(n-1)\,(n-2)}{2}\,(c-a) + a - (n-2)\,b\right)^2 = \left(\frac{(n-1)\,(n-2)}{2}\,(c-a) + c - (n-2)\,b\right)^2$$

*tenglik o‘rinli bo‘ladi.*

Quyida 1-teoremadan $n$ ning ba’zi qiymatlari uchun natijalar olamiz

$n = 2$ uchun, $b^2 + a^2 = c^2$

$n = 3$ uchun, $(c-a)^2 + (a+b-c)^2 + (c-b)^2 = (2c-a-b)^2$

$n = 4$ uchun, $(c-a)^2 + (c-a)^2 + (b+2a-2c)^2 + (3c-2a-2b)^2 = (4c-3a-2b)^2$

$n = 5$ uchun, $(c-a)^2 + (c-a)^2 + (c-a)^2 + (b+3a-3c)^2 + (6c-5a-3b)^2 = (7c-6a-3b)^2$ kabi natijalarga ega bo’lamiz.

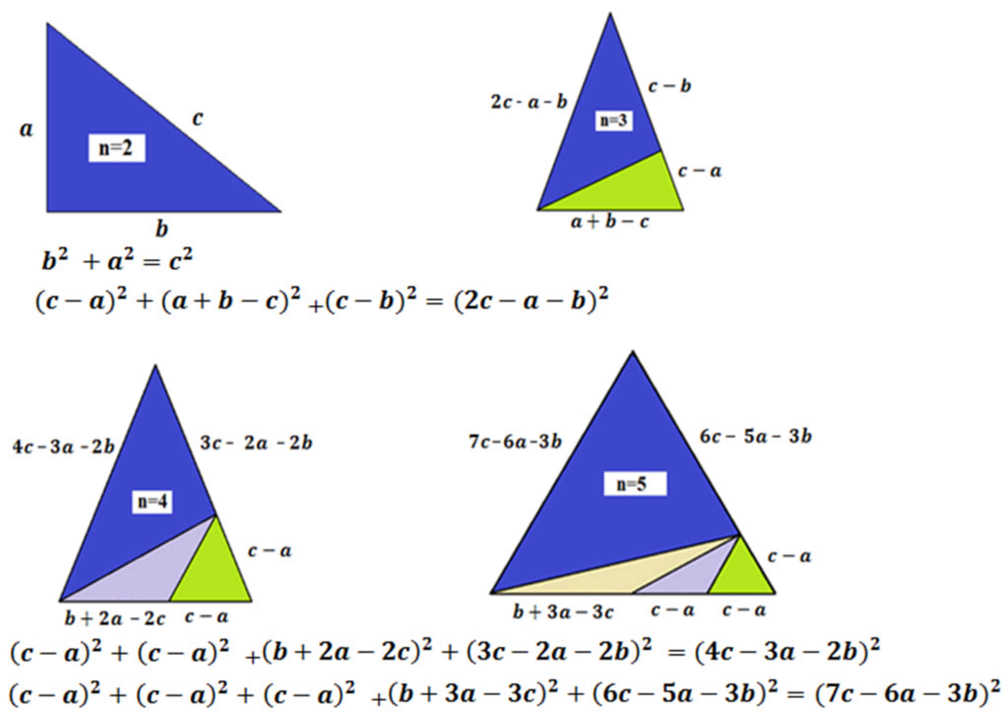


Рис. 8: 2-chizma

1-teoremadagi formuladan $n$ ning $n$ =2, $n$ =3, $n$ =4 va $n$ =5 qiymatlari uchun olingan natijalarning 2-chizmada tasvirlaymiz.

Endi 1-teoremadagi formuladan $n = 3$ holatdan foydalanib $n = 2$ holga o‘tishni chizmada ko‘rsatamiz:

$n = 3$ uchun, $(c-a)^2 + (a+b-c)^2 + (c-b)^2 = (2c-a-b)^2$ dan foydalanib

$$a + b - c + c - a = a$$

$$a + b - c + c - b = b$$

$$a + b - c + 2c - a - b = c$$

$a$, $b$ va $c$ ni topdik, va bulardan

$n = 2$ dagi $b^2 + a^2 = c^2$ kelib chiqadi (3-chizma).

Endi 1-teoremadagi formuladan $n = 5$ holdan $n = 4$ holga o‘tishni 3-chizmada ko‘rsatamiz.

$n = 5$ uchun, $(c-a)^2 + (c-a)^2 + (c-a)^2 + (b+3a-3c)^2 + (6c-5a-3b)^2 = (7c-6a-3b)^2$ dan foydalanib

$$b + 3a - 3c + c - a = b + 2a - 2c$$

$$b + 3a - 3c + 6c - 5a - 3b = 3c + 2a - 2b$$

$$b + 3a - 3c + 7c - 6a - 3b = 4c - 3a - 2b$$

$c - a$ larning ikkitasi o‘z-o‘ziga o‘tadi. $b + 2a - 2c$, $3c - 2a - 2b$ va $4c - 3a - 2b$ ni topdik, va bulardan

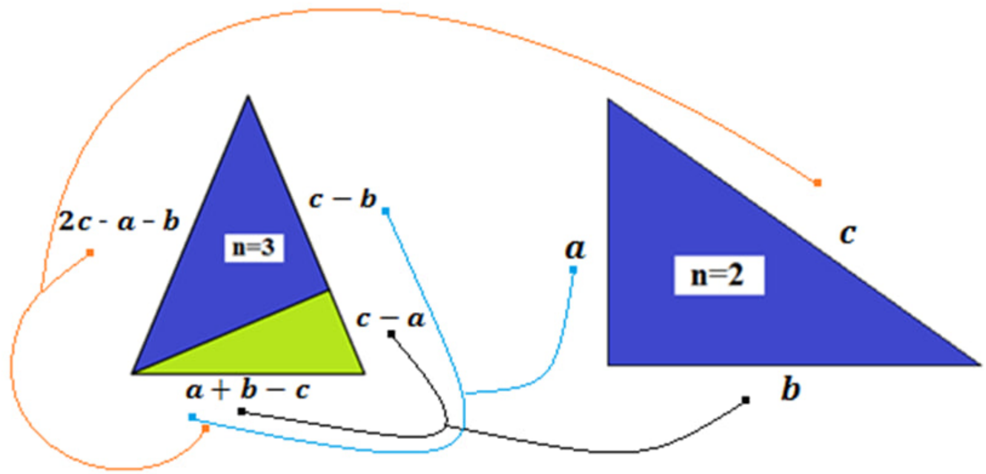


Рис. 9: 3-chizma

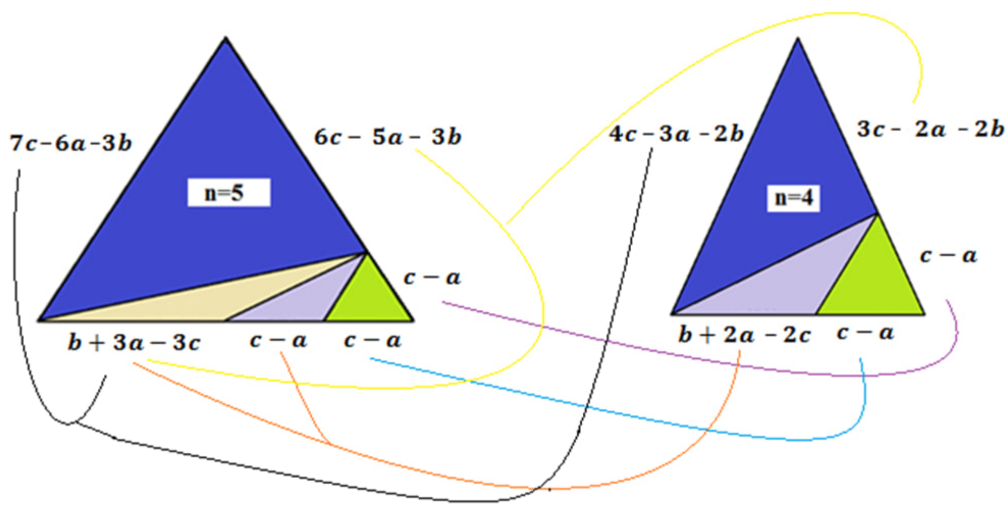


Рис. 10: 4-chizma

$n = 4$ uchun, $(c-a)^2 + (c-a)^2 + (b+2a-2c)^2 + (3c-2a-2b)^2 = (4c-3a-2b)^2$ kelib chiqadi (4- chizma).

**2-teorema.** *Agar $a$ , $b$ va $c$ Pifagor sonlari bo'lsa, u holda ixtiyoriy $n$ uchun quyidagi*

$$(n-2)(c+a)^2 + (b+(n-2)(c+a))^2 +$$

$$+ \left(\frac{(n-1)(n-2)}{2}(c+a) - a + (n-2)b\right)^2 =$$

$$= \left(\frac{(n-1)(n-2)}{2}(c+a) + c + (n-2)b\right)^2$$

*tenglik o'rinli bo'ladi.*

Endi 2-teoremadan $n$ ning ba'zi qiymatlari uchun natijalar yozamiz

$n = 2$ uchun, $b^2 + a^2 = c^2$ Pifagor teoremasi.

$n = 3$ uchun, $(c+a)^2 + (a+b+c)^2 + (c+b)^2 = (2c+a+b)^2$

$n = 4$ uchun, $(c+a)^2 + (c+a)^2 + (b+2a+2c)^2 + (3c+2a+2b)^2 = (4c+3a+2b)^2$

$n=5$ uchun, $(c+a)^2+(c+a)^2+(c+a)^2+(b+3a+3c)^2+(3b+5a+6c)^2=(7c+6a+3b)^2$

Endi 2-teoremadan foydalanib $(3,4;5)$ va $(4,3;5)$ Pifagor uchliklari yordamida qurish mumkin bo‘lgan ikki seriya Pifagor g‘ishtlari qirralari hamda diagonalini 1- jadvalda keltiramiz [2-17 bet]:

| P.G‘ $\mathbb{R}^n$ | 2-teorema yordamida qurilgan Pifagor g‘ishtlari $(3,4;5)$ | | 2-teorema yordamida qurilgan Pifagor g‘ishtlari $(4,3;5)$ | |
|---|---|---|---|---|
| | Qirralari | Diagonali | Qirralari | Diagonali |
| $\mathbb{R}^3$ | $(8,12,9)$ | 17 | $(9,12,8)$ | 17 |
| $\mathbb{R}^4$ | $(8,8,20,29)$ | 37 | $(9,9,21,29)$ | 38 |
| $\mathbb{R}^5$ | $(8,8,8,28,57)$ | 65 | $(9,9,9,30,59)$ | 68 |
| $\mathbb{R}^6$ | $(8,8,8,8,36,93)$ | 101 | $(9,9,9,9,39,98)$ | 107 |
| $\mathbb{R}^7$ | $(8,8,8,8,8,44,137)$ | 145 | $(9,9,9,9,9,48,149)$ | 155 |

1-jadval.

Endi 2-teoremadagi formuladan $n$ ning $n=2, n=3, n=4$ va $n=5$ qiymatlari uchun olingan natijalarning 5-chizma bilan ko‘rsatamiz [2-20 bet].

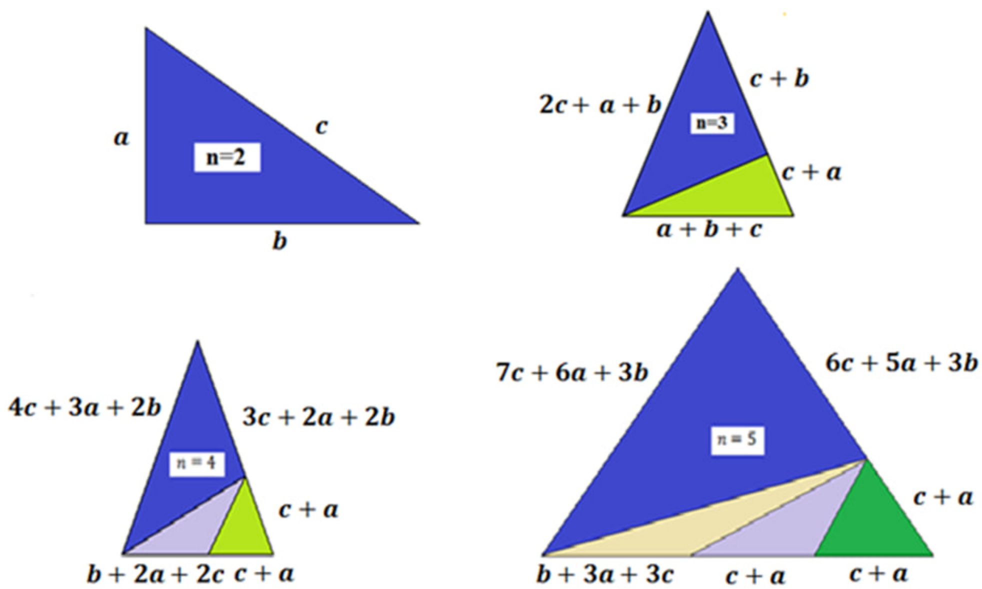


Рис. 11: 5-chizma

## Xulosa.

Endi 2-teoremadan $n$ ning ba’zi qiymatlari uchun:

$n=2$ *uchun,* $b^2+a^2=c^2$ $(a,b;c)$ - *Pifagor uchligini,*

$n=3$ *dan* $(c+a)^2+(a+b+c)^2+(c+b)^2=(2c+a+b)^2$

$(c+a,a+b+c,c+b;2c+a+b)$ - *Pifagor to‘rtligini,*

$n=4$ *dan* $(c+a)^2+(c+a)^2+(b+2a+2c)^2+(3c+2a+2b)^2=(4c+3a+2b)^2$

$(c+a,c+a,b+2a+2c,3c+2a+2b;4c+3a+2b)$ - *Pifagor beshligini hamda*

$$(\underbrace{c-a,c-a,\ \ldots,c-a}_{(n-3)ta},b+(n-2)(c+a),$$

$\frac{(n-1)(n-2)}{2}(c+a)-a+(n-2)b;\frac{(n-1)(n-2)}{2}(c+a)+c+(n-2)b)$ - *Pifagor n ligini yozishimiz mumkin.* Demak, bitta Pifagor uchligidan qirralari butun musbat Pifagor $n$ ligini, ya’ni n olchamli Pifagor gishtlatini qurish mumkin. Misol uchun, 2-teoremadan $n = 3$ uchun olingan,

$$(c+a)^2 + (a+b+c)^2 + (c+b)^2 = (2c+a+b)^2$$

natija teng yonli $ABC$ uchburchak va $TNKODHFS$ to'g'ri parallelepiped, ya'ni "Pifagor g'ishti" orasidagi boglanish 6-chizmada quyidaguicha ko'rsatish mumkin [2-20 bet].

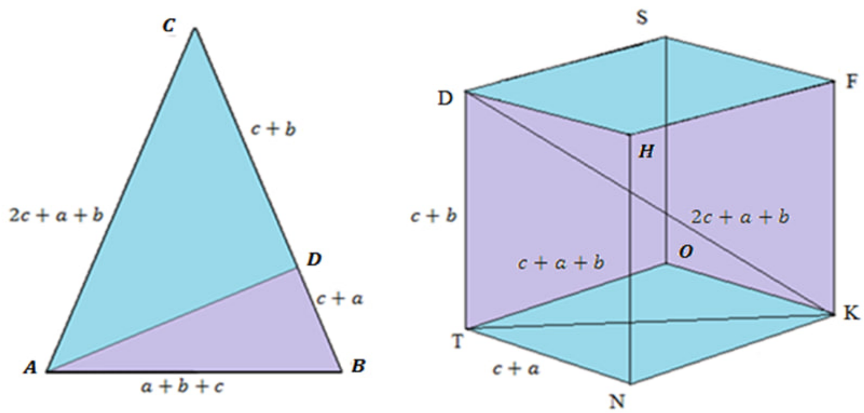


Рис. 12: 6-chizma

Bizga $\alpha$ tekislikda teng yonli $CTD$ uchburchak berilgan bo'lsin,
$AB = a + b + c$, $BD = c + a$, $DC = c + b$ va $AC = 2c + a + b$ ni biz $TNKODHFS$ to'g'ri parallelepipedning

$$OT = SD = FH = KN = a + b + c$$

$$TN = OK = SF = DH = c + a,$$

$$TD = NH = KF = OS = c + b$$

$DK = 2c + a + b$ ekanligidan $\mathbb{R}^3$ ga taaluqli "Pifagor g'ishti" qirralari va diagonali bo'lardi.

**3-teorema.** *Agar $(a, b; c)$ Pifagor uchligi bo'lsa, u orqali bir juft Pifagor uchligi $(a_{11}, b_{11}; c_{11})$ va $(a_{12}, b_{12}; c_{12})$ ni ushbu:*

$$a_{11} = 2a + b + 2c, \quad b_{11} = a + 2b + 2c, \qquad c_{11} = 2a + 2b + 3c$$

$$a_{12} = 2a - b + 2c, \quad b_{12} = a - 2b + 2c, \qquad c_{12} = 2a - 2b + 3c$$

*qoida yordamida hosil qilsh mumkin.*

Buni quyidagicha 1-sxemada tasvirlaymiz (*izoh: $\overset{\rightarrow}{+}$ ushbu belgi, o'rniga qo'yishda qo'shishdan, $\overset{\rightarrow}{-}$ ushbu belgi o'rniga qo'yishda ayirishshdan kelib chiqqan ma'nosi o'rnida):*

$$(a, b; c)^{(2a\pm b+2c,\quad a\pm 2b+2c,\quad 2a\pm 2b+3c)} \longrightarrow \begin{cases} \overset{\rightarrow}{+}(a_{11}, b_{11}; c_{11}) \\ \overset{\rightarrow}{-}(a_{12}, b_{12}; c_{12}) \end{cases}$$

1-sxema.

Xuddi shuningdek, olingan $(a_{11}, b_{11}; c_{11})$ va $(a_{12}, b_{12}; c_{12})$ larni har biriga 3-teoremadan foydalanib:

$$\begin{cases} (a_{11}, b_{11}; c_{11})^{(2a_{11}\pm b_{11}+2c_{11},\quad a_{11}\pm 2b_{11}+2c_{11},\quad 2a_{11}\pm 2b_{11}+3c_{11})} \longrightarrow \begin{cases} \overset{\rightarrow}{+}(a_{21}, b_{21}; c_{21}) \ldots \\ \overset{\rightarrow}{-}(a_{22}, b_{22}; c_{22}) \ldots \end{cases} \\ (a_{12}, b_{12}; c_{12})^{(2a_{12}\pm b_{12}+2c_{12},\quad a_{12}\pm 2b_{12}+2c_{12},\quad 2a_{12}\pm 2b_{12}+3c_{12})} \longrightarrow \begin{cases} \overset{\rightarrow}{+}(a_{23}, b_{23}; c_{23}) \ldots \\ \overset{\rightarrow}{-}(a_{24}, b_{24}; c_{24}) \ldots \end{cases} \end{cases}$$

ga ega bo'lamiz.

Agar $(a_{21}, b_{21}; c_{21})$, $(a_{22}, b_{22}; c_{22})$, $(a_{23}, b_{23}; c_{23})$ *va* $(a_{24}, b_{24}; c_{24})$ larni har biriga 3-teoremadan foydalanib:

$$\begin{cases} (a_{21},b_{21};c_{21})^{(2a_{21}\pm b_{21}+2c_{21},\quad a_{21}\pm 2b_{21}+2c_{21},\quad 2a_{21}\pm 2b_{21}+3c_{21})} \longrightarrow \begin{cases} \overrightarrow{+}\,(a_{31}, b_{31}; c_{31})\ldots \\ \overrightarrow{-}\,(a_{32}, b_{32}; c_{32})\ldots \end{cases} \\ (a_{22},b_{22};c_{22})^{(2a_{22}\pm b_{22}+2c_{22},\quad a_{22}\pm 2b_{22}+2c_{22},\quad 2a_{22}\pm 2b_{22}+3c_{22})} \longrightarrow \begin{cases} \overrightarrow{+}\,(a_{33}, b_{33}; c_{33})\ldots \\ \overrightarrow{-}\,(a_{34}, b_{34}; c_{34})\ldots \end{cases} \\ (a_{23},b_{23};c_{23})^{(2a_{23}\pm b_{23}+2c_{23},\quad a_{23}\pm 2b_{23}+2c_{23},\quad 2a_{23}\pm 2b_{23}+3c_{23})} \longrightarrow \begin{cases} \overrightarrow{+}\,(a_{35}, b_{35}; c_{35})\ldots \\ \overrightarrow{-}\,(a_{36}, b_{36}; c_{36})\ldots \end{cases} \\ (a_{24},b_{24};c_{24})^{(2a_{24}\pm b_{24}+2c_{24},\quad a_{24}\pm 2b_{24}+2c_{24},\quad 2a_{24}\pm 2b_{24}+3c_{24})} \longrightarrow \begin{cases} \overrightarrow{+}\,(a_{37}, b_{37}; c_{37})\ldots \\ \overrightarrow{-}\,(a_{38}, b_{38}; c_{38})\ldots \end{cases} \end{cases}$$

ga ega bo‘lamiz.
Demak, ushbu fraktal jarayonni 3-teoremadan foydalanib cheksiz davom ettirish mumkinligi ko’rinib qoldi.
Misol uchun $(3, 4; 5)$ Pifagor uchligi yordamida uch bo’g’in fractal jarayonni 2-sxemada korsatamiz:

$$(3,4;5)\begin{cases} \nearrow (20,21;29)\begin{cases} \nearrow (119,120;169)\begin{cases} \nearrow (696,697;985)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \\ \searrow (456,217;505)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \end{cases} \\ \searrow (77,36;85)\begin{cases} \nearrow (360,319;481)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \\ \searrow (288,175;;377)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \end{cases} \end{cases} \\ \searrow (12,5;13)\begin{cases} \nearrow (55,48;73)\begin{cases} \nearrow (284,257;385)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \\ \searrow (228,145;273)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \end{cases} \\ \searrow (45,28,53)\begin{cases} \nearrow (224,207;305)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \\ \searrow (168,95;;193)\begin{cases}\nearrow \cdots \\ \searrow \cdots\end{cases} \end{cases} \end{cases} \end{cases}$$

2-sxema.

Endi quyida hajmi eng kichik Eyler g‘ishtini keltiramiz [1,3-15 bet].

$$\begin{cases} 44^2 + 117^2 = 125^2 \\ 117^2 + 240^2 = 267^2 \\ 240^2 + 44^2 = 244^2 \end{cases}$$

**4-teorema**. *Agar* $(a,\ b;;c)$ *Pifagor uchligi bo‘lsa, u holda ixtiyoriy* $r \in Z$ *uchun*

$$x = a\left[(br)^2 - c^2\right]\cdot\left\{\left(2b\left[(br)^2 - c^2(2r-1)\right]\right)^2 - \left(c\left[c^2 + \left(r^2 - 2r\right)b^2\right]\right)^2\right\},$$

$$y = b\left[(br)^2 - c^2(2r-1)\right]\cdot\left\{\left(2a\left[(br)^2 - c^2\right]\right)^2 - \left(c\left[c^2 + \left(r^2 - 2r\right)b^2\right]\right)^2\right\},$$

$$z = 4abc\left[(br)^2 - c^2\right]\cdot\left[(br)^2 - c^2(2r-1)\right]\cdot\left[c^2 + \left(r^2 - 2r\right)b^2\right]$$

*noldan farqli* $x,\ y$ *va* $z$ *sonlari Eyler uchligi bo‘ladi.*

**Isbot**. Teoremaning isboti quyidagi

$$x^2 + y^2 = d_{xy}^2$$

$$x^2 + z^2 = d_{xz}^2$$

$$y^2 + z^2 = d_{yz}^2$$

tengliklarni tekshirish orqali amalga oshiriladi.

Bunda, Eyler g‘ishtining yon yoqlarining diagonallari quyidagilar bo‘ladi:

$$d_{xy} = \left(c[c^2 + (r^2 - 2r)b^2]\right)^3,$$

$$d_{xz} = a\left[(br)^2 - c^2\right]\cdot\left\{\left(2b\left[(br)^2 - c^2(2r-1)\right]\right)^2 + \left(c\left[c^2 + \left(r^2 - 2r\right)b^2\right]\right)^2\right\},$$

$$d_{yz}=(b\left[(br)^2{-}c^2\left(2r-1\right)\right]\cdot\{\left(2a\left[(br)^2{-}c^2\right]\right)^2+\left(c\left[c^2{+}\left(r^2-2r\right)b^2\right]\right)^2\}.$$

Eyler uchliklarini beruvchi formulada $r\in Z$ ga qiymatlar berib Eyler uchliklari beruvchi cheksiz ko‘p parametrik tenglamalarni hosil qilamiz.

Agar parametrik tenglamada $r=1$ ga mos keluvchi parametrik tenglamani topsak

$$x=a\left[b^2{-}c^2\right]\cdot\left\{\left(2b\left[b^2{-}c^2\right]\right)^2-\left(c\left[c^2{-}b^2\right]\right)^2\right\},$$

$$y=b\left[b^2{-}c^2\right]\cdot\left\{\left(2a\left[b^2{-}c^2\right]\right)^2-\left(c\left[c^2{-}b^2\right]\right)^2\right\},$$

$$z=4abc[b^2{-}c^2]\cdot[b^2{-}c^2]\cdot[c^2{-}b^2].$$

Hosil qilingan $x,\ y,\ z$ sonlarni ularning umumiy bo‘luvchisi $\left(b^2-c^2\right)^3$ ga qisqartirib quyidagiga ega bo‘lamiz:

$$x=a(4b^2-c^2),\qquad y=b(4a^2-c^2),\qquad z=4abc.$$

Bu esa Eylerning birinchi parametrik tenglamasi formulasi bilan ustma-ust tushadi.

Agar parametrik tenglamada $r=2$ desak, Eyler g‘ishtining qirralari:

$$x=a\left[4b^2{-}c^2\right]\left\{\left(2b\left[4b^2{-}3c^2\right]\right)^2{-}c^6\right\},$$

$$y=b\left[4b^2-3c^2\right]\left\{\left(2a\left[4b^2-c^2\right]\right)^2-c^6\right\},$$

$$z=4abc^3[4b^2-c^2]{\cdot}(4b^2-3c^2).$$

Endi $a=3,\ b=4,\quad c=5$ Pifagor uchliklariga mos Eyler g‘ishtining qirralarini hisoblaymiz.

$$x=3\cdot 39\cdot\left\{(2\cdot 4\cdot 11)^2-(125)^2\right\},$$

$$y=4\cdot 11\cdot\left\{(2\cdot 3\cdot 39)^2-(125)^2\right\},$$

$$z=4\cdot 3\cdot 4\cdot 125\cdot 39\cdot 11,$$

bundan esa

$$x=3\cdot 39\left(88^2-125^2\right),$$

$$y=4\cdot 11\left(234^2-125^2\right),$$

$$z=4\cdot 3\cdot 4\cdot 125\cdot 39\cdot 11.$$

Natijada

$$x=922077,\qquad y=1721764,\qquad z=2574000$$

Eyler uchligini olamiz.

Tekshirish:

$$922077^2{+}1721764^2{=}1953125^2,$$

$$922077^2{+}2574000^2{=}2734173^2,$$

$$1721764^2{+}2574000^2{=}3096764^2,$$

Qirralari (922077, 1721764, 2574000) bo‘lgan, Eyler g‘ishtining yon tomonining diagonallari bo‘lib 1953125, 2734173, 3096764 sonlar xizmat qiladi

[5-20-bet, 4-33,34-betlar].

Demak, 4-teoremadan ko‘rish mumkinki $(a,\ b;c)$ Pifagor uchligidan, har bir

$r\in Z$ uchun, uchta $(x,\ y;d_{xy})$, $(x,\ z;d_{xz})$ va $(y,\ z;d_{yz})$ Pifagor uchligi hosil bo‘ladi

va bu fraktal jarayonni cheksiz davom ettirish mumkin(3-sxema).

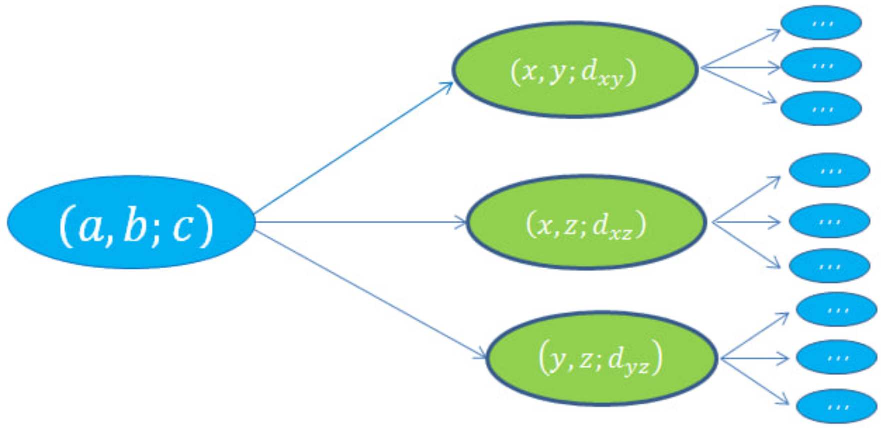


Рис. 13: 3-sxema

**Foydalanilgan adabiyotlar ro‘yxati**

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**РЕЗЮМЕ**

Показаны связи между гипотенузой прямоугольного треугольника и равносторонним треугольником, связи между равносторонним треугольником и $n$ - мерным кирпичиком Пифагора в пространстве $\mathbb{R}^n$. В данной работе показаны равносторонние и фрактальные связи между прямоугольными треугольниками на рисунках и схемах. Приведено уравнение с дополнительным параметром $r \in Z$, порождающее ребра и диагонали кирпича Эйлера, и приведен один пример.

***Ключевые слова:*** число Пифагора, доска Пифагора, кирпич Пифагора, треугольник Эйлера, кирпич Эйлера, диагональ, прямоугольный треугольник, равносторонний треугольник, параметрическое уравнение.

**RESUME**

The paper shows the relationships between the hypotenuse of a right triangle and an equilateral triangle, the relationships between an equilateral triangle and an n-dimensional Pythagorean brick in the space $R^n$. This paper shows equilateral and fractal relationships between right triangles in figures and diagrams. An equation with an additional parameter $r \in \mathbb{Z}$ generating the edges and diagonals of the Euler brick is given, and one example is given.

***Key words:*** Pythagorean number, Pythagorean board, Pythagorean brick, Euler triangle, Euler brick, diagonal, right triangle, equilateral triangle, parametric equation.