UDC 512.554

$\mathbb{Z}_2$ MAYDONDA IKKI VA UCH O‘LCHAMLI EVOLYUTSION ALGEBRALARNING TASNIFI

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Rezyume

Hozirgi vaqtda zamonaviy algebraning muhim yo‘nalishlaridan biri evolyutsion algebralar nazariyasini tadqiq qilishdir. Evolyutsion algebralar matematikaning boshqa sohalari, jumladan graflar nazariyasi, Markov jarayonlari, dinamik sistemalar bilan bog‘liq. Evolyutsion algebralar sinfi birinchi marta Lyubich tomonidan 1992 yilda kiritilgan. 2008 yilda Tian o‘zining monografiyasida evolyutsion algebra atamasini ishlatib, bu algebralarning tadbiqlari va ba’zi xossalarini keltirib o‘tgan. Bu maqolada $\mathbb{Z}_2$ maydonda ikki va uch o‘lchamli evolyutsion algebralar tasnifi keltirilgan.

Kalit so‘zlar: Evolyutsion algebra, tabiiy bazis, gomomorfizm, izomorfizm, maydon, o‘lcham.

2008 yilda J.P. Tian tadqiqotlarining asosiy yo‘nalishi evolyutsion algebralarning tadbiqiy muammolariga bag‘ishlangan edi. Shu bilan birga chekli o‘lchovli algebralarning biror ko‘pxilligini o‘rganishga tabiiy yondashish, masalan, strukturaviy nazariyani qurish u tomonidan qaralmagan edi [1]. Strukturaviy nazariyani o‘rganishga bag‘ishlangan dastlabki natijalar B.A.Omirov, U.A.Rozikov, M.Ladra, J.M.Casas, M.V.Velasco va boshqalarning ishlarida paydo bo‘ldi. Xususan, kompleks, haqiqiy sonlar maydonida ikki va uch o‘lchamli evolyutsion algebralarining tasnifi olingan, nilpotent evolyutsion algebralarining klassik xossalari tasniflangan [2,3].

Bizga biror K maydon berilgan bo‘lsin.

Ta’rif 1.[1] *(E, ·)* biror K maydon ustidagi algebra bo‘lsin. Agar

$$e_i \cdot e_j = 0, \quad \text{agar} \quad i \neq j, \quad \text{barcha} \quad i \quad larda \quad e_i \cdot e_i = \sum_k a_{ik} e_k$$

shartni qanoatlantiruvchi $e_1, e_2, ..., e_n, ...$ bazis mavjud bo‘lsa, u holda bu algebraga *evolyutsion algebra* deyiladi. Bu bazisga esa *tabiiy bazis* deyiladi.

Eslatma. Ushbu maqolada evolyutsion algebrani tashkil etuvchi konstantalari $\mathbb{Z}_2$ maydonda o‘rganiladi.

Ikki va uch o‘lchamli kompleks evolyutsion algebralar tasnifi [2], [3] da, ikki va uch o‘lchamli haqiqiy evolyutsion algebralar tasnifi [4], [5] da keltirilgan.

E-evolyutsion algebra uchun quyi markaziy qator aniqlaymiz:

$$E^1 = E, \quad E^k = \sum_{i=1}^{k-1} E^i E^{k-i}, \quad k \geq 1$$

Bizga $(E, \cdot)$ evolyutsion algebra va e_i uning tabiiy bazisi berilgan bo‘lsin.

Ta’rif 2.[2] E va E' evolyutsion algebralar va $\varphi : E \to E'$ chiziqli akslantirish bo‘lsin. Agar $\varphi(e_i)$ E' da tabiiy bazis bo‘lsa, φ- gomomorfizm deyiladi. Bundan tashqari, agar φ- biektiv bo‘lsa, u holda φ- izomorfizm deyiladi.

$\mathbb{Z}_2$ maydonda ikki o‘lchamli evolyutsion algebralarning tasnifi

$\mathbb{Z}_2$ maydonda ikki o‘lchamli evolyutsion algebra va $\{e_1, e_2\}$ uning tabiiy bazisi bo‘lsin.

Ma’lumki, agar $E^2 = 0$ bo‘lsa, u holda E *abel* algebrasi deyiladi, ya’ni nol ko‘paytmali algebra deyiladi.

Teorema 1. $\mathbb{Z}_2$ maydonda ikki o‘lchamli evolyutsion algebralar o‘zaro izomorf bo‘lmagan quyidagi algebralarning biri bilan izomorfdir.

(i) $dimE^2 = 1$ bo‘lgan holda

- $E_1 : \quad e_1 \cdot e_1 = e_1$
- $E_2 : \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1$
- $E_3 : \quad e_1 \cdot e_1 = e_2$

(ii) $dimE^2 = 2$ bo‘lgan holda

- $E_4 : \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1 + e_2$
- $E_5 : \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1$
- $E_6 : \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1 + e_2$

Isbot. Ikki o‘lchamli evolyutsion algebra uchun ko‘paytmaning umumiy ko‘rinishi
$e_1 \cdot e_1 = a_1e_1 + a_2e_2, \quad e_2 \cdot e_2 = a_3e_1 + a_4e_2, \quad e_1 \cdot e_2 = e_2 \cdot e_1 = 0, \quad a_i \in \mathbb{Z}_2 \quad (\mathbb{Z}_2 = \{\bar{0}, \bar{1}\}.)$

(i) $\quad dimE^2 = 1$ **bo‘lgan holda;**
$e_1 \cdot e_1 = c_1(a_1e_1 + a_2e_2), \quad e_2 \cdot e_2 = c_2(a_1e_1 + a_2e_2), \quad e_1 \cdot e_2 = e_2 \cdot e_1 = 0$. Ravshanki, $(c_1, c_2) \neq (0, 0)$ aks holda bizning algebra abel algebrasi bo‘lar edi. e_1 va e_2 simmetrik ekanligidan $c_1 \neq 0$ deb faraz qila olamiz, u holda biz bazisni sodda almashtirishlar orqali $c_1 = 1$ qila olamiz.

1-hol. $a_1 \neq 0$. U holda biz bazisni quyidagi o‘zgarishini qabul qilamiz.
$e_1' = e_1 + a_2e_2, \quad e_2' = Ae_1 + Be_2$, bu yerda $B - a_2A \neq 0$. Natijani hisoblasak

$0 = e_1' \cdot e_2' = (e_1 + a_2e_2)(Ae_1 + Be_2) = A(e_1 + a_2e_2) + a_2Bc_2(e_1 + a_2e_2) =$
$= (A + a_2Bc_2)(e_1 + a_2e_2)$

Shuning uchun, $A + a_2Bc_2 = 0$ ya’ni $A = -a_2Bc_2$ va $B - a_2A = B + a_2^2Bc_2 \neq 0$.

Bu shuni bildiradiki, $B \neq 0$ $(B = 1)$ va $1 + a_2^2c_2 \neq 0$ bo‘lganda biz yuqoridagi almashtirishlarni olamiz. Natijani ko’rib chiqsak

$e_1' \cdot e_1' = (e_1 + a_2e_2)(e_1 + a_2e_2) = (e_1 + a_2e_2) + a_2^2c_2(e_1 + a_2e_2) = (1 + a_2^2c_2)(e_1 + a_2e_2) =$
$= (1 + a_2^2c_2)e_1'$

$e_2' \cdot e_2' = (Ae_1 + e_2)(Ae_1 + e_2) = A^2(e_1 + a_2e_2) + c_2(e_1 + a_2e_2) = (A^2 + c_2)(e_1 + a_2e_2) =$
$= (a_2^2c_2^2 + c_2)e_1' = c_2(1 + a_2^2c_2)e_1'$

1.1-hol. $c_2 = 0$. U holda $e_1' \cdot e_1' = e_1', \quad e_2 \cdot e_2 = e_1 \cdot e_2 = e_2 \cdot e_1 = 0$, bizga E_1 algebrani beradi.

1.2-hol. $c_2 \neq 0$. U holda $e_1' \cdot e_1' = (1 + a_2^2)e_1', \quad e_2' \cdot e_2' = (1 + a_2^2)e_1'$ ga ega bo‘lamiz. Agar $(1 + a_2^2) \neq 0$ ya’ni $a_2 \neq 1, a_2 = 0$ bo‘lganda bajariladi. Bizga $e_1 \cdot e_1 = e_1, \quad e_2 \cdot e_2 = e_1$ ya’ni E_2 algebrani beradi.

2-hol. $a_1 = 0$. U holda biz $e_1 \cdot e_1 = a_2e_1; \quad e_2 \cdot e_2 = c_2a_2e_2$ natijalarga ega bo’lamiz. Bu yerda $a_2 \neq 0$, aks holda evolyutsion algebra bo‘lmaydi.

Agar $c_2 = 0$ bo’lsa, bazisni $e_1' = \dfrac{e_1}{\sqrt{a_2}}$ almashtirish orqali $e_1' \cdot e_1' = \dfrac{e_1}{\sqrt{a_2}} \cdot \dfrac{e_1}{\sqrt{a_2}} = e_2$, $e_2 \cdot e_2 = 0$ ya’ni E_3 algebraga ega bo‘lamiz.

Agar $c_2 \neq 0$ $(c_2 = 1)$ bo‘lsa, bazisni $e_1' = \dfrac{e_1}{\sqrt{a_2}}, \quad e_2' = \dfrac{e_2}{\sqrt{a_2}}$ almashtirishlar orqali $e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_2$ ya’ni E_2 algebraga izomorf bo‘lgan algebrani hosil qilamiz.

(ii) $\quad dimE^2 = 2$ **bo‘lgan holda;**
Ko‘paytmaning umumiy ko‘rinishi $e_1 \cdot e_1 = a_1e_1 + a_2e_2, \quad e_2 \cdot e_2 = a_3e_1 + a_4e_2$ bu yerda $a_1a_4 - a_2a_3 \neq 0, a_i \in \mathbb{Z}_2$

1-hol. $a_1 \neq 0$ va $a_4 \neq 0$. Bazisning o‘zgarishi $e_1 = a_1^{-1}e_1, e_1 = a_2^{-1}e_2$ orqali $a_1 = a_4 = 1$ deb faraz qila olamiz. Shu sababli biz 2 ta parametrik oilaga ega bo‘lamiz ya’ni
$E_4(a_2, a_3): \quad e_1 \cdot e_1 = e_1 + a_2e_2, \quad e_2 \cdot e_2 = a_3e_1 + e_2, \quad 1 - a_2a_3 \neq 0$ bundan $(a_2, a_3) \neq (1, 1)$.

Bazisning umumiy o‘zgarishini quyidagicha olaylik,
$e_1' = A_1e_1 + A_2e_2, \quad e_2' = B_1e_1 + B_2e_2, \quad A_1B_2 - A_2B_1 \neq 0$. Natijani ko‘rib chiqsak,
$0 = e_1'e_2' = (A_1e_1 + A_2e_2)(B_1e_1 + B_2e_2) = A_1B_1(e_1 + a_2e_2) + A_2B_2(a_3e_1 + e_2) = (A_1B_1 + A_2B_2a_3)e_1 + (A_1B_1a_2 + A_2B_2)e_2$

Ushbu yangi bazisdagi algebra ham evolyutsion algebra bo‘lishi kerakligidan bizga $A_1B_1 + A_2B_2a_3 = 0$va $A_1B_1a_2 + A_2B_2 = 0$. Bundan $A_2B_2(1 - a_2a_3) = 0$ va $A_1B_1(1 - a_2a_3) = 0$. Bunda $1 - a_2a_3 \neq 0$ ekanligidan, $A_1B_1 = A_2B_2 = 0$ ga ega bo‘lamiz.

1.1-hol. $A_2 = 0$. U holda $B_1 = 0$ bo‘lsin.
$e_1'e_1' = A_1^2e_1 \cdot e_1 = A_1^2(e_1 + a_2e_2) = e_1' + a_2'e_2' = A_1e_1 + a_2'B_2e_2$
$\Rightarrow A_1^2 = A_1, A_1^2a_2 = a_2'B_2 \Rightarrow A_1 = 1$
$e_2'e_2' = B_2^2(a_3e_1 + e_2) = a_3'e_1' + e_2' = a_3'A_1e_1 + B_2e_2$
$\Rightarrow B_2^2a_3 = a_3'A_1, B_2^2 = B_2 \Rightarrow B_2 = 1$
Demak, $e_1'e_1' = e_1 + a_2'e_2, \quad e_2'e_2' = a_3'e_1 + e_2$ ekanligidan

$a_2' = 0$ bo‘lsa $a_3' = 1$, bundan $e_1 \cdot e_1 = e_1$, $e_2 \cdot e_2 = e_1 + e_2$ ya’ni E_4 ni beradi.

$a_2' = 1$ bo‘lsa $a_3' = 0$, bundan $e_1 \cdot e_1 = e_1 + e_2$, $e_2 \cdot e_2 = e_2$ ya’ni E_5 ni beradi.

1.2-hol. $A_1 = 0$. U holda $B_2 = 0$ bo’sin va bizga $E_4(a_2, a_3) \cong E_4'(a_3, a_2)$ izomorf bo‘lgan algebra hosil bo‘ladi.

2-hol. $a_1 = 0$ yoki $a_4 = 0$ bo’lsin. e_1 va e_2 simmetrik ekanligidan umumiylikka zarar yetkazmasdan $a_1 = 0$ deb olishimiz mumkin.
$e_1e_1 = a_2e_2$ va $e_2e_2 = a_3e_1 + a_4e_2$ bu yerda $a_2a_3 \neq 0, (a_2a_3) \neq (0, 0)$, Z_2 da $a_2 = a_3 = \bar{1}$ bo‘lgan holda bajariladi.
Bazisni o‘zgarishini quyidagicha qabul qilib $e_1' = \sqrt[3]{\dfrac{1}{a_2^2a_3}}e_1, \quad e_2' = \sqrt[3]{\dfrac{1}{a_2a_3^2}}e_2$ bundan,

$$e_1' \cdot e_1' = \sqrt[3]{\frac{1}{a_2^2a_3} \cdot \frac{1}{a_2^2a_3}}e_1 \cdot e_1 = \sqrt[3]{\frac{1}{a_2a_3^2}}e_2; \quad e_2' \cdot e_2' = \sqrt[3]{\frac{1}{a_2^2a_3^4}}(a_3e_1 + a_4e_2) = \sqrt[3]{\frac{1}{a_2^2a_3}}e_1 + \sqrt[3]{\frac{1}{a_2^2a_3^4}}a_4e_2$$

Biz algebralarning bir parametrik oilasini $E_6(a_4): \quad e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_1 + a_4e_2$ ni olamiz. Bundan,

$a_4 = 0$ bo‘lsa, $e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_1$ ya’ni E_6 ni beradi.

$a_4 = 1$ bo‘lsa, $e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_1 + e_2$ ya’ni $E_7 \cong E_4$ o‘zaro izomorf bo‘lgan algebrani beradi. **Teorema isbotlandi.**

$\mathbb{Z}_2$ maydonda uch o‘lchamli evolyutsion algebralarning tasnifi

$\mathbb{Z}_2$ maydonda uch o‘lchamli evolyutsion algebra va $\{e_1, e_2, e_3\}$ uning tabiiy bazisi bo‘lsin.

Teorema 2. $\mathbb{Z}_2$ maydonda uch o‘lchamli E evolyutsion algebralar $dimE^2 = 1$ bo‘lganda quyidagi o‘zaro izomorf bo‘lmagan algebralarning biri bilan izomorfdir.

- $E_1: \quad e_1 \cdot e_1 = e_1$
- $E_2: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1$
- $E_3: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1; \quad e_3 \cdot e_3 = e_1$
- $E_4: \quad e_1 \cdot e_1 = e_2$
- $E_5: \quad e_1 \cdot e_1 = e_2; \quad e_3 \cdot e_3 = e_2$
- $E_6: \quad e_1 \cdot e_1 = e_3; \quad e_3 \cdot e_3 = e_3$

Isbot. Uch o‘lchamli evolyutsion algebra uchun ko‘paytmaning umumiy ko‘rinishi
$e_1 \cdot e_1 = a_1e_1 + a_2e_2 + a_3e_3$
$e_2 \cdot e_2 = b_1e_1 + b_2e_2 + b_3e_3$

$e_3 \cdot e_3 = c_1e_1 + c_2e_2 + c_3e_3$

Ma’lumki, $e_1 \cdot e_2 = e_1 \cdot e_3 = e_2 \cdot e_3 = 0$.

Bu yerda $B = \{e_1, e_2, e_3\}$- tabiiy bazis, $a_i, b_i, c_i \in \mathbb{Z}_2$

Bizda $dimE^2 = 1$ bo‘lganligi sababli,

$e_1 \cdot e_1 = k_0(a_1e_1 + a_2e_2 + a_3e_3)$

$e_2 \cdot e_2 = k_1(a_1e_1 + a_2e_2 + a_3e_3)$

$e_3 \cdot e_3 = k_2(a_1e_1 + a_2e_2 + a_3e_3)$

Ma’lumki, $(k_0, k_1, k_2) \neq (0, 0, 0)$ aks holda bizning algebra abel algebrasi bo‘lar edi. e_1, e_2, e_3 simmetrik ekanligidan $k_0 \neq 0$ deb faraz qila olamiz, u holda $k_0 = \bar{1}$ qila olamiz.

$e_1^2 \neq 0$; $e_1 \cdot e_1 = a_1e_1 + a_2e_2 + a_3e_3$

$e_2 \cdot e_2 = k_1(a_1e_1 + a_2e_2 + a_3e_3)$

$e_3 \cdot e_3 = k_2(a_1e_1 + a_2e_2 + a_3e_3)$

$M_B = \begin{pmatrix} a_1 & a_2 & a_3 \\ k_1a_1 & k_1a_2 & k_1a_3 \\ k_2a_1 & k_2a_2 & k_2a_3 \end{pmatrix}$ E evolyutsion algebrani B tabiiy bazisga nisbatan strukturaviy konstantalari

Biz tabiiy bazisni $B' = \{e'_1, e'_2, e'_3\}$ quyidagicha almashtiramiz:

$e'_1 = e_1^2 = a_1e_1 + a_2e_2 + a_3e_3$

$e'_2 = A_1e_1 + A_2e_2 + A_3e_3$

$e'_3 = B_1e_1 + B_2e_2 + B_3e_3$

$|P_{B'B}| = \begin{vmatrix} a_1 & a_2 & a_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} \neq 0$. Natijani hisoblasak,

$e'_1 \cdot e'_2 = 0, \quad e'_1 \cdot e'_3 = 0, \quad e'_2 \cdot e'_3 = 0$ ekanligidan

$\begin{cases} a_1A_1 + a_2A_2k_1 + a_3A_3k_2 = 0 \\ a_1B_1 + a_2B_2k_1 + a_3B_3k_2 = 0 \qquad (1) \\ A_1B_1 + A_2B_2k_1 + A_3B_3k_2 = 0 \end{cases}$ ifodaga ega bo‘lamiz. Natijani ko‘rib chiqsak,

$e'_1 \cdot e'_1 = (a_1^2 + a_2^2k_1 + a_3^2k_2)e'_1$

$e'_2 \cdot e'_2 = (A_1^2 + A_2^2k_1 + A_3^2k_2)e'_1$

$e'_3 \cdot e'_3 = (B_1^2 + B_2^2k_1 + B_3^2k_2)e'_1$

1-hol. $a_1 \neq 0$

$e_1 \cdot e_1 = e_1 + a_2e_2 + a_3e_3$

$\begin{cases} A_1 = -(a_2A_2k_1 + a_3A_3k_2) \\ B_1 = -(a_2B_2k_1 + a_3B_3k_2) \end{cases} \qquad (2)$

1.1-hol. $k_1 = k_2 = 0$ bo‘lsin. U holda $e'_1 \cdot e'_1 = e'_1, \quad e'_2 \cdot e'_2 = A_1^2e'_1, \quad e'_3 \cdot e'_3 = B_1^2e'_1$ (2) ifodadan $A_1 = 0, \quad B_1 = 0$ bundan $e_1 \cdot e_1 = e_1$ bizga E_1 algebrani beradi.

1.2-hol. $k_1 \neq 0, \quad k_2 = 0$ bo‘lsin. U holda
$e'_1 \cdot e'_1 = (1 + a_2^2)e'_1, \quad e'_2 \cdot e'_2 = (A_1^2 + A_2^2)e'_1, \quad e'_3 \cdot e'_3 = (B_1^2 + B_2^2)e'_1$ (2) ifodadan $A_1 = -a_2A_2, \quad B_1 = -a_2B_2$. Bundan

$e'_2 \cdot e'_2 = (a_2^2A_2^2 + A_2^2)e'_1 = A_2^2(1 + a_2^2)e'_1,$

$e'_3 \cdot e'_3 = (a_2^2B_2^2 + B_2^2)e'_1 = B_2^2(1 + a_2^2)e'_1$

1.2.1-hol. $1 + a_2^2 \neq 0$. Aks holda Abel algebra, bundan $a_2 = 0$ bo‘ladi.

$e'_1 \cdot e'_1 = e'_1, \quad e'_2 \cdot e'_2 = A_2^2e'_1, \quad e'_3 \cdot e'_3 = B_2^2e'_1$, (2) ifodadan $A_1 = 0, B_1 = 0$; (1) ifodadan $A_2B_2 = 0$, u holda $A_2 = 1, B_2 = 0$ bo‘lganda bajariladi. Bizga $e_1 \cdot e_1 = e_1, \quad e_2 \cdot e_2 = e_1$ ya’ni E_2 algebrani beradi.

1.3-hol. $k_1 \neq 0, \quad k_2 \neq 0$ bo‘lsin. U holda biz quyidagi natijalarga ega bo‘lamiz.
$e'_1 \cdot e'_1 = (1 + a_2^2 + a_3^2)e'_1, \quad e'_2 \cdot e'_2 = (A_1^2 + A_2^2 + A_3^2)e'_1, \quad e'_3 \cdot e'_3 = (B_1^2 + B_2^2 + B_3^2)e'_1$

(3.2.1.2) ifodadan $\begin{cases} A_1 = -(a_2A_2 + a_3A_3) \\ B_1 = -(a_2B_2 + a_3B_3) \end{cases}$

1.3.1-hol. $a_2 = 0, \quad a_3 = 0$. U holda $A_1 = 0, \quad B_1 = 0$ (1) ifodadan $\begin{cases} A_2B_2 + A_3B_3 = 0 \\ A_2B_3 - A_3B_2 \neq 0 \end{cases}$ bundan $A_2 = 0, A_3 = 1; \quad B_2 = 1, B_3 = 0$ bo'lganda o'rinli. Bizga $e_1 \cdot e_1 = e_1, \quad e_2 \cdot e_2 = e_1, \quad e_3 \cdot e_3 = e_1$ ya'ni E_3 algebrani beradi.

2-hol. $a_1 = 0$ bo'lsin. U holda

$e_1 \cdot e_1 = a_2e_2 + a_3e_3$

$e_2 \cdot e_2 = k_1(a_2e_2 + a_3e_3)$

$e_3 \cdot e_3 = k_2(a_2e_2 + a_3e_3)$

$a_2 \neq 0$ dan tabiiy bazisni quyidagicha almashtiramiz.

$B' = \{e_1, a_2e_2 + a_3e_3, e_3\}$, $e_1' = e_1, \quad e_2' = a_2e_2 + a_3e_3, \quad e_3' = e_3$

$|P_{B'B}| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & a_2 & a_3 \\ 0 & 0 & 1 \end{vmatrix} = a_2 \neq 0$. Natijani hisoblasak,

$e_2'e_3' = a_3e_3^2 = a_3k_2e_2' = 0$ tenglikdan $a_3k_2 = 0$ (3) ifodaga ega bo'lamiz.

$e_1'e_1' = e_1e_1 = e_2'$

$e_2'e_2' = (e_2 + a_3e_3)(e_2 + a_3e_3) = e_2e_2 + a_3^2e_3e_3 = (k_1 + a_3^2k_2)e_2'$

$e_3'e_3' = e_3e_3 = k_2e_2'$

2.1.1-hol. $k_1 = 0, \quad k_2 = 0$ bo'lsin. U holda

$e_1'e_1' = e_2', \quad e_2'e_2' = e_3'e_3' = 0 \Rightarrow e_1 \cdot e_1 = e_2$ bizga E_4 algebrani beradi.

2.1.2-hol. $k_1 = 0, \quad k_2 = 1$ bo'lsin. u holda

$e_1'e_1' = e_2'$

$e_2'e_2' = a_3^2e_2'$

$e_3'e_3' = e_2'$

(3) ifodadan $a_3 = 0$ ekanligidan bizga $e_1 \cdot e_1 = e_2, \quad e_3 \cdot e_3 = e_2$ ya'ni E_5 algebrani beradi.

2.1.3-hol. $k_1 = 1, \quad k_2 = 0$ bo'lsin. u holda

$e_1'e_1' = e_2'$

$e_2'e_2' = (1 + a_3^2 \cdot 0)e_2' = e_2'$

$e_3'e_3' = 0$

Bundan bizga $e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_2$, $E_6 \cong E_2$ algebrani beradi.

2.1.4-hol. $k_1 = 1, \quad k_2 = 1$ bo'lsin. u holda

$e_1'e_1' = e_2'$

$e_2'e_2' = (1 + a_3^2)e_2'$

$e_3'e_3' = e_2'$

(3) ifodadan $a_3 = 0$ ekanligidan bizga $e_1 \cdot e_1 = e_2, \quad e_2 \cdot e_2 = e_2 \quad e_3 \cdot e_3 = e_2$ ya'ni $E_7 \cong E_3$ izomorf bo'lgan algebrani beradi.

2.2-hol. $a_3 \neq 0$ bo'lsin. Tabiiy bazisni quyidagicha almashtiramiz.

$B - \{e_1, e_2, a_2e_2 + a_3e_3\}$, $e_1' = e_1, \quad e_2' = e_2 \quad e_3' = a_2e_2 + a_3e_3$

$|P_{B"B}| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & a_2 & a_3 \end{vmatrix} = a_3 \neq 0$. Natijani hisoblasak,

$e_2'e_3' = a_3e_3^2 = a_2k_1e_3' = 0$ tenglikdan $a_2k_1 = 0$ (4) ifodaga ega bo'lamiz.

$e_1'e_1' = e_1e_1 = a_2e_2 + a_3e_3 = e_3'$

$e_2'e_2' = e_2e_2 = k_1e_3'$

$e_3'e_3' = (a_2e_2 + a_3e_3)(a_2e_2 + a_3e_3) = (k_1a_2^2 + a_3^2k_2)e_3' = (k_1a_2^2 + k_2)e_3'$

2.2.1-hol. $k_1 = 0, \quad k_2 = 0$ bo'lsin. U holda

$e_1'e_1' = e_3', \quad e_2'e_2' = e_3'e_3' = 0$. Bizga $e_1 \cdot e_1 = e_3$ ya'ni $E_8 \cong E_4$ izomorf bo'lgan algebrani beradi.

2.2.2-hol. $k_1 = 0, \quad k_2 = 1$ bo'lsin. u holda

$e_1'e_1' = e_3'$

$e_2'e_2' = 0$
$e_3'e_3' = e_3'$
Bizga $e_1 \cdot e_1 = e_3, \quad e_3 \cdot e_3 = e_3$ ya’ni E_9 algebrani beradi.

2.2.3-hol. $k_1 = 1, \quad k_2 = 0$ bo‘lsin. u holda
$e_1'e_1' = e_3'$
$e_2'e_2' = e_3'$
$e_3'e_3' = a_2^2e_3'$
(4) ifodadan $a_2 = 0$ ekanligidan bizga $e_1 \cdot e_1 = e_3, \quad e_2 \cdot e_2 = e_3$ ya’ni $E_{10} \cong E_5$ izomorf bo‘lgan algebrani beradi.

2.2.4-hol. $k_1 = 1, \quad k_2 = 1$ bo‘lsin. u holda
$e_1'e_1' = e_3'$
$e_2'e_2' = e_3'$
$e_3'e_3' = (a_2^2 + 1)e_3'$
(4) ifodadan $a_2 = 0$ ekanligidan bizga $e_1 \cdot e_1 = e_3, \quad e_2 \cdot e_2 = e_3 \quad e_3 \cdot e_3 = e_3$ ya’ni $E_{11} \cong E_3$ izomorf bo‘lgan algebrani beradi. **Teorema isbotlandi.**

Teorema 3. $\mathbb{Z}_2$ maydonda uch o‘lchamli E evolyutsion algebralar $dimE^2 = 2$ bo‘lganda quyidagi o‘zaro izomorf bo‘lmagan algebralarning biri bilan izomorfdir.

- $E_1: \quad e_2 \cdot e_2 = e_2; \quad e_3 \cdot e_3 = e_1$
- $E_2: \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1$
- $E_3: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_2; \quad e_3 \cdot e_3 = e_1$
- $E_4: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1$
- $E_5: \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_2; \quad e_3 \cdot e_3 = e_1$
- $E_6: \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1; \quad e_3 \cdot e_3 = e_1$
- $E_7: \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1$
- $E_8: \quad e_1 \cdot e_1 = e_2; \quad e_3 \cdot e_3 = e_1$
- $E_9: \quad e_1 \cdot e_1 = e_1 + e_2; \quad e_2 \cdot e_2 = e_2; \quad e_3 \cdot e_3 = e_1$
- $E_{10}: \quad e_1 \cdot e_1 = e_1 + e_2; \quad e_2 \cdot e_2 = e_1; \quad e_3 \cdot e_3 = e_1$
- $E_{11}: \quad e_1 \cdot e_1 = e_1 + e_2; \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1$
- $E_{12}: \quad e_1 \cdot e_1 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1$
- $E_{13}: \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1 + e_2$
- $E_{14}: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1 + e_2; \quad e_3 \cdot e_3 = e_1 + e_2$
- $E_{15}: \quad e_1 \cdot e_1 = e_1; \quad e_2 \cdot e_2 = e_1; \quad e_3 \cdot e_3 = e_1 + e_2$
- $E_{16}: \quad e_1 \cdot e_1 = e_2; \quad e_2 \cdot e_2 = e_1; \quad e_3 \cdot e_3 = e_1 + e_2$
- $E_{17}: \quad e_1 \cdot e_1 = e_1; \quad e_3 \cdot e_3 = e_1 + e_2$
- $E_{18}: \quad e_1 \cdot e_1 = e_2; \quad e_3 \cdot e_3 = e_1 + e_2$

Isbot. $dimE^2 = 2$ da $E^2 =< e_1, e_2 >, \quad E^2 =< e_1, e_3 >, \quad E^2 =< e_2, e_3 >$ ni qaraymiz.
$\mathbb{Z}_2$ maydonda $\alpha_i + \alpha_i = 0, \quad \alpha_i^2 = \alpha_i$ ekanligini eslatib o‘tamiz.
Uch o‘lchamli evolyutsion algebra uchun ko‘paytmaning umumiy ko‘rinishi

$e_1 \cdot e_1 = a_1e_1 + a_2e_2 + a_3e_3$
$e_2 \cdot e_2 = b_1e_1 + b_2e_2 + b_3e_3$

$e_3 \cdot e_3 = c_1e_1 + c_2e_2 + c_3e_3$

ma’lumki, $e_1 \cdot e_2 = e_1 \cdot e_3 = e_2 \cdot e_3 = 0$

Bu yerda $B = \{e_1, e_2, e_3\}$- tabiiy bazis, $a_i, b_i, c_i \in \mathbb{Z}_2$

Bizda $dimE^2 = 2$ bo‘lganligi sababli,

$e_1 \cdot e_1 = \alpha_1e_1 + \alpha_2e_2$

$e_2 \cdot e_2 = \beta_1e_1 + \beta_2e_2$

$e_3 \cdot e_3 = \gamma_1e_1 + \gamma_2e_2 \Longrightarrow \gamma_1 \neq 0, \quad \gamma_1 = 0$ bittasi noldan farqli bo‘lishi kerak, aks holda 2-o‘lchamli evolyutsion algebra hosil bo‘ladi.

1-hol. $\gamma_1 = 1, \quad \gamma_2 = 0$ bo‘lsin. u holda $e_3 \cdot e_3 = e_1$.

Tabiiy bazisni quyidagicha almashtiramiz.

$e'_3 = A_1e_1 + A_2e_2 + e_3$

$e'_2 = B_1e_1 + B_2e_2$

$e'_1 = e'_3e'_3 = (A_1e_1 + A_2e'_2 + e_3)(A_1e_1 + A_2e'_2 + e_3) = A_1(\alpha_1e_1 + \alpha_2e_2) + A_2(\beta_1e_1 + \beta_2e_2) + e_1 = (1 + A_1\alpha_1 + A_2\beta_1)e_1 + (A_1\alpha_2 + A_2\beta_2)e_2$

$$|P_{B'B}| = \begin{vmatrix} (1 + A_1\alpha_1 + A_2\beta_1) & (A_1\alpha_2 + A_2\beta_2) & 0 \\ B_1 & B_2 & 0 \\ A_1 & A_2 & 1 \end{vmatrix} \neq 0 \Rightarrow$$

$$\begin{vmatrix} (1 + A_1\alpha_1 + A_2\beta_1) & (A_1\alpha_2 + A_2\beta_2) \\ B_1 & B_2 \end{vmatrix} \neq 0 \Rightarrow$$

$B_2(1 + A_1\alpha_1 + A_2\beta_1) \neq B_1(A_1\alpha_2 + A_2\beta_2) \quad (5)$ ifodaga ega bo‘lamiz.

$e'_1e'_2 = e'_1e'_3 = e'_2e'_3 = 0$ ekanligidan

$$\begin{cases} B_1\alpha_1(1 + A_1\alpha_1 + A_2\beta_1) + B_2\beta_1(A_1\alpha_2 + A_2\beta_2) = 0 \\ B_1\alpha_2(1 + A_1\alpha_1 + A_2\beta_1) + B_2\beta_2(A_1\alpha_2 + A_2\beta_2) = 0 \\ A_1\alpha_1(1 + A_1\alpha_1 + A_2\beta_1) + A_2\beta_1(A_1\alpha_2 + A_2\beta_2) = 0 \\ A_1\alpha_2(1 + A_1\alpha_1 + A_2\beta_1) + A_2\beta_2(A_1\alpha_2 + A_2\beta_2) = 0 \\ A_1B_1\alpha_1 + A_2B_2\beta_1 = 0 \\ A_1B_1\alpha_2 + A_2B_2\beta_1 = 0 \end{cases} \quad (6)$$

ifodaga ega bo‘lamiz. Natijani ko‘rib chiqsak,

$e'_1e'_1 = (\alpha_1(1 + A_1\alpha_1 + A_2\beta_1) + \beta_1(A_1\alpha_2 + A_2\beta_2))e_1 + (\alpha_2(1 + A_1\alpha_1 + A_2\beta_1) + \beta_2(A_1\alpha_2 + A_2\beta_2))e_2$

$e'_2e'_2 = (B_1\alpha_1 + B_2\beta_1)e_1 + (B_1\alpha_2 + B_2\beta_2)e_2$

$e'_3e'_3 = e'_1 = (1 + A_1\alpha_1 + A_2\beta_1)e_1 + (A_1\alpha_2 + A_2\beta_2)e_2$

1.1-hol. $B_2(1 + A_1\alpha_1 + A_2\beta_1) \neq B_1(A_1\alpha_2 + A_2\beta_2)$ (5) ifodadan $B_1(A_1\alpha_2 + A_2\beta_2) = 0$ bo‘lsin. Bundan $B_2(1 + A_1\alpha_1 + A_2\beta_1) = 1 \Rightarrow$ $B_2 = 1, \quad (1 + A_1\alpha_1 + A_2\beta_1) = 1$

1.1.1-hol. $B_1 = 0, \quad (A_1\alpha_2 + A_2\beta_2) = 1$ bo‘lsin.

(6) ifodadan
$$\begin{cases} \beta_1 = 0 \\ \beta_2 = 0 \\ A_1\alpha_1 = A_2\beta_1 \\ A_1\alpha_2 = A_2\beta_2 \Rightarrow \quad ziddiyat \quad (A_1\alpha_2 + A_2\beta_2) = 0 \\ A_2\beta_1 = 0 \\ A_2\beta_2 = 0 \end{cases}$$
sistema bajarilmaydi.

1.1.2-hol. $B_1 = 1, \quad (A_1\alpha_2 + A_2\beta_2) = 0$ bo‘lsin.

$$(6) \text{ ifodadan } \begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \\ A_1\alpha_1 = 0 \\ A_1\alpha_2 = 0 \\ A_1\alpha_1 = A_2\beta_1 = 0 \\ A_1\alpha_2 = A_2\beta_2 = 0 \end{cases}$$

$e'_1e'_1 = 0, \quad e'_2e'_2 = \beta_1e_1 + \beta_2e_2, \quad e'_3e'_3 = e_1$ ko‘paytmani hosil qilamiz.
Agar $\beta_1 = 0, \quad \beta_2 = 1 \Rightarrow e_2e_2 = e_2 \quad e_3e_3 = e_1$ E_1 algebrani beradi;
Agar $\beta_1 = 1, \quad \beta_2 = 0 \Rightarrow e_2e_2 = e_1 \quad e_3e_3 = e_1$ E_2 algebrani beradi;
Agar $\beta_1 = 1, \quad \beta_2 = 1 \Rightarrow e_2e_2 = e_1 + e_2 \quad e_3e_3 = e_1$ E_3 algebrani beradi;
Agar $\beta_1 = 0, \quad \beta_2 = 0 \Rightarrow e_2e_2 = 0 \quad e_3e_3 = e_1$ E_4 algebrani beradi.

1.1.3-hol. $B_1 = 0, \quad (A_1\alpha_2 + A_2\beta_2) = 0$ bo‘lsin.

$$(6) \text{ ifodadan } \begin{cases} 0 = 0 \\ 0 = 0 \\ A_1\alpha_1 = 0 \\ A_1\alpha_2 = 0 \\ A_2\beta_1 = 0 \\ A_2\beta_2 = 0 \end{cases} \text{ ekanligidan}$$

$e'_1e'_1 = \alpha_1e_1, \quad e'_2e'_2 = \beta_1e_1 + \beta_2e_2, \quad e'_3e'_3 = e_1$ ni hosil qilamiz. Bunda $(\alpha_1; \alpha_2) \neq 0$ aks holda **1.1.2-holga** kelamiz va bundan quyidagi holatlar hosil boladi.

1.1.3.1-hol. $\alpha_1 = 1, \quad \alpha_2 = 0$ bo‘lsin. Bundan
$e_1e_1 = e_1, \quad e_2e_2 = e_2, \quad e_3e_3 = e_1$ E_5 algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = e_1, \quad e_3e_3 = e_1$ E_6 algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_1$ E_7 algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = 0, \quad e_3e_3 = e_1$ E_8 algebrani beradi.

1.1.3.2-hol. $\alpha_1 = 0, \quad \alpha_2 = 1$ bo‘lsin. Bundan
$e_1e_1 = e_2, \quad e_2e_2 = e_2, \quad e_3e_3 = e_1$ E_9 algebrani beradi;
$e_1e_1 = e_2, \quad e_2e_2 = e_1, \quad e_3e_3 = e_1$ E_{10} algebrani beradi;
$e_1e_1 = e_2, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_1$ E_{11} algebrani beradi;
$e_1e_1 = e_2, \quad e_2e_2 = 0, \quad e_3e_3 = e_1$ E_{12} algebrani beradi.

1.1.3.3-hol. $\alpha_1 = 1, \quad \alpha_2 = 1$ bo‘lsin. Bundan
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_2, \quad e_3e_3 = e_1$ E_13 algebrani beradi;
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_1, \quad e_3e_3 = e_1$ E_{14} algebrani beradi;
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_1$ E_{15} algebrani beradi;
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = 0, \quad e_3e_3 = e_1$ E_{16} algebrani beradi.

1.2-hol. $B_2(1 + A_1\alpha_1 + A_2\beta_1) \neq B_1(A_1\alpha_2 + A_2\beta_2)$ (5) ifodadan
$B_2(1 + A_1\alpha_1 + A_2\beta_1) = 0$ bo‘lsin. Bundan $B_1(A_1\alpha_2 + A_2\beta_2) = 1 \Rightarrow$
$B_1 = 1, \quad (A_1\alpha_2 + A_2\beta_2) = 1$ kelib chiqadi.
Ushbu holatdan bolishi mumkin bo‘lgan holatlarni hisobga olib, (6) sistemaga qo‘yganimizda biz ziddiyatga ega bo‘lamiz ya’ni sistema birgalikda emas.

2-hol. $\gamma_1 = 0, \quad \gamma_2 = 1$ bo‘lsin. u holda $e_3 \cdot e_3 = e_2$.

Tabiiy $B' = \{e'_1, e'_2, e'_3\}$ bazisni quyidagicha almashtiramiz.

$e'_3 = A_1e_1 + A_2e_2 + e_3$

$e'_1 = C_1e_1 + C_2e_2$

$e'_2 = e'_3e'_3 = (A_1e_1 + A_2e_2 + e_3)(A_1e_1 + A_2e_2 + e_3) = A_1(\alpha_1e_1 + \alpha_2e_2) + A_2(\beta_1e_1 + \beta_2e_2) + e_2 = (A_1\alpha_1 + A_2\beta_1)e_1 + (1 + A_1\alpha_2 + A_2\beta_2)e_2$

$$|P_{B'B}| = \begin{vmatrix} C_1 & C_2 & 0 \\ (A_1\alpha_1 + A_2\beta_1) & (1 + A_1\alpha_2 + A_2\beta_2) & 0 \\ A_1 & A_2 & 1 \end{vmatrix} \neq 0 \Rightarrow$$

$$\begin{vmatrix} C_1 & C_2 \\ (A_1\alpha_1 + A_2\beta_1) & (1 + A_1\alpha_2 + A_2\beta_2) \end{vmatrix} \neq 0 \Rightarrow$$

$C_1(1 + A_1\alpha_2 + A_2\beta_2) \neq C_2(A_1\alpha_1 + A_2\beta_1) \qquad (7)$ ifodaga ega bo‘lamiz.

$e_1'e_2' = e_1'e_3' = e_2'e_3' = 0$ ekanligidan

$$\begin{cases} C_1\alpha_1(A_1\alpha_1 + A_2\beta_1) + C_2\beta_1(1 + A_1\alpha_2 + A_2\beta_2) = 0 \\ C_1\alpha_2(A_1\alpha_1 + A_2\beta_1) + C_2\beta_2(1 + A_1\alpha_2 + A_2\beta_2) = 0 \\ A_1\alpha_1(A_1\alpha_1 + A_2\beta_1) + A_2\beta_1(1 + A_1\alpha_2 + A_2\beta_2) = 0 \\ A_1\alpha_2(A_1\alpha_1 + A_2\beta_1) + A_2\beta_2(1 + A_1\alpha_2 + A_2\beta_2) = 0 \\ C_1A_1\alpha_1 + C_2A_2\beta_1 = 0 \\ C_1A_1\alpha_2 + C_2A_2\beta_1 = 0 \end{cases} \qquad (8)$$

sistemaga ega bo‘lamiz. Natijani ko‘rib chiqsak,

$e_1'e_1' = (C_1\alpha_1 + C_2\beta_1)e_1 + (C_1\alpha_2 + C_2\beta_2)e_2$

$e_2'e_2' = (\alpha_1(A_1\alpha_1 + A_2\beta_1) + \beta_1(1 + A_1\alpha_2 + A_2\beta_2))e_1 + (\alpha_2(A_1\alpha_1 + A_2\beta_1) + \beta_2(1 + A_1\alpha_2 + A_2\beta_2))e_2$

$e_3'e_3' = e_2' = (A_1\alpha_1 + A_2\beta_1)e_1 + (1 + A_1\alpha_2 + A_2\beta_2)e_2$

2.1-hol. $C_1(1 + A_1\alpha_2 + A_2\beta_2) \neq C_2(A_1\alpha_1 + A_2\beta_1)$ (7) ifodadan $C_2(A_1\alpha_1 + A_2\beta_1) = 0$ bo‘lsin. U holda $C_1(1 + A_1\alpha_2 + A_2\beta_2) = 1 \Rightarrow$ $C_1 = 1, \quad (1 + A_1\alpha_2 + A_2\beta_2) = 1$

2.1.1-hol. $C_2 = 0, \quad (A_1\alpha_1 + A_2\beta_1) = 1$ bo‘lsin.

(8) sistemadan $\begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \\ A_1\alpha_1 = A_2\beta_1 = 0 \Rightarrow ziddiyat (A_1\alpha_1 + A_2\beta_1) = 0 \\ A_1\alpha_2 = A_2\beta_2 = 0 \\ A_1\alpha_1 = 0 \\ A_2\alpha_2 = 0 \end{cases}$ sistema bajarilmaydi.

2.1.2-hol. $C_2 = 1, \quad (A_1\alpha_1 + A_2\beta_1) = 0$ bo‘lsin.

(8) sistemadan $\begin{cases} \beta_1 = 0 \\ \beta_2 = 0 A_2\beta_1 = 0 \\ A_2\beta_2 = 0 \\ A_1\alpha_1 = A_2\beta_1 \\ A_1\alpha_2 = A_2\beta_2 \end{cases}$

$e_1'e_1' = \alpha_1e_1 + \alpha_2e_2, \quad e_2'e_2' = 0, \quad e_3'e_3' = e_2$ ko‘paytmani hosil qilamiz.

Agar $\alpha_1 = 0, \quad \alpha_2 = 1 \Rightarrow e_1e_1 = e_2 \quad e_3e_3 = e_2 \qquad E_{17} \cong E_2$ algebrani beradi;
Agar $\alpha_1 = 1, \quad \alpha_2 = 0 \Rightarrow e_1e_1 = e_1 \quad e_3e_3 = e_2 \qquad E_{18} \cong E_1$ algebrani beradi;
Agar $\alpha_1 = 1, \quad \alpha_2 = 1 \Rightarrow e_1e_1 = e_1 + e_2 \quad e_3e_3 = e_2 \qquad E_{19} \cong E_3$ algebrani beradi;
Agar $\alpha_1 = 0, \quad \alpha_2 = 0 \Rightarrow e_1e_1 = 0 \quad e_3e_3 = e_2 \qquad E_{20}$ algebrani beradi;

2.1.3-hol. $C_2 = 0, \quad (A_1\alpha_1 + A_2\beta_1) = 0$ bo‘lsin.

(8) sistemadan $\begin{cases} 0 = 0 \\ 0 = 0 \\ A_2\beta_1 = 0 \\ A_2\beta_2 = 0 \\ A_1\alpha_1 = 0 \\ A_1\alpha_2 = 0 \end{cases}$ ekanligidan

$e_1'e_1' = \alpha_1e_1 + \alpha_2e_2, \quad e_2'e_2' = \beta_1e_1 + \beta_2e_2, \quad e_3'e_3' = e_2$ ni hosil qilamiz. Bunda $(\beta_1; \beta_2) \neq 0$ aks holda **2.1.2-holga** kelamiz va bundan quyidagi holatlar hosil bo‘ladi.

2.1.3.1-hol. $\beta_1 = 1, \quad \beta_2 = 0$ bo‘lsin. U holda
$e_1e_1 = e_2, \quad e_2e_2 = e_1, \quad e_3e_3 = e_2 \qquad E_{21} \cong E_{10}$ algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = e_1, \quad e_3e_3 = e_2 \qquad E_{22} \cong E_9$ algebrani beradi;

$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_1, \quad e_3e_3 = e_2 \qquad E_{23} \cong E_{11}$ algebrani beradi;
$e_1e_1 = 0, \quad e_2e_2 = e_1, \quad e_3e_3 = e_2 \qquad E_{24} \cong E_{12}$algebrani beradi.

2.1.3.2-hol. $\beta_1 = 0, \quad \beta_2 = 1$ bo‘lsin. U holda
$e_1e_1 = e_2, \quad e_2e_2 = e_2, \quad e_3e_3 = e_2 \qquad E_{25} \cong E_6$ algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = e_2, \quad e_3e_3 = e_2 \qquad E_{26} \cong E_5$ algebrani beradi;
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_2, \quad e_3e_3 = e_2 \qquad E_{27} \cong E_7$ algebrani beradi;
$e_1e_1 = 0, \quad e_2e_2 = e_2, \quad e_3e_3 = e_2 \qquad E_{28} \cong E_8$ algebrani beradi.

2.1.3.3-hol. $\beta_1 = 1, \quad \beta_2 = 1$ bo‘lsin. u holda
$e_1e_1 = e_2, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_2 \qquad E_{29} \cong E_{14}$ algebrani beradi;
$e_1e_1 = e_1, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_2 \qquad E_{30} \cong E_{13}$ algebrani beradi;
$e_1e_1 = e_1 + e_2, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_2 \qquad E_{31} \cong E_{15}$ algebrani beradi;
$e_1e_1 = 0, \quad e_2e_2 = e_1 + e_2, \quad e_3e_3 = e_2 \qquad E_{32} \cong E_{16}$algebrani beradi.

2.2-hol. $C_1(1 + A_1\alpha_2 + A_2\beta_2) \neq C_2(A_1\alpha_1 + A_2\beta_1)$ (7) ifodadan
$C_1(1 + A_1\alpha_2 + A_2\beta_2) = 0$ bo‘lsin. U holda $C_2(A_1\alpha_1 + A_2\beta_1) = 1 \Rightarrow$
$C_1 = 1, \quad (A_1\alpha_1 + A_2\beta_1) = 1$ kelib chiqadi.
Ushbu holatdan bolishi mumkin bo‘lgan holatlarni hisobga olib, (8) sistemaga qo‘yganimizda biz ziddiyatga ega bo‘lamiz ya’ni sistema birgalikda emas.

3-hol. $\gamma_1 = 1, \quad \gamma_2 = 1$ bo‘lsin. u holda $e_3 \cdot e_3 = e_1 + e_2$.

Tabiiy $B' = \{e'_1, e'_2, e'_3\}$ bazisni quyidagicha almashtiramiz.

$e'_3 = A_1e_1 + A_2e_2 + e_3$

$e'_2 = B_1e_1 + B_2e_2$

$e'_1 = (1 + B_1 + A_1\alpha_1 + A_2\beta_1)e_1 + (1 + B_2 + A_1\alpha_2 + A_2\beta_2)e_2$

bu yerda $\quad (e'_3e'_3 = e'_1 + e'_2 \quad \Rightarrow e'_1 = e'_3e'_3 - e'_2)$

$$|P_{B'B}| = \begin{vmatrix} (1 + B_1 + A_1\alpha_1 + A_2\beta_1) & (1 + B_2 + A_1\alpha_2 + A_2\beta_2) & 0 \\ B_1 & B_2 & 0 \\ A_1 & A_2 & 1 \end{vmatrix} \neq 0 \Rightarrow$$

$$\begin{vmatrix} (1 + B_1 + A_1\alpha_1 + A_2\beta_1) & (1 + B_2 + A_1\alpha_2 + A_2\beta_2) \\ B_1 & B_2 \end{vmatrix} \neq 0 \Rightarrow$$

$B_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) \neq B_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \qquad (9) \qquad$ ifodaga ega bo‘lamiz.

$e'_1e'_2 = e'_1e'_3 = e'_2e'_3 = 0$ ekanligidan

$$\begin{cases} B_1\alpha_1(1 + B_1 + A_1\alpha_1 + A_2\beta_1) = B_2\beta_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \\ B_1\alpha_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) = B_2\beta_2(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \\ A_1\alpha_1(1 + B_1 + A_1\alpha_1 + A_2\beta_1) = A_2\beta_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \\ A_1\alpha_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) = A_2\beta_2(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \\ A_1B_1\alpha_1 = A_2B_2\beta_1 \\ A_1B_1\alpha_2 = A_2B_2\beta_2 \end{cases} \qquad (10) \qquad \text{sistemaga ega bo‘lamiz.}$$

Natijani ko‘rib chiqsak,

$e'_1e'_1 = ((\alpha_1(1 + B_1 + A_1\alpha_1 + A_2\beta_1) + \beta_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2))e_1 + (\alpha_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) + \beta_2(1 + B_2 + A_1\alpha_2 + A_2\beta_2))e_2$

$e'_2e'_2 = (B_1\alpha_1 + B_2\beta_1)e_1 + (B_1\alpha_2 + B_2\beta_2)e_2$

$e'_3e'_3 = e'_1 + e'_2 = (1 + A_1\alpha_1 + A_2\beta_1)e_1 + (1 + A_1\alpha_2 + A_2\beta_2)e_2$

3.1-hol. $B_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) \neq B_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2) \qquad (9) \qquad$ ifodadan $B_1(1 + B_2 + A_1\alpha_2 + A_2\beta_2) = 0$ bo‘lsin. U holda $B_2(1 + B_1 + A_1\alpha_1 + A_2\beta_1) = 1 \Rightarrow B_2 = 1, \quad (1 + B_1 + A_1\alpha_1 + A_2\beta_1) = 1$

3.1.1-hol. $B_1 = 0, \quad (1 + B_2 + A_1\alpha_2 + A_2\beta_2) = 1$ bo‘lsin.

(10) sistemadan $\begin{cases}\beta_1=0\\ \beta_2=0\\ A_1\alpha_1=A_2\beta_1\\ A_1\alpha_2=A_2\beta_2\\ A_2\beta_1=0\\ A_2\beta_2=0\end{cases}$ ziddiyat hosil bo‘ladi. Sistema bajarilmaydi.

3.1.2-hol. $B_1=1,(1+B_2+A_1\alpha_2+A_2\beta_2)=0 \quad \Rightarrow (A_1\alpha_2+A_2\beta_2)=0$ bo‘lsin.

(10) sistemadan $\begin{cases}\alpha_1=0\\ \alpha_2=0\\ A_1\alpha_1=0\\ A_2\alpha_2=0\\ A_1\alpha_1=A_2\beta_1=0\Rightarrow ziddiyat(A_1\alpha_1+A_2\beta_1)=0\\ A_1\alpha_2=A_2\beta_2=0\end{cases}$ sistema bajarilmaydi.

3.1.3-hol. $B_1=0,(1+B_2+A_1\alpha_2+A_2\beta_2)=0 \quad \Rightarrow (A_1\alpha_2+A_2\beta_2)=0$ bo‘lsin.

(10) sistemadan $\begin{cases}0=0\\ 0=0\\ A_1\alpha_1=0\\ A_1\alpha_2=0\\ A_2\beta_1=0\\ A_2\beta_2=0\end{cases}$ ekanligidan

$e_1'e_1'=\alpha_1e_1+\alpha_2e_2, \quad e_2'e_2'=\beta_1e_1+\beta_2e_2, \quad e_3'e_3'=e_1+e_2$ ko‘paytmani hosil qilamiz. $M_B=\begin{pmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{pmatrix}$ E evolyutsion algebrani B' tabiiy bazisga nisbatan strukturaviy konstantalari uchun $\alpha_i, \quad \beta_i$ larni $\mathbb{Z}_2$ maydondan tanlasak, quyidagicha 16 ta evolyutsion algebralar hosil bo‘ladi.

$e_1\cdot e_1=e_2; \quad e_2\cdot e_2=e_1+e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{33}$
$e_1\cdot e_1=e_1+e_2; \quad e_2\cdot e_2=e_1+e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{34}$
$e_1\cdot e_1=e_1; \quad e_2\cdot e_2=e_1+e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{35}$
$e_1\cdot e_1=e_1+e_2; \quad e_2\cdot e_2=e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{36}\cong E_{35}$
$e_1\cdot e_1=e_1+e_2; \quad e_2\cdot e_2=e_1; \quad e_3\cdot e_3=e_1+e_2 \quad E_{37}\cong E_{33}$
$e_1\cdot e_1=e_1+e_2; \quad e_2\cdot e_2=0; \quad e_3\cdot e_3=e_1+e_2 \quad E_{38}$
$e_1\cdot e_1=e_1; \quad e_2\cdot e_2=e_1; \quad e_3\cdot e_3=e_1+e_2 \quad E_{39}$
$e_1\cdot e_1=e_2; \quad e_2\cdot e_2=e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{40}\cong E_{39}$
$e_1\cdot e_1=0; \quad e_2\cdot e_2=e_1+e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{41}\cong E_{38}$
$e_1\cdot e_1=e_2; \quad e_2\cdot e_2=e_1; \quad e_3\cdot e_3=e_1+e_2 \quad E_{42}$
$e_1\cdot e_1=e_1; \quad e_2\cdot e_2=e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{43}\cong E_{42}$
$e_1\cdot e_1=e_1; \quad e_2\cdot e_2=0; \quad e_3\cdot e_3=e_1+e_2 \quad E_{44}$
$e_1\cdot e_1=0; \quad e_2\cdot e_2=e_2; \quad e_3\cdot e_3=e_1+e_2 \quad E_{45}\cong E_{44}$
$e_1\cdot e_1=e_2; \quad e_2\cdot e_2=0; \quad e_3\cdot e_3=e_1+e_2 \quad E_{46}$
$e_1\cdot e_1=0; \quad e_2\cdot e_2=e_1; \quad e_3\cdot e_3=e_1+e_2 \quad E_{47}\cong E_{45}$
$e_1\cdot e_1=0; \quad e_2\cdot e_2=0; \quad e_3\cdot e_3=e_1+e_2 \quad E_{48}$

3.2-hol. $B_2(1+B_1+A_1\alpha_1+A_2\beta_1)\neq B_1(1+B_2+A_1\alpha_2+A_2\beta_2)$ (9) ifodadan $B_2(1+B_1+A_1\alpha_1+A_2\beta_1)=0$ bo‘lsin. U holda $B_1(1+B_2+A_1\alpha_2+A_2\beta_2)=1 \quad \Rightarrow$
$B_1=1, \quad (1+B_2+A_1\alpha_2+A_2\beta_2)=1$
Ushbu holatni ko‘rib chiqqanimizda **3.1-holatda** hosil bo‘lgan evolyutsion algebralarga izomorf evolyutsion algebralar paydo bo‘ldi. **Teorema isbotlandi.**

Teorema 4. $\mathbb{Z}_2$ maydonda uch o‘lchamli E evolyutsion algebralar $dimE^2=3$ bo‘lganda quyidagi o‘zaro izomorf bo‘lmagan algebralarning biri bilan izomorfdir.

- $E_1:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_3$
- $E_2:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_2+e_3$
- $E_3:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_1+e_3$
- $E_4:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_5:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_2$
- $E_6:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_2+e_3$
- $E_7:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_8:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_9:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_2+e_3;\quad e_3\cdot e_3=e_3$
- $E_{10}:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_2+e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{11}:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_2;\quad e_3\cdot e_3=e_3$
- $E_{12}:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_2;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{13}:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{14}:\quad e_1\cdot e_1=e_1;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{15}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_1$
- $E_{16}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_2$
- $E_{17}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_1+e_2$
- $E_{18}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1+e_2;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{19}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_2$
- $E_{20}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{21}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_2$
- $E_{22}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{23}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{24}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_2+e_3;\quad e_3\cdot e_3=e_1$
- $E_{25}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_2+e_3;\quad e_3\cdot e_3=e_1+e_2+e_3$
- $E_{26}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1+e_2+e_3;\quad e_3\cdot e_3=e_1$
- $E_{27}:\quad e_1\cdot e_1=e_1+e_3;\quad e_2\cdot e_2=e_1+e_2+e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{28}:\quad e_1\cdot e_1=e_1+e_2+e_3;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_1$
- $E_{29}:\quad e_1\cdot e_1=e_1+e_2+e_3;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{30}:\quad e_1\cdot e_1=e_1+e_2+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_2$
- $E_{31}:\quad e_1\cdot e_1=e_1+e_2+e_3;\quad e_2\cdot e_2=e_3;\quad e_3\cdot e_3=e_2$
- $E_{32}:\quad e_1\cdot e_1=e_3;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_1+e_2$
- $E_{33}:\quad e_1\cdot e_1=e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_1+e_2$

- $E_{34}:\quad e_1\cdot e_1=e_3;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_2$
- $E_{35}:\quad e_1\cdot e_1=e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_2$
- $E_{36}:\quad e_1\cdot e_1=e_2+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_2$
- $E_{37}:\quad e_1\cdot e_1=e_2+e_3;\quad e_2\cdot e_2=e_1+e_3;\quad e_3\cdot e_3=e_2$
- $E_{38}:\quad e_1\cdot e_1=e_2+e_3;\quad e_2\cdot e_2=e_1;\quad e_3\cdot e_3=e_1+e_2$

Isbot. $dimE^2=3$ bo'lganda $E^2=<e_1,e_2,e_3>$ ni qaraymiz.
$\mathbb{Z}_2$ maydonda $\alpha_i+\alpha_i=0,\quad \alpha_i^2=\alpha_i$ ekanligini eslatib o'tamiz.
Uch o'lchamli evolyutsion algebra uchun ko'paytmaning umumiy ko'rinishi

$e_1\cdot e_1=a_1e_1+a_2e_2+a_3e_3$
$e_2\cdot e_2=b_1e_1+b_2e_2+b_3e_3$
$e_3\cdot e_3=c_1e_1+c_2e_2+c_3e_3$
ma'lumki, $e_1\cdot e_2=e_1\cdot e_3=e_2\cdot e_3=0$
Bu yerda $B=\{e_1,e_2,e_3\}$- tabiiy bazis, $a_i,b_i,c_i\in\mathbb{Z}_2$

$M_B=\begin{pmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{pmatrix}$ E evolyutsion algebrani B tabiiy bazisga nisbatan strukturaviy konstantalari

$|P_B|=\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}\neq 0$. Natijani hisoblaymiz.

1-hol. $a_1=b_2=c_3=0$ bo'lsin.

$|P_B|=\begin{vmatrix}0&a_2&a_3\\b_1&0&b_3\\c_1&c_2&0\end{vmatrix}\neq 0.\Rightarrow a_2b_3c_1+a_3b_1c_2\neq 0\Rightarrow a_2b_3c_1\neq a_3b_1c_2$

1.1-hol. $a_2b_3c_1=0$ bo'lsin. U holda $a_3b_1c_2=1\Rightarrow a_3=b_1=c_2=1$
Agar $a_2=0$ bo'lsa;
$b_3=1,\quad c_1=1;\quad e_1e_1=e_3,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_1+e_2\quad E_1$ algebrani beradi;
$b_3=0,\quad c_1=1;\quad e_1e_1=e_3,\quad e_2e_2=e_1,\quad e_3e_3=e_1+e_2\quad E_2$ algebrani beradi;
$b_3=1,\quad c_1=0;\quad e_1e_1=e_3,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_2\quad E_3$ algebrani beradi;
$b_3=0,\quad c_1=0;\quad e_1e_1=e_3,\quad e_2e_2=e_1,\quad e_3e_3=e_2\quad E_4$ algebrani beradi;
Agar $a_2=1$ bo'lsa;
$b_3=0,\quad c_1=0;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_1,\quad e_3e_3=e_2\quad E_5$ algebrani beradi;
$b_3=1,\quad c_1=0;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_2\quad E_6$ algebrani beradi;
$b_3=0,\quad c_1=1;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_1,\quad e_3e_3=e_1+e_2\quad E_7$ algebrani beradi.

1.2-hol. $a_2b_3c_1=1$ bo'lsin. U holda $a_3b_1c_2=0\Rightarrow a_2=b_3=c_1=1$
Agar $a_3=0$ bo'lsa;
$b_1=1,\quad c_2=1;\quad e_1e_1=e_2,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_1+e_2\quad E_8\cong E_7$ algebrani beradi;
$b_1=0,\quad c_2=1;\quad e_1e_1=e_2,\quad e_2e_2=e_3,\quad e_3e_3=e_1+e_2\quad E_9\cong E_2$ algebrani beradi;
$b_1=1,\quad c_2=0;\quad e_1e_1=e_2,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_1\quad E_{10}\cong E_5$ algebrani beradi;
$b_1=0,\quad c_2=0;\quad e_1e_1=e_2,\quad e_2e_2=e_3,\quad e_3e_3=e_1\quad E_{11}\cong E_4$ algebrani beradi;
Agar $a_3=1$ bo'lsa;
$b_1=0,\quad c_2=0;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_3,\quad e_3e_3=e_1\quad E_{12}\cong E_3$ algebrani beradi;
$b_1=1,\quad c_2=0;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_1+e_3,\quad e_3e_3=e_1\quad E_{13}\cong E_6$ algebrani beradi;
$b_1=0,\quad c_2=1;\quad e_1e_1=e_2+e_3,\quad e_2e_2=e_3,\quad e_3e_3=e_1+e_2\quad E_{14}\cong E_1$ algebrani beradi.

2-hol. $a_1=1$ bo'lsin.

U holda $|P_B|=\begin{vmatrix}1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}\neq 0.$

Ushbu evolyutsion algebraning strukturaviy konstantalarining determinanti noldan farqli bo'ladigan holatlarni ko'rib chiqqanimizda jami 96 ta evolyutsion algebra hosil bo'ldi. Bunda o'zaro izomorflarini ajratib chiqqanimizda 31 ta o'zaro izomorf bo'lmagan evolyutsion algebralar hosil bo'ldi. **Teorema isbotlandi.**

Rezyume

At present, one of the important directions in modern algebra is the study of the theory of evolution algebras. Evolution algebras are connected to various other areas of mathematics, including graph theory, Markov processes, and dynamical systems. The class of evolution algebras was first introduced by Lyubich in 1992. In 2008, Tian, in his monograph, used the term evolution algebra and presented certain applications and properties of these algebras. This paper provides a classification of two- and three-dimensional evolution algebras over the field $\mathbb{Z}_2$

Key words: Evolution algebra, natural basis, homomorphism, isomorphism, field, dimension.

Rezyume

На современном этапе одним из актуальных направлений в развитии алгебры является теория эволюционных алгебра. Данный класс алгебра представляет собой относительно новое направление, обладающее широкой областью применения и тесно связанное с другими разделами математики, такими как теория графов, марковские процессы и теория динамических систем. Понятие эволюционной алгебры было впервые введено Любичем в 1992 году. Существенный вклад в развитие данной теории внёс Тиан, который в своей монографии, изданной в 2008 году, систематизировал основные свойства эволюционных алгебр и продемонстрировал их приложения. Настоящая работа посвящена классификации двух и трёхмерных эволюционных алгебра над полем $\mathbb{Z}_2$

Ключевые слова: Эволюционная алгебра, натуральный базис, гомоморфизм, изоморфизм, поля, размерность.

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